Computer Algebra Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/173-6.3.7-d-hyper-^m-a+b-c-tanh-^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [263]. This is test number [173].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (263)	0.00 (0)
Mathematica	100.00 (263)	0.00 (0)
Fricas	100.00 (263)	0.00 (0)
Maple	94.68 (249)	5.32 (14)
Giac	76.05 (200)	23.95 (63)
Mupad	70.34 (185)	29.66 (78)
Maxima	67.30 (177)	32.70 (86)
Sympy	15.21 (40)	84.79 (223)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

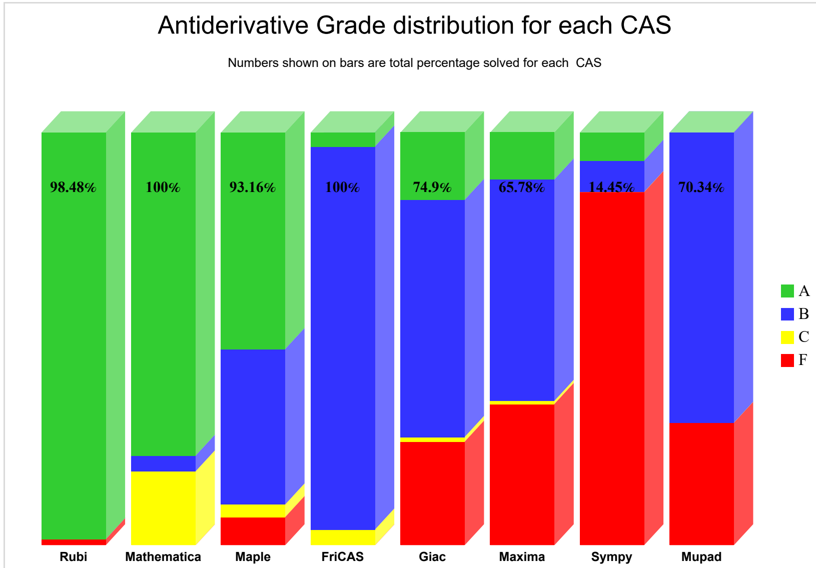
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

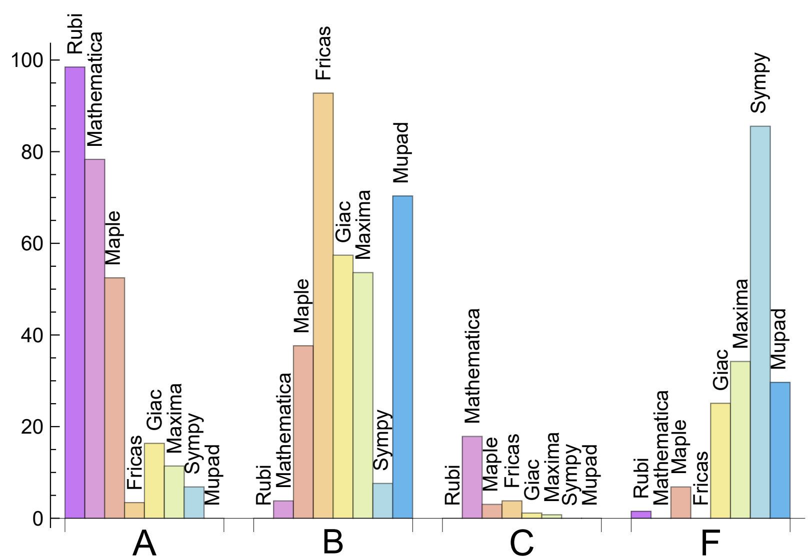
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.479	0.000	0.000	1.521
Mathematica	78.327	3.802	17.871	0.000
Maple	52.471	37.643	3.042	6.844
Giac	16.350	57.414	1.141	25.095
Maxima	11.407	53.612	0.760	34.221
Sympy	6.844	7.605	0.000	85.551
Fricas	3.422	92.776	3.802	0.000
Mupad	0.000	70.342	0.000	29.658

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	14	100.00	0.00	0.00
Giac	63	98.41	0.00	1.59
Mupad	78	0.00	100.00	0.00
Maxima	86	98.84	0.00	1.16
Sympy	223	87.44	12.56	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.09
Maxima	0.30
Fricas	0.39
Giac	0.48
Mathematica	1.71
Mupad	2.47
Sympy	6.36
Maple	9.51

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	91.52	1.00	78.00	1.00
Mathematica	126.30	1.64	86.00	1.00
Sympy	147.45	2.45	115.50	1.91
Maple	178.94	2.13	142.00	1.55
Giac	275.22	3.32	204.50	2.52
Maxima	417.18	4.13	235.00	2.92
Mupad	434.32	6.79	177.00	2.67
Fricas	3902.31	52.85	1942.00	23.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

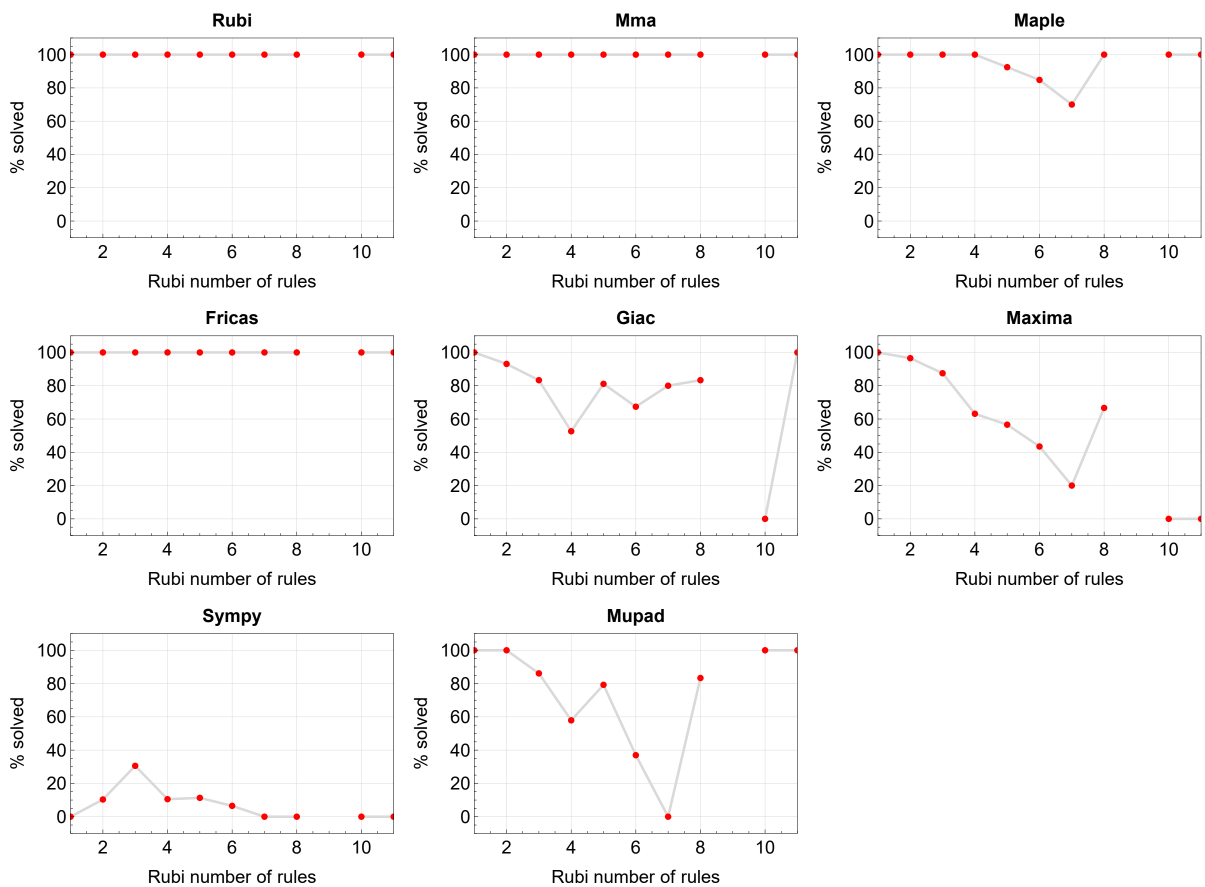


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

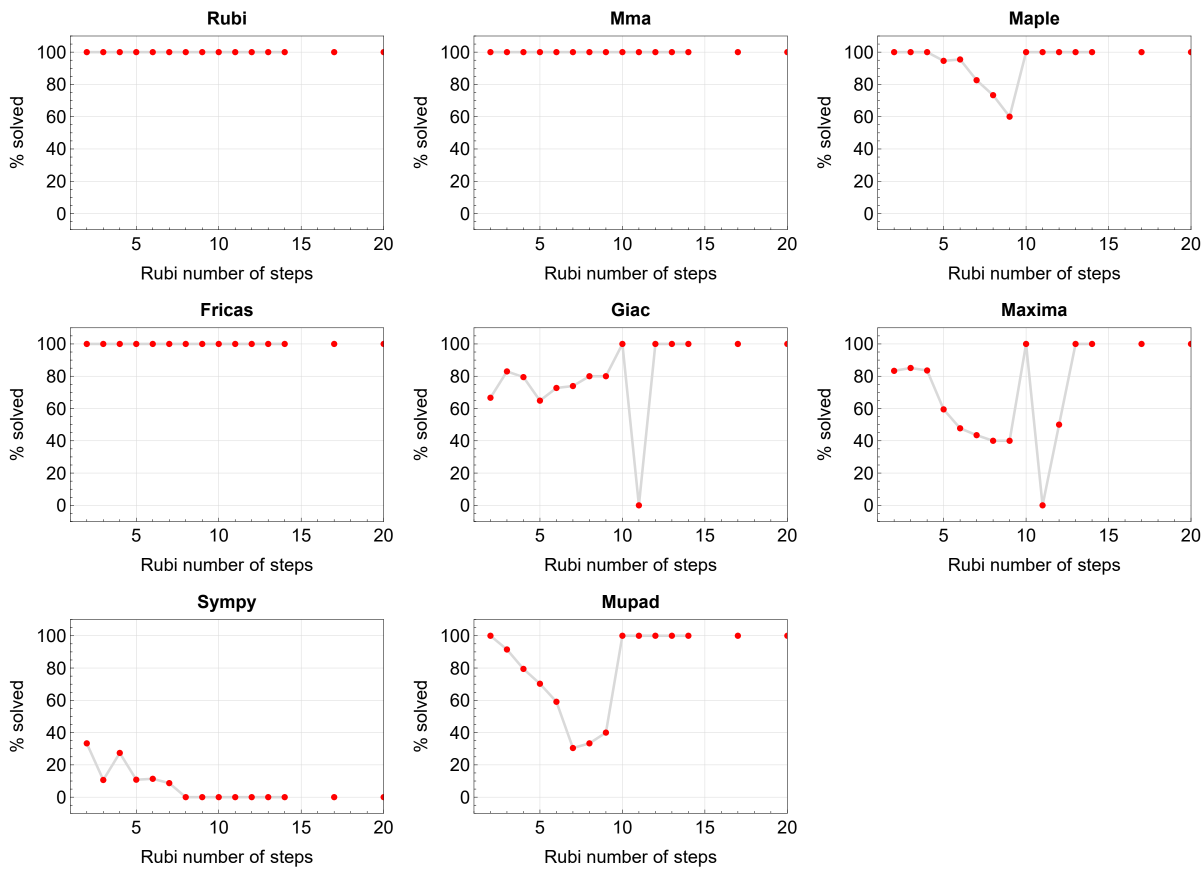


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

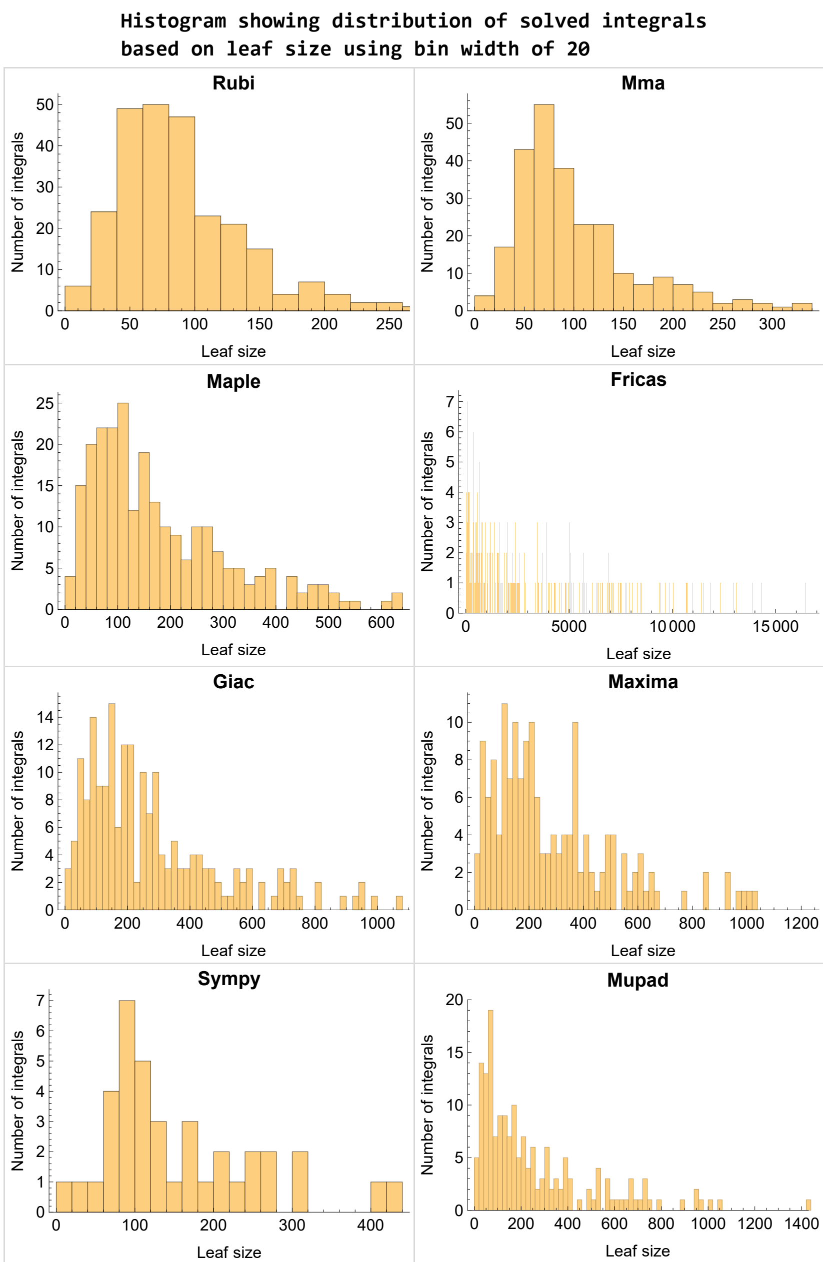


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

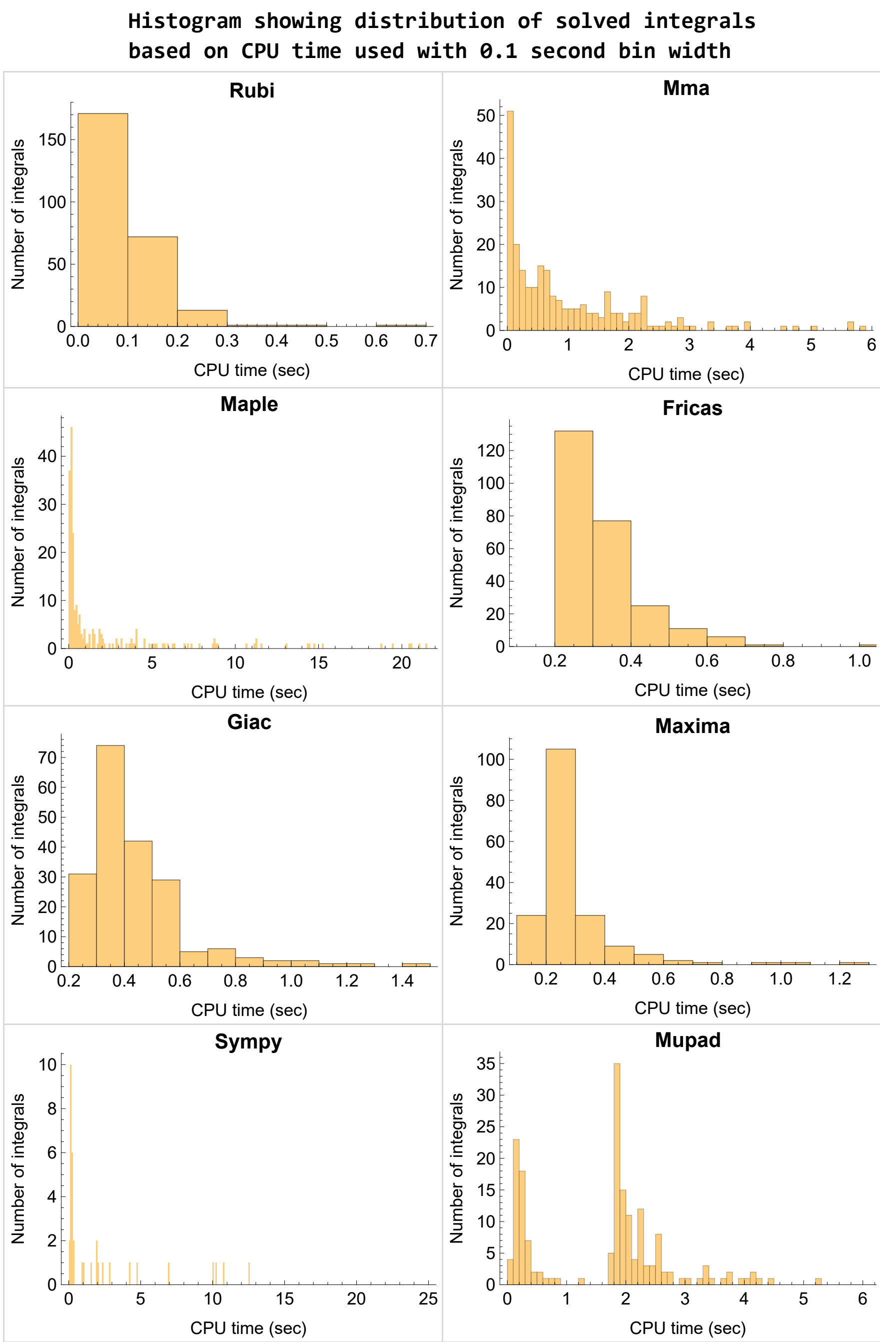


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

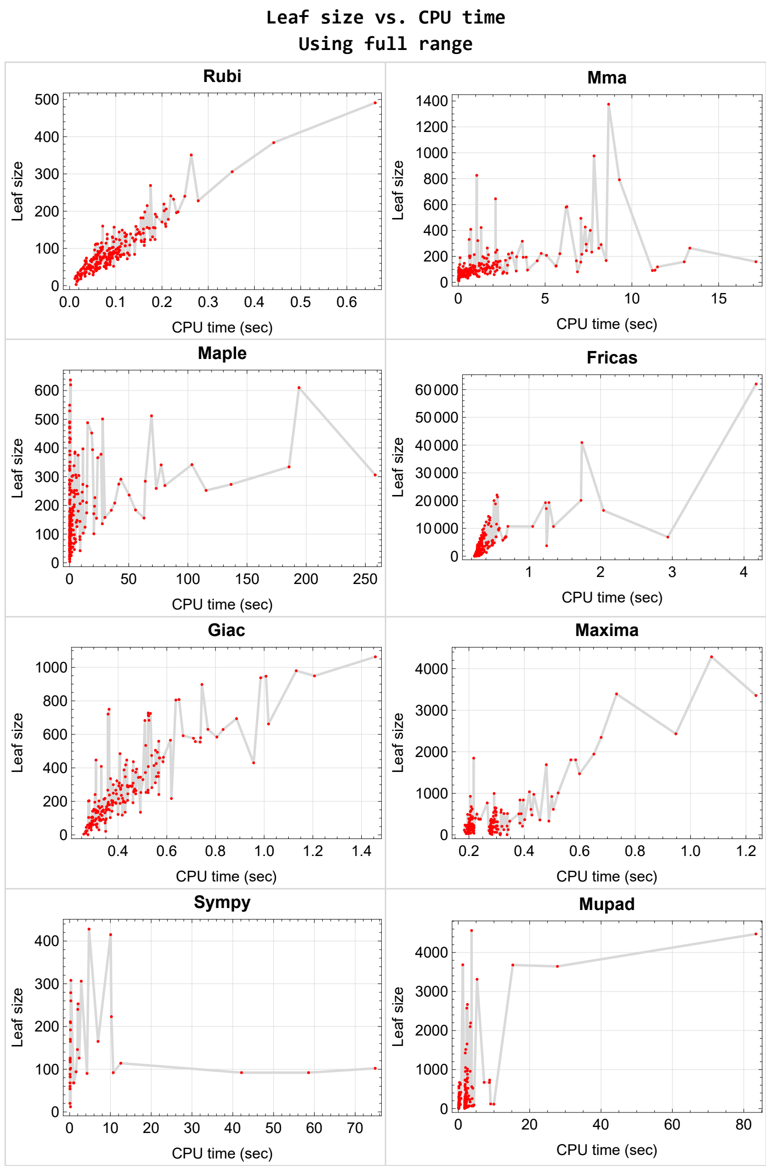


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{74, 76, 77, 79\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{74, 76, 77, 79\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{74, 76, 77, 79\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{76, 77, 79\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{93, 95, 98, 217, 236, 243, 245, 250, 252, 254\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

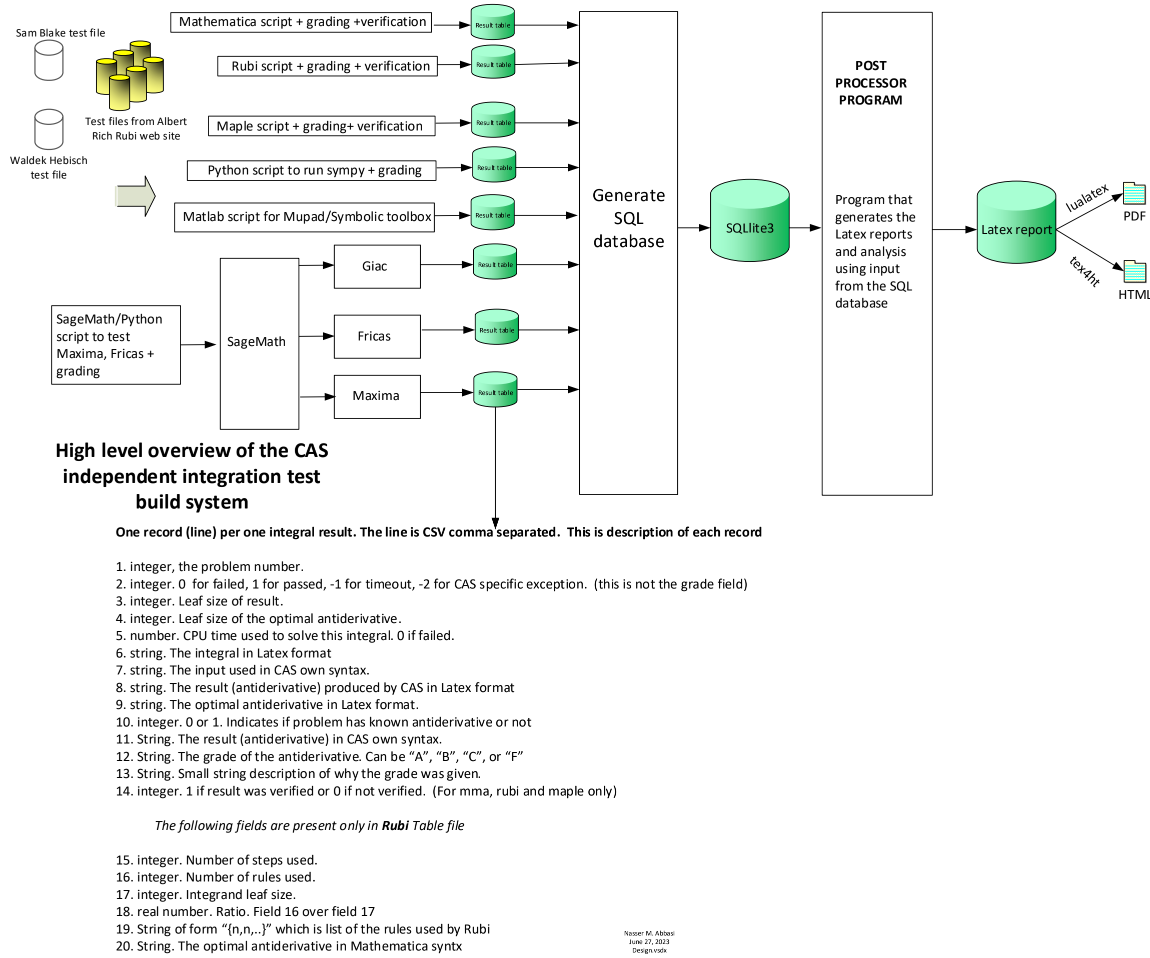
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 216, 218, 219, 221, 222, 223, 225, 226, 227, 228, 229, 231, 233, 234, 235, 237, 240, 248, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { 7, 74, 76, 77, 79, 104, 144, 146, 202, 241 }

C grade { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 93, 95, 98, 133, 140, 142, 150, 154, 209, 211, 215, 217, 220, 224, 230, 232, 236, 238, 239, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 21, 23, 24, 29, 31, 37, 39, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 95, 98, 101, 103, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 257, 258, 260, 261 }

B grade { 10, 12, 18, 20, 22, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 40, 41, 42, 43, 44, 45, 46, 48, 62, 70, 91, 94, 96, 97, 99, 100, 102, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 156, 157, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 255, 256 }

C grade { 73, 75, 78, 80, 111, 259, 262, 263 }

F normal fail { 214, 215, 216, 217, 218, 223, 224, 235, 236, 237, 244, 245, 253, 254 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 3, 4, 81, 82, 83, 89, 138, 206 }

B grade { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263 }

C grade { 73, 74, 75, 76, 77, 79, 80, 226, 228, 256 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 6, 30, 49, 50, 51, 52, 53, 54, 66, 68, 81, 86, 94, 102, 110, 112, 121, 138, 139, 140, 150, 172, 174, 175, 176, 178, 202, 204, 206 }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 32, 33, 35, 38, 40, 41, 43, 46, 48, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 114, 116, 119, 123, 125, 128, 130, 132, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 207, 257, 258 }

C grade { 203, 205 }

F normal fail { 26, 28, 29, 31, 34, 36, 37, 39, 44, 45, 47, 73, 75, 78, 80, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { 42 }

Giac

A grade { 6, 8, 14, 23, 49, 50, 51, 52, 53, 54, 58, 60, 61, 63, 66, 68, 78, 80, 81, 84, 93, 137, 138, 139, 140, 141, 150, 153, 171, 173, 174, 175, 176, 177, 178, 189, 202, 203, 205, 206, 207, 257, 258 }

B grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 24, 55, 56, 57, 59, 62, 64, 65, 67, 69, 70, 71, 72, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

C grade { 226, 228, 256 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 73, 75, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 214, 223, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { 79 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 219, 221, 225, 226, 227, 229, 231, 233, 234, 238, 240, 242, 247, 249, 251, 255, 256, 257, 258 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 74, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 186, 187, 188, 189, 197, 198, 199, 200, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 228, 230, 232, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 254, 259, 260, 261, 262, 263 }

F(-2) exception fail { }

Sympy

A grade { 134, 136, 138, 148, 160, 180, 182, 184, 207, 208, 210, 212, 219, 221, 233, 242, 251, 257 }

B grade { 135, 137, 140, 144, 145, 146, 147, 156, 157, 158, 159, 168, 169, 170, 171, 172, 173, 174, 175, 258 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 72, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 139, 141, 142, 143, 149, 150, 151, 152, 153, 154, 161, 162, 163, 164, 176, 177, 178, 179, 185, 186, 187, 188, 189, 197, 198, 199, 200, 202, 203, 204, 205, 206, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { 35, 41, 42, 43, 44, 65, 66, 73, 74, 75, 76, 125, 126, 133, 155, 165, 166, 167, 181, 183, 190, 191, 192, 193, 194, 195, 196, 201 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	96	154	120	0	142	101
N.S.	1	1.00	0.77	1.32	2.11	1.64	0.00	1.95	1.38
time (sec)	N/A	0.055	0.685	1.469	0.196	0.267	0.000	0.316	0.248

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	136	91	0	105	99
N.S.	1	1.00	1.55	1.60	2.89	1.94	0.00	2.23	2.11
time (sec)	N/A	0.043	0.300	0.634	0.213	0.258	0.000	0.320	1.845

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	66	101	71	0	107	64
N.S.	1	1.00	0.93	1.50	2.30	1.61	0.00	2.43	1.45
time (sec)	N/A	0.037	0.433	0.319	0.218	0.246	0.000	0.295	1.786

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	44	67	42	0	63	27
N.S.	1	1.00	1.80	1.76	2.68	1.68	0.00	2.52	1.08
time (sec)	N/A	0.024	0.219	0.212	0.214	0.246	0.000	0.287	1.735

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	27	40	167	0	68	64
N.S.	1	1.00	2.00	1.04	1.54	6.42	0.00	2.62	2.46
time (sec)	N/A	0.025	0.039	0.201	0.206	0.251	0.000	0.284	0.137

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	39	88	0	45	43
N.S.	1	1.00	1.00	1.04	1.62	3.67	0.00	1.88	1.79
time (sec)	N/A	0.024	0.019	0.425	0.218	0.242	0.000	0.289	0.102

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	123	50	152	924	0	142	156
N.S.	1	1.00	2.41	0.98	2.98	18.12	0.00	2.78	3.06
time (sec)	N/A	0.046	0.278	1.258	0.205	0.271	0.000	0.295	1.856

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	61	55	113	244	0	80	173
N.S.	1	1.00	1.39	1.25	2.57	5.55	0.00	1.82	3.93
time (sec)	N/A	0.033	0.346	2.637	0.206	0.257	0.000	0.296	1.809

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	94	166	295	394	0	293	293
N.S.	1	1.00	0.80	1.41	2.50	3.34	0.00	2.48	2.48
time (sec)	N/A	0.095	2.232	3.167	0.200	0.267	0.000	0.403	1.982

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	71	148	265	259	0	205	215
N.S.	1	1.00	0.92	1.92	3.44	3.36	0.00	2.66	2.79
time (sec)	N/A	0.072	1.827	1.966	0.211	0.247	0.000	0.373	0.305

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	70	118	217	291	0	213	248
N.S.	1	1.00	0.89	1.49	2.75	3.68	0.00	2.70	3.14
time (sec)	N/A	0.076	1.546	0.876	0.218	0.258	0.000	0.342	1.872

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	46	98	171	167	0	139	154
N.S.	1	1.00	0.94	2.00	3.49	3.41	0.00	2.84	3.14
time (sec)	N/A	0.042	1.109	0.527	0.197	0.271	0.000	0.332	1.828

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	64	63	196	890	0	123	160
N.S.	1	1.00	1.25	1.24	3.84	17.45	0.00	2.41	3.14
time (sec)	N/A	0.048	0.979	0.451	0.200	0.258	0.000	0.327	0.169

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	68	136	264	0	86	209
N.S.	1	1.00	0.93	1.48	2.96	5.74	0.00	1.87	4.54
time (sec)	N/A	0.041	1.199	1.838	0.200	0.252	0.000	0.348	1.866

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	127	67	181	2462	0	153	261
N.S.	1	1.00	1.55	0.82	2.21	30.02	0.00	1.87	3.18
time (sec)	N/A	0.087	5.638	3.711	0.202	0.287	0.000	0.360	0.177

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	81	210	393	0	143	143
N.S.	1	1.00	0.82	1.12	2.92	5.46	0.00	1.99	1.99
time (sec)	N/A	0.059	2.067	8.706	0.202	0.260	0.000	0.336	1.930

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	125	246	480	879	0	505	730
N.S.	1	1.00	0.69	1.35	2.64	4.83	0.00	2.77	4.01
time (sec)	N/A	0.154	5.624	10.637	0.204	0.266	0.000	0.554	2.120

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	239	439	540	0	330	361
N.S.	1	1.00	0.87	2.28	4.18	5.14	0.00	3.14	3.44
time (sec)	N/A	0.110	11.183	5.004	0.212	0.255	0.000	0.502	2.033

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	139	95	180	377	725	0	393	668
N.S.	1	1.14	0.78	1.48	3.09	5.94	0.00	3.22	5.48
time (sec)	N/A	0.132	3.990	2.975	0.211	0.274	0.000	0.476	0.328

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	170	321	383	0	236	308
N.S.	1	1.00	0.89	2.43	4.59	5.47	0.00	3.37	4.40
time (sec)	N/A	0.052	0.546	1.809	0.202	0.280	0.000	0.413	1.870

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	93	118	560	2277	0	196	317
N.S.	1	1.00	1.11	1.40	6.67	27.11	0.00	2.33	3.77
time (sec)	N/A	0.061	11.331	1.218	0.205	0.276	0.000	0.420	0.266

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	70	141	348	572	0	202	590
N.S.	1	1.00	1.09	2.20	5.44	8.94	0.00	3.16	9.22
time (sec)	N/A	0.048	2.803	5.620	0.203	0.257	0.000	0.422	1.885

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	159	103	403	5037	0	206	412
N.S.	1	1.00	1.05	0.68	2.65	33.14	0.00	1.36	2.71
time (sec)	N/A	0.160	17.146	11.550	0.214	0.305	0.000	0.439	0.299

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	136	493	925	0	257	622
N.S.	1	1.00	0.89	1.39	5.03	9.44	0.00	2.62	6.35
time (sec)	N/A	0.071	3.337	27.663	0.195	0.253	0.000	0.464	1.932

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	93	215	514	2024	0	0	250
N.S.	1	1.00	0.79	1.82	4.36	17.15	0.00	0.00	2.12
time (sec)	N/A	0.122	0.885	11.143	0.339	0.298	0.000	0.000	2.316

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	135	202	0	1367	0	0	955
N.S.	1	1.00	1.80	2.69	0.00	18.23	0.00	0.00	12.73
time (sec)	N/A	0.104	1.228	3.896	0.000	0.285	0.000	0.000	3.336

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	151	316	916	0	0	198
N.S.	1	1.00	0.86	1.94	4.05	11.74	0.00	0.00	2.54
time (sec)	N/A	0.074	0.668	1.190	0.303	0.291	0.000	0.000	2.207

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	107	104	0	666	0	0	520
N.S.	1	1.00	2.02	1.96	0.00	12.57	0.00	0.00	9.81
time (sec)	N/A	0.045	0.658	0.475	0.000	0.308	0.000	0.000	2.694

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	135	67	0	587	0	0	284
N.S.	1	1.00	2.45	1.22	0.00	10.67	0.00	0.00	5.16
time (sec)	N/A	0.058	0.686	0.357	0.000	0.290	0.000	0.000	2.598

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	112	62	618	0	0	136
N.S.	1	1.00	1.00	2.33	1.29	12.88	0.00	0.00	2.83
time (sec)	N/A	0.046	0.586	0.488	0.293	0.279	0.000	0.000	1.976

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	198	111	0	1790	0	0	787
N.S.	1	1.00	2.33	1.31	0.00	21.06	0.00	0.00	9.26
time (sec)	N/A	0.085	1.414	0.737	0.000	0.305	0.000	0.000	2.599

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	246	134	1628	0	0	254
N.S.	1	1.00	1.01	3.51	1.91	23.26	0.00	0.00	3.63
time (sec)	N/A	0.067	0.690	1.570	0.305	0.276	0.000	0.000	2.281

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	132	512	1690	7366	0	0	0
N.S.	1	1.00	0.69	2.67	8.80	38.36	0.00	0.00	0.00
time (sec)	N/A	0.185	1.679	69.258	0.479	0.376	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	160	267	0	5025	0	0	0
N.S.	1	1.00	1.29	2.15	0.00	40.52	0.00	0.00	0.00
time (sec)	N/A	0.162	2.283	14.742	0.000	0.343	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	105	381	840	3918	0	0	0
N.S.	1	1.00	0.80	2.89	6.36	29.68	0.00	0.00	0.00
time (sec)	N/A	0.120	1.496	4.502	0.396	0.336	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	133	167	0	2252	0	0	0
N.S.	1	1.00	1.45	1.82	0.00	24.48	0.00	0.00	0.00
time (sec)	N/A	0.061	1.474	1.719	0.000	0.308	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	188	161	0	2614	0	0	0
N.S.	1	1.00	1.83	1.56	0.00	25.38	0.00	0.00	0.00
time (sec)	N/A	0.107	1.501	0.951	0.000	0.348	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	252	212	2562	0	0	0
N.S.	1	1.00	1.05	3.07	2.59	31.24	0.00	0.00	0.00
time (sec)	N/A	0.057	1.134	1.546	0.338	0.311	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	221	187	0	6335	0	0	0
N.S.	1	1.00	1.57	1.33	0.00	44.93	0.00	0.00	0.00
time (sec)	N/A	0.160	5.852	2.454	0.000	0.348	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	338	282	5062	0	0	0
N.S.	1	1.00	1.01	2.99	2.50	44.80	0.00	0.00	0.00
time (sec)	N/A	0.112	1.625	3.720	0.384	0.330	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	184	610	3392	18818	0	0	0
N.S.	1	1.00	0.77	2.54	14.13	78.41	0.00	0.00	0.00
time (sec)	N/A	0.249	1.643	193.922	0.733	0.533	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	227	341	0	13095	0	0	0
N.S.	1	1.00	1.37	2.05	0.00	78.89	0.00	0.00	0.00
time (sec)	N/A	0.207	3.094	77.406	0.000	0.449	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	158	488	1806	12965	0	0	0
N.S.	1	1.00	0.85	2.64	9.76	70.08	0.00	0.00	0.00
time (sec)	N/A	0.188	2.286	15.259	0.585	0.453	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	157	252	0	7119	0	0	0
N.S.	1	1.00	1.25	2.00	0.00	56.50	0.00	0.00	0.00
time (sec)	N/A	0.081	2.294	5.912	0.000	0.370	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	249	304	0	10716	0	0	0
N.S.	1	1.00	1.60	1.95	0.00	68.69	0.00	0.00	0.00
time (sec)	N/A	0.181	2.234	2.878	0.000	0.473	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	109	306	478	8312	0	0	0
N.S.	1	1.00	0.97	2.73	4.27	74.21	0.00	0.00	0.00
time (sec)	N/A	0.061	1.838	5.215	0.427	0.379	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	401	304	0	21301	0	0	0
N.S.	1	1.00	2.05	1.55	0.00	108.68	0.00	0.00	0.00
time (sec)	N/A	0.231	7.605	7.892	0.000	0.563	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	149	397	615	14334	0	0	0
N.S.	1	1.00	0.99	2.63	4.07	94.93	0.00	0.00	0.00
time (sec)	N/A	0.147	2.026	11.279	0.505	0.435	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	92	100	194	1530	0	203	156
N.S.	1	1.00	0.70	0.76	1.47	11.59	0.00	1.54	1.18
time (sec)	N/A	0.121	0.206	1.491	0.277	0.282	0.000	0.321	0.288

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	112	104	174	1070	0	132	171
N.S.	1	1.00	1.14	1.06	1.78	10.92	0.00	1.35	1.74
time (sec)	N/A	0.085	0.040	0.975	0.282	0.276	0.000	0.309	0.247

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	68	141	924	0	140	115
N.S.	1	1.00	0.69	0.68	1.41	9.24	0.00	1.40	1.15
time (sec)	N/A	0.086	0.107	0.426	0.280	0.323	0.000	0.317	2.008

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	73	105	528	0	86	128
N.S.	1	1.00	1.22	1.16	1.67	8.38	0.00	1.37	2.03
time (sec)	N/A	0.061	0.031	0.410	0.283	0.334	0.000	0.300	0.150

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	56	83	522	0	74	233
N.S.	1	1.00	1.53	1.14	1.69	10.65	0.00	1.51	4.76
time (sec)	N/A	0.059	0.018	0.440	0.279	0.278	0.000	0.293	3.242

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	44	141	0	45	79
N.S.	1	1.00	1.00	0.97	1.52	4.86	0.00	1.55	2.72
time (sec)	N/A	0.026	0.024	0.672	0.202	0.251	0.000	0.313	0.171

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	114	54	156	1188	0	143	173
N.S.	1	1.00	1.61	0.76	2.20	16.73	0.00	2.01	2.44
time (sec)	N/A	0.068	0.027	1.961	0.278	0.269	0.000	0.310	3.304

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	74	46	184	1739	0	149	162
N.S.	1	1.00	1.32	0.82	3.29	31.05	0.00	2.66	2.89
time (sec)	N/A	0.041	0.214	4.062	0.294	0.268	0.000	0.308	1.991

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	206	156	180	379	5034	0	374	359
N.S.	1	1.21	0.92	1.06	2.23	29.61	0.00	2.20	2.11
time (sec)	N/A	0.209	7.056	7.300	0.281	0.313	0.000	0.472	0.480

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	121	195	348	3341	0	289	397
N.S.	1	1.00	0.66	1.07	1.91	18.36	0.00	1.59	2.18
time (sec)	N/A	0.159	2.228	3.698	0.290	0.293	0.000	0.456	0.411

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	159	137	130	301	3649	0	296	306
N.S.	1	1.23	1.06	1.01	2.33	28.29	0.00	2.29	2.37
time (sec)	N/A	0.144	2.204	2.022	0.284	0.280	0.000	0.416	2.054

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	90	145	253	2230	0	203	338
N.S.	1	1.00	0.73	1.18	2.06	18.13	0.00	1.65	2.75
time (sec)	N/A	0.108	1.564	1.427	0.279	0.278	0.000	0.378	0.311

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	125	111	447	2498	0	178	522
N.S.	1	1.00	1.28	1.13	4.56	25.49	0.00	1.82	5.33
time (sec)	N/A	0.102	0.955	1.036	0.288	0.286	0.000	0.377	4.052

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	94	98	256	518	0	122	483
N.S.	1	1.00	2.00	2.09	5.45	11.02	0.00	2.60	10.28
time (sec)	N/A	0.041	0.975	4.037	0.206	0.252	0.000	0.400	2.003

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	159	91	378	4642	0	192	561
N.S.	1	1.00	1.49	0.85	3.53	43.38	0.00	1.79	5.24
time (sec)	N/A	0.121	1.741	8.677	0.292	0.318	0.000	0.391	3.771

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	147	101	468	4125	0	249	344
N.S.	1	1.00	1.52	1.04	4.82	42.53	0.00	2.57	3.55
time (sec)	N/A	0.069	1.207	20.507	0.297	0.282	0.000	0.380	0.276

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	306	294	274	647	12323	0	694	682
N.S.	1	1.11	1.07	1.00	2.35	44.81	0.00	2.52	2.48
time (sec)	N/A	0.352	7.365	41.602	0.298	0.403	0.000	0.887	2.550

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	291	366	604	8462	0	580	757
N.S.	1	1.00	0.83	1.04	1.72	24.11	0.00	1.65	2.16
time (sec)	N/A	0.263	8.227	23.771	0.317	0.358	0.000	0.739	2.458

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	241	244	206	544	9862	0	577	617
N.S.	1	1.10	1.11	0.94	2.47	44.83	0.00	2.62	2.80
time (sec)	N/A	0.219	7.376	8.875	0.316	0.369	0.000	0.710	0.582

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	233	297	484	6410	0	463	707
N.S.	1	1.00	0.87	1.10	1.80	23.83	0.00	1.72	2.63
time (sec)	N/A	0.175	7.689	5.718	0.297	0.317	0.000	0.586	2.211

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	168	245	654	7127	0	414	671
N.S.	1	1.00	0.77	1.12	2.99	32.54	0.00	1.89	3.06
time (sec)	N/A	0.204	8.517	3.445	0.294	0.345	0.000	0.547	7.234

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	113	171	679	1192	0	311	1515
N.S.	1	1.00	1.59	2.41	9.56	16.79	0.00	4.38	21.34
time (sec)	N/A	0.048	2.387	20.423	0.208	0.273	0.000	0.553	2.085

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	264	208	586	10985	0	426	731
N.S.	1	1.00	1.14	0.90	2.53	47.35	0.00	1.84	3.15
time (sec)	N/A	0.225	13.337	38.158	0.294	0.382	0.000	0.557	8.786

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	213	156	997	9459	0	437	646
N.S.	1	1.00	1.54	1.13	7.22	68.54	0.00	3.17	4.68
time (sec)	N/A	0.081	1.737	62.879	0.291	0.353	0.000	0.584	0.538

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	645	452	0	17123	0	0	3313
N.S.	1	1.00	1.31	0.92	0.00	34.87	0.00	0.00	6.75
time (sec)	N/A	0.661	2.145	18.727	0.000	1.241	0.000	0.000	5.254

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	F(-1)	N/A	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	826	289	533	62017	0	3	0
N.S.	1	1.00	35.91	12.57	23.17	2696.39	0.00	0.13	0.00
time (sec)	N/A	0.032	1.064	7.246	0.844	4.169	0.000	3.880	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	423	301	0	10695	0	0	2100
N.S.	1	1.00	1.10	0.78	0.00	27.85	0.00	0.00	5.47
time (sec)	N/A	0.442	1.315	2.074	0.000	1.052	0.000	0.000	3.332

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	F(-1)	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	409	159	250	40923	0	3	4474
N.S.	1	1.00	19.48	7.57	11.90	1948.71	0.00	0.14	213.05
time (sec)	N/A	0.020	0.719	1.616	0.506	1.738	0.000	3.415	83.407

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	331	96	160	20085	19	3	3679
N.S.	1	1.00	15.76	4.57	7.62	956.43	0.90	0.14	175.19
time (sec)	N/A	0.029	0.628	0.632	0.397	1.726	0.436	2.970	15.272

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	190	116	0	640	0	21	669
N.S.	1	1.00	1.21	0.74	0.00	4.08	0.00	0.13	4.26
time (sec)	N/A	0.096	0.663	0.668	0.000	0.281	0.000	0.349	8.731

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	F(-2)	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	214	136	188	6846	20	0	3643
N.S.	1	1.00	9.30	5.91	8.17	297.65	0.87	0.00	158.39
time (sec)	N/A	0.037	0.919	1.233	0.448	2.937	0.351	0.000	27.782

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	322	173	0	3726	0	180	4563
N.S.	1	1.00	1.50	0.80	0.00	17.33	0.00	0.84	21.22
time (sec)	N/A	0.168	1.122	1.599	0.000	1.247	0.000	0.365	3.728

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	82	104	63	0	104	74
N.S.	1	1.00	0.70	1.30	1.65	1.00	0.00	1.65	1.17
time (sec)	N/A	0.035	0.132	4.873	0.203	0.248	0.000	0.321	0.188

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	37	83	45	0	84	74
N.S.	1	1.00	1.47	1.23	2.77	1.50	0.00	2.80	2.47
time (sec)	N/A	0.026	0.013	1.898	0.198	0.259	0.000	0.303	0.196

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	69	30	0	78	27
N.S.	1	1.00	0.97	1.64	2.09	0.91	0.00	2.36	0.82
time (sec)	N/A	0.027	0.050	0.623	0.194	0.262	0.000	0.293	1.852

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	47	32	55	102	0	47	66
N.S.	1	1.00	1.74	1.19	2.04	3.78	0.00	1.74	2.44
time (sec)	N/A	0.023	0.018	0.328	0.281	0.268	0.000	0.283	0.126

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	56	80	323	0	84	125
N.S.	1	1.00	1.20	1.40	2.00	8.08	0.00	2.10	3.12
time (sec)	N/A	0.023	0.014	0.615	0.275	0.255	0.000	0.293	0.121

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	34	159	0	59	59
N.S.	1	1.00	1.00	0.89	1.21	5.68	0.00	2.11	2.11
time (sec)	N/A	0.022	0.013	1.415	0.193	0.257	0.000	0.296	1.872

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	93	85	181	1046	0	153	280
N.S.	1	1.00	1.41	1.29	2.74	15.85	0.00	2.32	4.24
time (sec)	N/A	0.036	0.010	4.028	0.281	0.265	0.000	0.300	1.896

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	86	42	371	345	0	95	304
N.S.	1	1.00	1.79	0.88	7.73	7.19	0.00	1.98	6.33
time (sec)	N/A	0.030	0.021	8.915	0.205	0.257	0.000	0.309	0.154

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	63	124	171	95	0	186	102
N.S.	1	1.00	0.74	1.46	2.01	1.12	0.00	2.19	1.20
time (sec)	N/A	0.062	1.683	13.067	0.204	0.264	0.000	0.359	0.252

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	72	72	161	519	0	152	130
N.S.	1	1.00	1.33	1.33	2.98	9.61	0.00	2.81	2.41
time (sec)	N/A	0.045	0.622	5.132	0.282	0.264	0.000	0.351	1.982

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	96	140	105	0	167	77
N.S.	1	1.00	1.06	1.88	2.75	2.06	0.00	3.27	1.51
time (sec)	N/A	0.058	1.080	1.943	0.193	0.255	0.000	0.353	1.930

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	97	152	774	0	160	182
N.S.	1	1.00	0.90	1.62	2.53	12.90	0.00	2.67	3.03
time (sec)	N/A	0.062	0.212	0.961	0.279	0.279	0.000	0.325	0.210

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	427	134	199	1373	0	170	303
N.S.	1	1.00	4.69	1.47	2.19	15.09	0.00	1.87	3.33
time (sec)	N/A	0.065	7.314	2.113	0.289	0.262	0.000	0.319	0.157

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	126	53	391	0	169	482
N.S.	1	1.00	1.00	2.57	1.08	7.98	0.00	3.45	9.84
time (sec)	N/A	0.038	0.216	6.236	0.201	0.272	0.000	0.333	1.957

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	792	174	345	2824	0	267	572
N.S.	1	1.00	6.34	1.39	2.76	22.59	0.00	2.14	4.58
time (sec)	N/A	0.100	9.279	14.466	0.278	0.277	0.000	0.358	0.169

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	158	928	677	0	238	732
N.S.	1	1.00	1.09	2.08	12.21	8.91	0.00	3.13	9.63
time (sec)	N/A	0.052	1.855	29.886	0.205	0.254	0.000	0.364	1.869

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	81	184	267	227	0	282	133
N.S.	1	1.00	0.89	2.02	2.93	2.49	0.00	3.10	1.46
time (sec)	N/A	0.095	6.865	55.729	0.205	0.259	0.000	0.539	2.039

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	494	155	284	1840	0	264	232
N.S.	1	1.00	5.68	1.78	3.26	21.15	0.00	3.03	2.67
time (sec)	N/A	0.075	7.062	22.770	0.280	0.282	0.000	0.473	0.333

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	148	256	369	0	265	243
N.S.	1	1.00	0.88	1.90	3.28	4.73	0.00	3.40	3.12
time (sec)	N/A	0.066	2.558	6.308	0.206	0.257	0.000	0.466	2.014

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	89	193	295	2411	0	272	355
N.S.	1	1.00	0.90	1.95	2.98	24.35	0.00	2.75	3.59
time (sec)	N/A	0.087	0.437	2.813	0.289	0.277	0.000	0.410	0.287

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	119	240	362	3465	0	310	535
N.S.	1	1.00	0.80	1.61	2.43	23.26	0.00	2.08	3.59
time (sec)	N/A	0.114	11.476	6.984	0.280	0.282	0.000	0.405	1.996

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	227	71	786	0	347	1050
N.S.	1	1.00	1.00	3.39	1.06	11.73	0.00	5.18	15.67
time (sec)	N/A	0.045	0.253	21.438	0.187	0.271	0.000	0.430	2.036

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	158	291	553	6114	0	485	951
N.S.	1	1.00	0.80	1.47	2.79	30.88	0.00	2.45	4.80
time (sec)	N/A	0.163	13.015	43.363	0.298	0.318	0.000	0.408	1.988

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	218	269	1847	1185	0	447	1424
N.S.	1	1.00	2.14	2.64	18.11	11.62	0.00	4.38	13.96
time (sec)	N/A	0.067	2.891	80.580	0.217	0.264	0.000	0.436	1.956

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	115	273	514	2180	0	0	967
N.S.	1	1.00	0.96	2.28	4.28	18.17	0.00	0.00	8.06
time (sec)	N/A	0.121	0.948	11.204	0.327	0.303	0.000	0.000	2.503

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	230	0	1850	0	0	2194
N.S.	1	1.00	0.99	2.88	0.00	23.12	0.00	0.00	27.42
time (sec)	N/A	0.086	0.564	3.968	0.000	0.298	0.000	0.000	3.439

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	159	316	948	0	0	880
N.S.	1	1.00	1.00	2.06	4.10	12.31	0.00	0.00	11.43
time (sec)	N/A	0.079	0.745	1.238	0.298	0.288	0.000	0.000	2.556

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	149	0	766	0	0	154
N.S.	1	1.00	1.00	2.81	0.00	14.45	0.00	0.00	2.91
time (sec)	N/A	0.057	0.136	0.576	0.000	0.286	0.000	0.000	2.240

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	102	0	511	0	0	147
N.S.	1	1.00	1.00	2.83	0.00	14.19	0.00	0.00	4.08
time (sec)	N/A	0.036	0.046	0.904	0.000	0.268	0.000	0.000	0.279

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	114	36	455	0	0	81
N.S.	1	1.00	1.00	3.56	1.12	14.22	0.00	0.00	2.53
time (sec)	N/A	0.043	0.550	3.195	0.276	0.286	0.000	0.000	2.056

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	144	0	540	0	0	449
N.S.	1	1.00	1.00	2.62	0.00	9.82	0.00	0.00	8.16
time (sec)	N/A	0.058	0.312	8.707	0.000	0.293	0.000	0.000	2.298

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	197	63	649	0	0	176
N.S.	1	1.00	1.00	3.94	1.26	12.98	0.00	0.00	3.52
time (sec)	N/A	0.051	0.567	21.085	0.300	0.284	0.000	0.000	2.221

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	236	0	1584	0	0	1012
N.S.	1	1.00	0.92	2.74	0.00	18.42	0.00	0.00	11.77
time (sec)	N/A	0.095	0.769	50.330	0.000	0.294	0.000	0.000	2.597

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	252	140	2032	0	0	252
N.S.	1	1.00	0.95	3.36	1.87	27.09	0.00	0.00	3.36
time (sec)	N/A	0.067	0.868	115.364	0.301	0.298	0.000	0.000	2.238

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	111	378	0	6934	0	0	0
N.S.	1	1.00	0.87	2.95	0.00	54.17	0.00	0.00	0.00
time (sec)	N/A	0.136	1.304	26.642	0.000	0.344	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	110	386	840	4324	0	0	0
N.S.	1	1.00	0.79	2.76	6.00	30.89	0.00	0.00	0.00
time (sec)	N/A	0.148	1.606	4.574	0.384	0.344	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	84	286	0	3502	0	0	0
N.S.	1	1.00	0.83	2.83	0.00	34.67	0.00	0.00	0.00
time (sec)	N/A	0.105	0.853	1.817	0.000	0.355	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	252	0	2041	0	0	0
N.S.	1	1.00	0.94	3.04	0.00	24.59	0.00	0.00	0.00
time (sec)	N/A	0.059	0.340	4.077	0.000	0.295	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	210	125	1515	0	0	0
N.S.	1	1.00	0.95	3.18	1.89	22.95	0.00	0.00	0.00
time (sec)	N/A	0.048	1.028	14.326	0.326	0.296	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	183	0	1555	0	0	0
N.S.	1	1.00	0.96	2.54	0.00	21.60	0.00	0.00	0.00
time (sec)	N/A	0.054	0.193	35.269	0.000	0.286	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	83	259	127	1443	0	0	0
N.S.	1	1.00	1.08	3.36	1.65	18.74	0.00	0.00	0.00
time (sec)	N/A	0.059	1.020	73.200	0.342	0.284	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	203	273	0	2140	0	0	0
N.S.	1	1.00	1.99	2.68	0.00	20.98	0.00	0.00	0.00
time (sec)	N/A	0.092	0.641	136.479	0.000	0.345	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	102	306	209	2869	0	0	0
N.S.	1	1.00	1.05	3.15	2.15	29.58	0.00	0.00	0.00
time (sec)	N/A	0.100	1.387	258.305	0.393	0.315	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	265	351	0	6396	0	0	0
N.S.	1	1.00	1.71	2.26	0.00	41.26	0.00	0.00	0.00
time (sec)	N/A	0.185	1.672	0.304	0.000	0.373	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	164	501	1806	13887	0	0	0
N.S.	1	1.00	0.83	2.53	9.12	70.14	0.00	0.00	0.00
time (sec)	N/A	0.235	2.408	27.890	0.568	0.458	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	136	375	0	11392	0	0	0
N.S.	1	1.00	0.88	2.44	0.00	73.97	0.00	0.00	0.00
time (sec)	N/A	0.173	2.223	7.195	0.000	0.433	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	134	394	0	7909	0	0	0
N.S.	1	1.00	0.93	2.74	0.00	54.92	0.00	0.00	0.00
time (sec)	N/A	0.107	1.223	19.445	0.000	0.370	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	77	284	366	5840	0	0	0
N.S.	1	1.00	0.80	2.96	3.81	60.83	0.00	0.00	0.00
time (sec)	N/A	0.054	2.613	63.990	0.401	0.344	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	123	342	0	6614	0	0	0
N.S.	1	1.00	0.95	2.65	0.00	51.27	0.00	0.00	0.00
time (sec)	N/A	0.093	0.832	103.406	0.000	0.364	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	334	360	5659	0	0	0
N.S.	1	1.00	1.00	2.90	3.13	49.21	0.00	0.00	0.00
time (sec)	N/A	0.069	1.962	185.475	0.456	0.336	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	276	0	5077	0	0	0
N.S.	1	1.00	0.84	2.65	0.00	48.82	0.00	0.00	0.00
time (sec)	N/A	0.072	0.466	0.221	0.000	0.323	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	128	376	332	5233	0	0	0
N.S.	1	1.00	0.98	2.87	2.53	39.95	0.00	0.00	0.00
time (sec)	N/A	0.093	1.960	0.189	0.489	0.340	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	317	389	0	8070	0	0	0
N.S.	1	1.00	2.03	2.49	0.00	51.73	0.00	0.00	0.00
time (sec)	N/A	0.168	3.688	0.207	0.000	0.377	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	97	68	199	339	82	134	50
N.S.	1	1.00	1.80	1.26	3.69	6.28	1.52	2.48	0.93
time (sec)	N/A	0.042	0.055	0.079	0.209	0.272	0.142	0.316	1.863

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	71	168	1205	88	92	53
N.S.	1	1.00	0.88	1.45	3.43	24.59	1.80	1.88	1.08
time (sec)	N/A	0.050	0.290	0.073	0.286	0.268	0.116	0.327	0.110

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	65	45	105	160	54	86	34
N.S.	1	1.00	1.81	1.25	2.92	4.44	1.50	2.39	0.94
time (sec)	N/A	0.033	0.036	0.064	0.201	0.271	0.104	0.305	1.803

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	41	49	76	399	60	57	37
N.S.	1	1.00	1.32	1.58	2.45	12.87	1.94	1.84	1.19
time (sec)	N/A	0.026	0.038	0.084	0.276	0.270	0.091	0.287	1.795

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	22	31	37	20	29	18
N.S.	1	1.00	1.47	1.16	1.63	1.95	1.05	1.53	0.95
time (sec)	N/A	0.012	0.001	0.040	0.190	0.255	0.089	0.290	0.069

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	24	35	70	0	46	228
N.S.	1	1.00	1.32	0.96	1.40	2.80	0.00	1.84	9.12
time (sec)	N/A	0.034	0.055	0.232	0.198	0.264	0.000	0.286	1.854

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	32	27	31	38	90	30	25
N.S.	1	1.00	1.78	1.50	1.72	2.11	5.00	1.67	1.39
time (sec)	N/A	0.024	0.035	0.115	0.199	0.271	4.275	0.321	1.784

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	35	106	407	0	58	76
N.S.	1	1.00	1.26	1.13	3.42	13.13	0.00	1.87	2.45
time (sec)	N/A	0.033	0.131	0.245	0.197	0.272	0.000	0.310	1.818

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	61	39	105	156	0	86	162
N.S.	1	1.00	1.69	1.08	2.92	4.33	0.00	2.39	4.50
time (sec)	N/A	0.036	0.050	0.212	0.200	0.254	0.000	0.330	1.814

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	56	206	1216	0	93	177
N.S.	1	1.00	1.04	1.14	4.20	24.82	0.00	1.90	3.61
time (sec)	N/A	0.050	0.377	0.273	0.190	0.269	0.000	0.338	1.827

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	190	135	369	796	165	300	91
N.S.	1	1.00	2.29	1.63	4.45	9.59	1.99	3.61	1.10
time (sec)	N/A	0.060	0.101	0.118	0.205	0.259	0.211	0.380	0.167

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	130	333	3441	170	191	100
N.S.	1	1.00	0.87	1.71	4.38	45.28	2.24	2.51	1.32
time (sec)	N/A	0.086	0.500	0.096	0.279	0.302	0.191	0.376	0.214

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	137	97	231	483	117	218	67
N.S.	1	1.00	2.17	1.54	3.67	7.67	1.86	3.46	1.06
time (sec)	N/A	0.059	0.082	0.093	0.198	0.259	0.155	0.346	1.829

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	92	186	1638	122	116	76
N.S.	1	1.00	0.88	1.61	3.26	28.74	2.14	2.04	1.33
time (sec)	N/A	0.056	0.370	0.103	0.280	0.285	0.127	0.334	1.835

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	65	60	114	201	68	103	47
N.S.	1	1.00	1.51	1.40	2.65	4.67	1.58	2.40	1.09
time (sec)	N/A	0.025	0.894	0.061	0.184	0.274	0.112	0.296	1.841

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	59	104	668	0	141	210
N.S.	1	1.00	0.98	1.20	2.12	13.63	0.00	2.88	4.29
time (sec)	N/A	0.058	0.162	0.176	0.272	0.266	0.000	0.337	1.961

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	64	36	64	97	0	71	59
N.S.	1	1.00	1.78	1.00	1.78	2.69	0.00	1.97	1.64
time (sec)	N/A	0.052	0.122	0.254	0.197	0.269	0.000	0.346	1.826

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	62	134	677	0	141	211
N.S.	1	1.00	0.96	1.19	2.58	13.02	0.00	2.71	4.06
time (sec)	N/A	0.075	0.190	0.196	0.197	0.263	0.000	0.375	0.236

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	65	43	114	197	0	103	175
N.S.	1	1.00	1.51	1.00	2.65	4.58	0.00	2.40	4.07
time (sec)	N/A	0.056	0.543	0.166	0.209	0.244	0.000	0.368	0.155

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	77	236	1649	0	118	197
N.S.	1	1.00	0.81	1.07	3.28	22.90	0.00	1.64	2.74
time (sec)	N/A	0.083	0.472	0.219	0.186	0.274	0.000	0.417	1.919

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	98	58	231	473	0	218	529
N.S.	1	1.00	1.56	0.92	3.67	7.51	0.00	3.46	8.40
time (sec)	N/A	0.066	0.128	0.168	0.191	0.255	0.000	0.418	0.184

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	74	92	390	3454	0	192	362
N.S.	1	1.00	0.80	1.00	4.24	37.54	0.00	2.09	3.93
time (sec)	N/A	0.096	0.566	0.244	0.199	0.305	0.000	0.461	0.244

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	123	218	583	1563	260	534	138
N.S.	1	1.00	1.08	1.91	5.11	13.71	2.28	4.68	1.21
time (sec)	N/A	0.079	1.723	0.169	0.208	0.273	0.304	0.513	0.229

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	98	205	540	7502	279	309	155
N.S.	1	1.00	0.92	1.92	5.05	70.11	2.61	2.89	1.45
time (sec)	N/A	0.120	0.354	0.144	0.289	0.336	0.275	0.462	1.881

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	164	400	1036	192	418	106
N.S.	1	1.00	1.15	1.74	4.26	11.02	2.04	4.45	1.13
time (sec)	N/A	0.068	2.004	0.130	0.207	0.273	0.224	0.431	1.865

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	76	151	351	4298	211	216	123
N.S.	1	1.00	0.92	1.82	4.23	51.78	2.54	2.60	1.48
time (sec)	N/A	0.088	0.308	0.130	0.282	0.296	0.186	0.401	1.854

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	95	112	239	567	126	241	86
N.S.	1	1.00	1.28	1.51	3.23	7.66	1.70	3.26	1.16
time (sec)	N/A	0.036	0.717	0.094	0.196	0.273	0.158	0.306	0.159

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	80	214	2381	0	267	380
N.S.	1	1.00	0.93	1.11	2.97	33.07	0.00	3.71	5.28
time (sec)	N/A	0.084	0.620	0.208	0.276	0.295	0.000	0.390	0.364

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	59	147	341	0	135	218
N.S.	1	1.00	1.37	1.00	2.49	5.78	0.00	2.29	3.69
time (sec)	N/A	0.062	2.613	0.199	0.197	0.262	0.000	0.429	1.868

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	77	203	1686	0	274	327
N.S.	1	1.00	0.88	1.07	2.82	23.42	0.00	3.81	4.54
time (sec)	N/A	0.081	0.487	0.246	0.285	0.268	0.000	0.468	2.913

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	82	56	147	341	0	135	219
N.S.	1	1.00	1.39	0.95	2.49	5.78	0.00	2.29	3.71
time (sec)	N/A	0.062	1.412	0.244	0.209	0.268	0.000	0.492	1.839

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	90	264	2393	0	267	381
N.S.	1	1.00	0.81	1.08	3.18	28.83	0.00	3.22	4.59
time (sec)	N/A	0.087	0.648	0.245	0.212	0.294	0.000	0.523	0.360

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	100	72	239	557	0	241	568
N.S.	1	1.00	1.35	0.97	3.23	7.53	0.00	3.26	7.68
time (sec)	N/A	0.073	1.620	0.216	0.205	0.265	0.000	0.567	1.893

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	76	106	420	4305	0	217	380
N.S.	1	1.00	0.74	1.03	4.08	41.80	0.00	2.11	3.69
time (sec)	N/A	0.094	0.266	0.270	0.208	0.311	0.000	0.619	0.274

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	128	179	410	1176	209	447	133
N.S.	1	1.00	1.16	1.63	3.73	10.69	1.90	4.06	1.21
time (sec)	N/A	0.056	1.195	0.134	0.207	0.279	0.228	0.309	0.184

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	170	262	624	2133	308	721	188
N.S.	1	1.00	1.06	1.64	3.90	13.33	1.92	4.51	1.18
time (sec)	N/A	0.072	2.720	0.206	0.212	0.297	0.319	0.359	1.950

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	79	133	742	415	132	72
N.S.	1	1.00	0.91	1.20	2.02	11.24	6.29	2.00	1.09
time (sec)	N/A	0.088	0.212	0.109	0.277	0.301	10.068	0.346	0.245

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	66	87	509	777	428	87	56
N.S.	1	1.00	1.12	1.47	8.63	13.17	7.25	1.47	0.95
time (sec)	N/A	0.082	0.242	0.136	0.389	0.289	4.762	0.325	1.864

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	49	82	118	306	96	46
N.S.	1	1.00	0.96	1.07	1.78	2.57	6.65	2.09	1.00
time (sec)	N/A	0.079	0.052	0.076	0.278	0.298	2.859	0.351	1.830

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	72	215	486	253	65	38
N.S.	1	1.00	1.02	1.57	4.67	10.57	5.50	1.41	0.83
time (sec)	N/A	0.064	0.039	0.120	0.302	0.288	2.081	0.313	0.099

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	44	58	82	146	61	43
N.S.	1	1.00	0.83	1.05	1.38	1.95	3.48	1.45	1.02
time (sec)	N/A	0.046	0.033	0.061	0.200	0.265	1.932	0.307	1.815

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	65	71	57	484	240	63	37
N.S.	1	1.00	1.44	1.58	1.27	10.76	5.33	1.40	0.82
time (sec)	N/A	0.055	0.108	0.104	0.278	0.283	1.999	0.296	0.085

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	65	101	118	0	97	194
N.S.	1	1.00	0.92	1.08	1.68	1.97	0.00	1.62	3.23
time (sec)	N/A	0.077	0.082	0.162	0.200	0.307	0.000	0.310	2.085

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	67	89	329	784	0	89	402
N.S.	1	1.00	1.12	1.48	5.48	13.07	0.00	1.48	6.70
time (sec)	N/A	0.080	0.234	0.196	0.347	0.307	0.000	0.323	2.209

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	60	88	159	747	0	133	313
N.S.	1	1.00	0.71	1.04	1.87	8.79	0.00	1.56	3.68
time (sec)	N/A	0.104	0.212	0.204	0.217	0.340	0.000	0.346	2.244

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	91	109	1038	2368	0	147	519
N.S.	1	1.00	1.11	1.33	12.66	28.88	0.00	1.79	6.33
time (sec)	N/A	0.129	0.748	0.228	0.419	0.308	0.000	0.352	2.206

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	91	217	1141	102	194	170
N.S.	1	1.00	0.83	1.10	2.61	13.75	1.23	2.34	2.05
time (sec)	N/A	0.112	0.557	0.114	0.299	0.329	74.908	0.430	2.317

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	104	1010	1950	0	195	1655
N.S.	1	1.00	1.01	1.17	11.35	21.91	0.00	2.19	18.60
time (sec)	N/A	0.087	0.616	0.192	0.522	0.316	0.000	0.380	2.440

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	84	170	629	92	149	210
N.S.	1	1.00	0.79	1.17	2.36	8.74	1.28	2.07	2.92
time (sec)	N/A	0.088	0.500	0.106	0.216	0.291	42.147	0.364	0.399

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	86	100	614	2025	0	177	106
N.S.	1	1.00	1.01	1.18	7.22	23.82	0.00	2.08	1.25
time (sec)	N/A	0.076	0.445	0.183	0.424	0.311	0.000	0.357	0.606

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	86	170	623	92	149	129
N.S.	1	1.00	0.81	1.26	2.50	9.16	1.35	2.19	1.90
time (sec)	N/A	0.069	0.447	0.102	0.218	0.284	58.576	0.339	2.048

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	97	103	206	1942	0	195	110
N.S.	1	1.00	1.09	1.16	2.31	21.82	0.00	2.19	1.24
time (sec)	N/A	0.064	0.587	0.183	0.319	0.316	0.000	0.304	2.144

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	110	235	1148	0	195	0
N.S.	1	1.00	0.87	1.16	2.47	12.08	0.00	2.05	0.00
time (sec)	N/A	0.108	2.130	0.254	0.217	0.356	0.000	0.359	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	120	976	3725	0	336	0
N.S.	1	1.00	0.93	1.01	8.20	31.30	0.00	2.82	0.00
time (sec)	N/A	0.140	1.851	0.272	0.434	0.337	0.000	0.369	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	93	131	402	3468	0	323	0
N.S.	1	1.00	0.75	1.06	3.24	27.97	0.00	2.60	0.00
time (sec)	N/A	0.143	1.023	0.332	0.223	0.451	0.000	0.395	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	139	142	2345	8482	0	281	0
N.S.	1	1.00	0.87	0.89	14.75	53.35	0.00	1.77	0.00
time (sec)	N/A	0.208	1.637	0.344	0.677	0.379	0.000	0.410	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	144	157	3354	7528	0	408	2669
N.S.	1	1.00	1.00	1.09	23.29	52.28	0.00	2.83	18.53
time (sec)	N/A	0.151	1.341	0.304	1.236	0.395	0.000	0.524	2.553

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	91	127	376	2584	0	245	416
N.S.	1	1.00	0.83	1.17	3.45	23.71	0.00	2.25	3.82
time (sec)	N/A	0.130	1.220	0.154	0.238	0.304	0.000	0.472	0.745

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	135	148	2432	7757	0	384	2574
N.S.	1	1.00	0.99	1.08	17.75	56.62	0.00	2.80	18.79
time (sec)	N/A	0.144	1.270	0.299	0.947	0.399	0.000	0.460	2.373

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	115	384	2611	0	245	397
N.S.	1	1.00	0.82	1.17	3.92	26.64	0.00	2.50	4.05
time (sec)	N/A	0.105	0.514	0.168	0.235	0.310	0.000	0.440	2.454

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	137	152	1472	7791	0	388	255
N.S.	1	1.00	1.00	1.11	10.74	56.87	0.00	2.83	1.86
time (sec)	N/A	0.123	1.257	0.302	0.599	0.374	0.000	0.424	3.907

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	378	2554	0	245	235
N.S.	1	1.00	0.82	1.23	4.02	27.17	0.00	2.61	2.50
time (sec)	N/A	0.082	0.602	0.146	0.242	0.315	0.000	0.393	2.754

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	147	156	507	7496	0	409	260
N.S.	1	1.00	1.04	1.10	3.57	52.79	0.00	2.88	1.83
time (sec)	N/A	0.119	0.345	0.264	0.380	0.366	0.000	0.331	0.809

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	117	161	498	4800	0	295	0
N.S.	1	1.00	0.85	1.17	3.61	34.78	0.00	2.14	0.00
time (sec)	N/A	0.153	1.773	0.526	0.229	0.518	0.000	0.438	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	166	167	1944	11865	0	437	0
N.S.	1	1.00	0.93	0.94	10.92	66.66	0.00	2.46	0.00
time (sec)	N/A	0.214	6.803	0.417	0.651	0.443	0.000	0.462	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	138	184	770	10720	0	474	0
N.S.	1	1.00	0.81	1.08	4.50	62.69	0.00	2.77	0.00
time (sec)	N/A	0.200	2.083	0.688	0.266	0.706	0.000	0.538	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	194	189	4285	22038	0	493	0
N.S.	1	1.00	0.85	0.83	18.79	96.66	0.00	2.16	0.00
time (sec)	N/A	0.278	3.911	0.527	1.076	0.554	0.000	0.562	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	203	219	925	20020	0	750	3685
N.S.	1	1.00	1.01	1.09	4.60	99.60	0.00	3.73	18.33
time (sec)	N/A	0.205	0.684	0.447	0.499	0.513	0.000	0.363	1.258

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	14	4	5	8	0	5	3
N.S.	1	1.00	4.67	1.33	1.67	2.67	0.00	1.67	1.00
time (sec)	N/A	0.014	0.010	0.136	0.338	0.238	0.000	0.259	1.773

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	5	96	0	1	14
N.S.	1	1.00	1.00	0.94	0.31	6.00	0.00	0.06	0.88
time (sec)	N/A	0.019	0.009	0.086	0.313	0.255	0.000	0.276	0.222

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	28	140	0	45	20
N.S.	1	1.00	1.05	0.95	1.27	6.36	0.00	2.05	0.91
time (sec)	N/A	0.016	0.023	0.062	0.312	0.258	0.000	0.269	0.086

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	28	32	385	0	41	27
N.S.	1	1.00	0.66	0.80	0.91	11.00	0.00	1.17	0.77
time (sec)	N/A	0.018	0.023	0.083	0.314	0.303	0.000	0.281	1.819

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	0	11	2
N.S.	1	1.00	1.00	1.27	1.00	0.18	0.00	1.00	0.18
time (sec)	N/A	0.016	0.037	0.078	0.283	0.262	0.000	0.267	0.132

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	25	36	12	21	14
N.S.	1	1.00	1.00	0.92	1.92	2.77	0.92	1.62	1.08
time (sec)	N/A	0.017	0.035	0.044	0.279	0.251	0.224	0.267	1.848

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	288	0	4529	126	980	119
N.S.	1	1.00	0.98	3.31	0.00	52.06	1.45	11.26	1.37
time (sec)	N/A	0.126	0.579	0.175	0.000	0.493	2.355	1.132	9.060

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	580	337	0	9360	0	938	0
N.S.	1	1.00	4.79	2.79	0.00	77.36	0.00	7.75	0.00
time (sec)	N/A	0.140	6.212	0.104	0.000	0.577	0.000	0.986	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	253	0	2329	94	630	66
N.S.	1	1.00	0.95	4.02	0.00	36.97	1.49	10.00	1.05
time (sec)	N/A	0.088	0.189	0.087	0.000	0.343	1.587	0.769	3.682

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	193	276	0	4825	0	554	0
N.S.	1	1.00	2.27	3.25	0.00	56.76	0.00	6.52	0.00
time (sec)	N/A	0.092	3.739	0.088	0.000	0.402	0.000	0.737	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	238	0	1543	68	349	51
N.S.	1	1.00	1.00	5.41	0.00	35.07	1.55	7.93	1.16
time (sec)	N/A	0.060	0.038	0.072	0.000	0.298	0.985	0.565	2.227

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	81	238	0	3443	0	253	0
N.S.	1	1.00	1.35	3.97	0.00	57.38	0.00	4.22	0.00
time (sec)	N/A	0.036	0.370	0.109	0.000	0.357	0.000	0.494	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	3467	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	61.91	0.00	0.00	0.00
time (sec)	N/A	0.088	0.038	0.000	0.000	0.358	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	0	1539	0	348	0
N.S.	1	1.00	0.88	0.00	0.00	32.06	0.00	7.25	0.00
time (sec)	N/A	0.066	0.141	0.000	0.000	0.303	0.000	0.557	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	4891	0	557	0
N.S.	1	1.00	1.00	0.00	0.00	58.93	0.00	6.71	0.00
time (sec)	N/A	0.117	0.243	0.000	0.000	0.406	0.000	0.717	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	161	0	0	2355	0	629	0
N.S.	1	1.00	2.06	0.00	0.00	30.19	0.00	8.06	0.00
time (sec)	N/A	0.101	2.162	0.000	0.000	0.354	0.000	0.831	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	111	0	0	9642	0	947	0
N.S.	1	1.00	0.92	0.00	0.00	79.69	0.00	7.83	0.00
time (sec)	N/A	0.155	0.737	0.000	0.000	0.573	0.000	1.008	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	488	0	4941	223	1063	112
N.S.	1	1.00	1.05	5.95	0.00	60.26	2.72	12.96	1.37
time (sec)	N/A	0.109	0.472	0.079	0.000	0.493	10.257	1.457	9.969

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	584	529	0	10046	0	949	0
N.S.	1	1.00	4.75	4.30	0.00	81.67	0.00	7.72	0.00
time (sec)	N/A	0.175	6.252	0.083	0.000	0.587	0.000	1.207	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	473	0	2385	165	662	64
N.S.	1	1.00	0.94	7.51	0.00	37.86	2.62	10.51	1.02
time (sec)	N/A	0.079	0.190	0.058	0.000	0.372	6.974	1.018	4.103

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	110	473	0	4841	0	584	0
N.S.	1	1.00	1.25	5.38	0.00	55.01	0.00	6.64	0.00
time (sec)	N/A	0.065	0.729	0.101	0.000	0.428	0.000	0.806	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	4039	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	56.89	0.00	0.00	0.00
time (sec)	N/A	0.103	0.081	0.000	0.000	0.416	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	197	0	0	3913	0	430	0
N.S.	1	1.00	2.56	0.00	0.00	50.82	0.00	5.58	0.00
time (sec)	N/A	0.092	3.360	0.000	0.000	0.403	0.000	0.957	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	51	97	0	679	0	104	68
N.S.	1	1.00	1.65	3.13	0.00	21.90	0.00	3.35	2.19
time (sec)	N/A	0.020	0.096	0.194	0.000	0.268	0.000	0.279	0.194

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	53	142	0	226	0	104	43
N.S.	1	1.00	1.18	3.16	0.00	5.02	0.00	2.31	0.96
time (sec)	N/A	0.026	0.057	0.160	0.000	0.245	0.000	0.290	1.938

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	74	158	0	1027	0	202	78
N.S.	1	1.00	1.48	3.16	0.00	20.54	0.00	4.04	1.56
time (sec)	N/A	0.030	0.180	0.081	0.000	0.272	0.000	0.278	0.255

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	76	211	0	361	0	204	0
N.S.	1	1.00	1.13	3.15	0.00	5.39	0.00	3.04	0.00
time (sec)	N/A	0.033	0.092	0.075	0.000	0.263	0.000	0.281	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	68	164	0	2827	0	592	65
N.S.	1	1.00	0.97	2.34	0.00	40.39	0.00	8.46	0.93
time (sec)	N/A	0.105	0.573	0.117	0.000	0.426	0.000	0.667	2.676

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	208	178	0	5494	0	559	0
N.S.	1	1.00	2.36	2.02	0.00	62.43	0.00	6.35	0.00
time (sec)	N/A	0.089	5.081	0.113	0.000	0.484	0.000	0.567	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	129	0	1625	0	345	39
N.S.	1	1.00	1.00	2.74	0.00	34.57	0.00	7.34	0.83
time (sec)	N/A	0.079	0.127	0.091	0.000	0.382	0.000	0.492	2.208

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	101	137	0	3361	0	252	0
N.S.	1	1.00	1.68	2.28	0.00	56.02	0.00	4.20	0.00
time (sec)	N/A	0.067	0.448	0.100	0.000	0.439	0.000	0.523	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	114	0	1361	68	188	23
N.S.	1	1.00	1.00	3.93	0.00	46.93	2.34	6.48	0.79
time (sec)	N/A	0.051	0.021	0.129	0.000	0.307	1.044	0.408	2.182

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	114	0	1287	0	188	25
N.S.	1	1.00	1.00	3.68	0.00	41.52	0.00	6.06	0.81
time (sec)	N/A	0.024	0.029	0.115	0.000	0.301	0.000	0.391	2.122

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	3527	0	254	0
N.S.	1	1.00	1.00	0.00	0.00	62.98	0.00	4.54	0.00
time (sec)	N/A	0.082	0.056	0.000	0.000	0.382	0.000	0.515	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	130	0	0	1565	0	343	0
N.S.	1	1.00	2.55	0.00	0.00	30.69	0.00	6.73	0.00
time (sec)	N/A	0.075	0.854	0.000	0.000	0.321	0.000	0.484	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	107	0	0	5711	0	565	0
N.S.	1	1.00	1.22	0.00	0.00	64.90	0.00	6.42	0.00
time (sec)	N/A	0.125	0.465	0.000	0.000	0.494	0.000	0.615	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	322	0	3991	0	460	70
N.S.	1	1.00	0.93	4.47	0.00	55.43	0.00	6.39	0.97
time (sec)	N/A	0.125	0.130	0.101	0.000	0.493	0.000	0.571	3.042

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	188	328	0	6973	0	371	0
N.S.	1	1.00	2.24	3.90	0.00	83.01	0.00	4.42	0.00
time (sec)	N/A	0.093	2.971	0.096	0.000	0.540	0.000	0.566	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	287	0	2525	0	287	45
N.S.	1	1.00	1.00	5.52	0.00	48.56	0.00	5.52	0.87
time (sec)	N/A	0.095	0.147	0.079	0.000	0.348	0.000	0.443	2.778

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	112	289	0	2281	0	293	0
N.S.	1	1.00	2.11	5.45	0.00	43.04	0.00	5.53	0.00
time (sec)	N/A	0.078	1.690	0.078	0.000	0.351	0.000	0.458	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	273	0	2277	92	292	41
N.S.	1	1.00	0.84	5.57	0.00	46.47	1.88	5.96	0.84
time (sec)	N/A	0.065	0.041	0.068	0.000	0.356	10.708	0.449	2.509

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	223	272	0	2509	0	288	0
N.S.	1	1.00	3.98	4.86	0.00	44.80	0.00	5.14	0.00
time (sec)	N/A	0.033	4.778	0.090	0.000	0.357	0.000	0.438	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	6955	0	372	0
N.S.	1	1.00	0.90	0.00	0.00	89.17	0.00	4.77	0.00
time (sec)	N/A	0.110	0.098	0.000	0.000	0.543	0.000	0.512	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	263	0	0	3929	0	459	0
N.S.	1	1.00	3.09	0.00	0.00	46.22	0.00	5.40	0.00
time (sec)	N/A	0.117	8.093	0.000	0.000	0.492	0.000	0.569	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	231	549	0	19265	0	805	0
N.S.	1	1.00	1.96	4.65	0.00	163.26	0.00	6.82	0.00
time (sec)	N/A	0.162	2.145	0.110	0.000	1.230	0.000	0.637	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	469	0	7033	0	711	92
N.S.	1	1.00	0.81	5.58	0.00	83.73	0.00	8.46	1.10
time (sec)	N/A	0.141	0.125	0.098	0.000	0.675	0.000	0.526	4.406

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	132	491	0	5719	0	684	0
N.S.	1	1.00	1.47	5.46	0.00	63.54	0.00	7.60	0.00
time (sec)	N/A	0.106	2.840	0.103	0.000	0.635	0.000	0.527	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	435	0	6621	0	725	82
N.S.	1	1.00	0.85	5.88	0.00	89.47	0.00	9.80	1.11
time (sec)	N/A	0.104	0.096	0.085	0.000	0.665	0.000	0.533	4.162

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	216	454	0	6507	0	728	0
N.S.	1	1.00	2.45	5.16	0.00	73.94	0.00	8.27	0.00
time (sec)	N/A	0.097	7.114	0.077	0.000	0.664	0.000	0.524	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	43	420	0	5779	114	683	76
N.S.	1	1.00	0.61	6.00	0.00	82.56	1.63	9.76	1.09
time (sec)	N/A	0.082	0.052	0.054	0.000	0.637	12.557	0.510	4.280

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	976	420	0	6933	0	714	0
N.S.	1	1.00	10.49	4.52	0.00	74.55	0.00	7.68	0.00
time (sec)	N/A	0.068	7.819	0.098	0.000	0.681	0.000	0.524	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	19305	0	808	0
N.S.	1	1.00	0.68	0.00	0.00	178.75	0.00	7.48	0.00
time (sec)	N/A	0.162	0.101	0.000	0.000	1.280	0.000	0.650	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	1375	0	0	10671	0	898	0
N.S.	1	1.00	10.50	0.00	0.00	81.46	0.00	6.85	0.00
time (sec)	N/A	0.180	8.660	0.000	0.000	1.342	0.000	0.744	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	35	62	0	543	0	58	63
N.S.	1	1.00	1.40	2.48	0.00	21.72	0.00	2.32	2.52
time (sec)	N/A	0.013	0.027	0.169	0.000	0.259	0.000	0.274	0.185

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	66	0	175	0	58	22
N.S.	1	1.00	1.37	2.44	0.00	6.48	0.00	2.15	0.81
time (sec)	N/A	0.016	0.025	0.167	0.000	0.250	0.000	0.282	1.867

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	112	95	91	194	2074	100	142	91
N.S.	1	1.26	1.07	1.02	2.18	23.30	1.12	1.60	1.02
time (sec)	N/A	0.058	0.750	0.088	0.273	0.289	0.147	0.298	1.869

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	41	73	95	102	25	38
N.S.	1	1.00	1.05	1.08	1.92	2.50	2.68	0.66	1.00
time (sec)	N/A	0.051	0.085	0.084	0.285	0.265	0.254	0.272	0.107

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	166	620	0	11528	0	0	0
N.S.	1	1.00	1.34	5.00	0.00	92.97	0.00	0.00	0.00
time (sec)	N/A	0.185	4.547	0.898	0.000	0.547	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	116	0	5136	0	0	0
N.S.	1	1.00	0.97	1.30	0.00	57.71	0.00	0.00	0.00
time (sec)	N/A	0.101	0.078	0.714	0.000	0.435	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	1286	0	0	0
N.S.	1	1.00	1.00	0.92	0.00	32.15	0.00	0.00	0.00
time (sec)	N/A	0.058	0.022	0.562	0.000	0.406	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	431	0	3914	0	0	0
N.S.	1	1.00	0.99	5.82	0.00	52.89	0.00	0.00	0.00
time (sec)	N/A	0.093	0.543	0.681	0.000	0.482	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	113	637	0	16463	0	0	0
N.S.	1	1.00	0.96	5.40	0.00	139.52	0.00	0.00	0.00
time (sec)	N/A	0.156	0.877	0.711	0.000	2.040	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [258] had the largest ratio of [.6250000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	21	0.238
2	A	3	2	1.00	21	0.095
3	A	4	4	1.00	21	0.190
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	3	2	1.00	21	0.095
7	A	4	4	1.00	21	0.190
8	A	3	2	1.00	21	0.095
9	A	6	5	1.00	23	0.217
10	A	3	2	1.00	23	0.087
11	A	5	5	1.00	23	0.217
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	21	0.143
14	A	3	2	1.00	23	0.087
15	A	5	5	1.00	23	0.217
16	A	3	2	1.00	23	0.087
17	A	6	5	1.00	23	0.217
18	A	3	2	1.00	23	0.087
19	A	6	5	1.14	23	0.217
20	A	3	2	1.00	21	0.095
21	A	4	3	1.00	21	0.143
22	A	3	2	1.00	23	0.087
23	A	6	5	1.00	23	0.217
24	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	6	6	1.00	23	0.261
26	A	4	4	1.00	23	0.174
27	A	5	5	1.00	23	0.217
28	A	3	3	1.00	21	0.143
29	A	4	4	1.00	21	0.190
30	A	3	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	4	4	1.00	23	0.174
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	23	0.174
35	A	6	6	1.00	23	0.261
36	A	4	4	1.00	21	0.190
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	23	0.174
39	A	6	6	1.00	23	0.261
40	A	5	4	1.00	23	0.174
41	A	8	6	1.00	23	0.261
42	A	6	5	1.00	23	0.217
43	A	7	6	1.00	23	0.261
44	A	5	4	1.00	21	0.190
45	A	6	6	1.00	21	0.286
46	A	5	4	1.00	23	0.174
47	A	7	6	1.00	23	0.261
48	A	6	5	1.00	23	0.217
49	A	8	5	1.00	21	0.238
50	A	9	6	1.00	21	0.286
51	A	7	5	1.00	21	0.238
52	A	7	6	1.00	19	0.316
53	A	5	3	1.00	19	0.158
54	A	3	2	1.00	21	0.095
55	A	6	3	1.00	21	0.143
56	A	3	2	1.00	21	0.095
57	A	8	5	1.21	23	0.217
58	A	12	8	1.00	23	0.348
59	A	7	5	1.23	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	10	8	1.00	21	0.381
61	A	8	5	1.00	21	0.238
62	A	3	2	1.00	23	0.087
63	A	9	5	1.00	23	0.217
64	A	3	2	1.00	23	0.087
65	A	8	5	1.11	23	0.217
66	A	20	8	1.00	23	0.348
67	A	7	5	1.10	23	0.217
68	A	17	8	1.00	21	0.381
69	A	13	5	1.00	21	0.238
70	A	3	2	1.00	23	0.087
71	A	14	6	1.00	23	0.261
72	A	3	2	1.00	23	0.087
73	A	11	10	1.00	23	0.435
74	N/A	0	0	1.00	23	0.000
75	A	11	10	1.00	23	0.435
76	N/A	0	0	1.00	21	0.000
77	N/A	0	0	1.00	21	0.000
78	A	8	8	1.00	23	0.348
79	N/A	0	0	1.00	23	0.000
80	A	12	11	1.00	23	0.478
81	A	4	4	1.00	21	0.190
82	A	2	1	1.00	21	0.048
83	A	3	3	1.00	21	0.143
84	A	3	3	1.00	19	0.158
85	A	3	3	1.00	19	0.158
86	A	2	1	1.00	21	0.048
87	A	4	4	1.00	21	0.190
88	A	3	2	1.00	21	0.095
89	A	4	4	1.00	23	0.174
90	A	4	3	1.00	23	0.130
91	A	5	4	1.00	23	0.174
92	A	5	4	1.00	21	0.190
93	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	3	2	1.00	23	0.087
95	A	5	5	1.00	23	0.217
96	A	3	2	1.00	23	0.087
97	A	6	5	1.00	23	0.217
98	A	5	4	1.00	23	0.174
99	A	5	4	1.00	23	0.174
100	A	6	5	1.00	21	0.238
101	A	5	5	1.00	21	0.238
102	A	3	2	1.00	23	0.087
103	A	6	6	1.00	23	0.261
104	A	3	2	1.00	23	0.087
105	A	6	6	1.00	23	0.261
106	A	4	3	1.00	23	0.130
107	A	5	5	1.00	23	0.217
108	A	3	3	1.00	21	0.143
109	A	2	2	1.00	21	0.095
110	A	2	2	1.00	23	0.087
111	A	4	4	1.00	23	0.174
112	A	3	3	1.00	23	0.130
113	A	5	5	1.00	23	0.217
114	A	4	3	1.00	23	0.130
115	A	5	4	1.00	23	0.174
116	A	6	6	1.00	23	0.261
117	A	5	4	1.00	21	0.190
118	A	3	3	1.00	21	0.143
119	A	3	3	1.00	23	0.130
120	A	3	3	1.00	23	0.130
121	A	3	3	1.00	23	0.130
122	A	5	5	1.00	23	0.217
123	A	5	4	1.00	23	0.174
124	A	6	6	1.00	23	0.261
125	A	7	6	1.00	23	0.261
126	A	6	5	1.00	21	0.238
127	A	4	4	1.00	21	0.190
128	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	4	4	1.00	23	0.174
130	A	4	4	1.00	23	0.174
131	A	4	3	1.00	23	0.130
132	A	4	4	1.00	23	0.174
133	A	6	6	1.00	23	0.261
134	A	4	3	1.00	21	0.143
135	A	3	3	1.00	21	0.143
136	A	3	3	1.00	21	0.143
137	A	2	2	1.00	19	0.105
138	A	3	2	1.00	12	0.167
139	A	3	2	1.00	19	0.105
140	A	2	2	1.00	21	0.095
141	A	3	3	1.00	21	0.143
142	A	4	4	1.00	21	0.190
143	A	4	4	1.00	21	0.190
144	A	4	3	1.00	23	0.130
145	A	4	3	1.00	23	0.130
146	A	4	3	1.00	23	0.130
147	A	4	3	1.00	21	0.143
148	A	4	3	1.00	14	0.214
149	A	4	3	1.00	21	0.143
150	A	4	3	1.00	23	0.130
151	A	4	3	1.00	23	0.130
152	A	4	3	1.00	23	0.130
153	A	4	3	1.00	23	0.130
154	A	4	3	1.00	23	0.130
155	A	4	3	1.00	23	0.130
156	A	4	3	1.00	23	0.130
157	A	4	3	1.00	23	0.130
158	A	4	3	1.00	23	0.130
159	A	4	3	1.00	21	0.143
160	A	4	3	1.00	14	0.214
161	A	4	3	1.00	21	0.143
162	A	4	3	1.00	23	0.130
163	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
164	A	4	3	1.00	23	0.130
165	A	4	3	1.00	23	0.130
166	A	4	3	1.00	23	0.130
167	A	4	3	1.00	23	0.130
168	A	4	3	1.00	14	0.214
169	A	4	3	1.00	14	0.214
170	A	4	3	1.00	23	0.130
171	A	5	5	1.00	23	0.217
172	A	4	3	1.00	23	0.130
173	A	4	4	1.00	23	0.174
174	A	5	4	1.00	21	0.190
175	A	3	3	1.00	14	0.214
176	A	4	3	1.00	21	0.143
177	A	5	5	1.00	23	0.217
178	A	4	3	1.00	23	0.130
179	A	6	6	1.00	23	0.261
180	A	4	3	1.00	23	0.130
181	A	5	5	1.00	23	0.217
182	A	4	3	1.00	23	0.130
183	A	5	5	1.00	23	0.217
184	A	4	3	1.00	21	0.143
185	A	5	5	1.00	14	0.357
186	A	4	3	1.00	21	0.143
187	A	6	6	1.00	23	0.261
188	A	4	3	1.00	23	0.130
189	A	7	6	1.00	23	0.261
190	A	6	6	1.00	23	0.261
191	A	4	3	1.00	23	0.130
192	A	6	6	1.00	23	0.261
193	A	4	3	1.00	23	0.130
194	A	6	6	1.00	23	0.261
195	A	4	3	1.00	21	0.143
196	A	6	6	1.00	14	0.429
197	A	4	3	1.00	21	0.143
198	A	7	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
199	A	4	3	1.00	23	0.130
200	A	8	7	1.00	23	0.304
201	A	7	6	1.00	14	0.429
202	A	3	3	1.00	12	0.250
203	A	4	4	1.00	10	0.400
204	A	4	4	1.00	12	0.333
205	A	5	5	1.00	10	0.500
206	A	3	3	1.00	12	0.250
207	A	3	3	1.00	10	0.300
208	A	7	6	1.00	17	0.353
209	A	8	7	1.00	17	0.412
210	A	6	6	1.00	17	0.353
211	A	7	6	1.00	17	0.353
212	A	5	5	1.00	15	0.333
213	A	6	5	1.00	12	0.417
214	A	7	5	1.00	15	0.333
215	A	5	5	1.00	17	0.294
216	A	8	6	1.00	17	0.353
217	A	6	6	1.00	17	0.353
218	A	9	7	1.00	17	0.412
219	A	7	6	1.00	17	0.353
220	A	8	7	1.00	17	0.412
221	A	6	5	1.00	15	0.333
222	A	7	6	1.00	12	0.500
223	A	8	6	1.00	15	0.400
224	A	7	6	1.00	17	0.353
225	A	5	5	1.00	10	0.500
226	A	6	5	1.00	12	0.417
227	A	6	6	1.00	10	0.600
228	A	7	6	1.00	12	0.500
229	A	6	5	1.00	17	0.294
230	A	7	6	1.00	17	0.353
231	A	5	5	1.00	17	0.294
232	A	6	5	1.00	17	0.294
233	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
234	A	3	3	1.00	12	0.250
235	A	7	5	1.00	15	0.333
236	A	5	5	1.00	17	0.294
237	A	8	6	1.00	17	0.353
238	A	6	5	1.00	17	0.294
239	A	7	6	1.00	17	0.353
240	A	5	5	1.00	17	0.294
241	A	4	4	1.00	17	0.235
242	A	5	5	1.00	15	0.333
243	A	4	4	1.00	12	0.333
244	A	8	6	1.00	15	0.400
245	A	6	6	1.00	17	0.353
246	A	8	7	1.00	17	0.412
247	A	6	5	1.00	17	0.294
248	A	6	6	1.00	17	0.353
249	A	6	6	1.00	17	0.353
250	A	6	6	1.00	17	0.353
251	A	6	5	1.00	15	0.333
252	A	6	6	1.00	12	0.500
253	A	9	7	1.00	15	0.467
254	A	7	7	1.00	17	0.412
255	A	3	3	1.00	10	0.300
256	A	3	3	1.00	12	0.250
257	A	6	4	1.26	14	0.286
258	A	6	5	1.00	8	0.625
259	A	9	8	1.00	15	0.533
260	A	8	7	1.00	15	0.467
261	A	4	4	1.00	15	0.267
262	A	6	6	1.00	15	0.400
263	A	7	7	1.00	15	0.467

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3}{8}(a+5b)x - \frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{b\tanh(c+dx)}{d}$$

```
[Out] 3/8*(a+5*b)*x-1/8*(5*a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)
^3*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3744, 466, 1171, 396, 212}

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d} - \frac{(5a+9b)\sinh(c+dx)\cosh(c+dx)}{8d} + \frac{3}{8}x(a+5b) - \frac{b\tanh(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (3*(a + 5*b)*x)/8 - ((5*a + 9*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a +
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) - (b*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-b-4(a+b)x^2-4bx^4}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= -\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{-3a-7b-8bx^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= -\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{b\tanh(c+dx)}{d} + \frac{(3(a+5b))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= \frac{3}{8}(a+5b)x - \frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{b\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{12(a+5b)(c+dx) - 8(a+2b)\sinh(2(c+dx)) + (a+b)\sinh(4(c+dx)) - 32b\tanh(c+dx)}{32d}$$

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
[Out] (12*(a + 5*b)*(c + d*x) - 8*(a + 2*b)*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c
 + d*x)] - 32*b*Tanh[c + d*x])/(32*d)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{15bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}a}{8d}-\frac{e^{2dx+2c}b}{4d}+\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{4d}-\frac{e^{-4dx-4c}a}{64d}-$ e

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-
15/8*tanh(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\sinh(dx+c)^5+\left(10\,(a+b)\cosh(dx+c)^2-7\,a-15\,b\right)\sinh(dx+c)^3+8\,(3\,(a+5\,b)dx+8\,b)\cos}{64\,d\cosh(dx}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/64*((a + b)*sinh(d*x + c)^5 + (10*(a + b)*cosh(d*x + c)^2 - 7*a - 15*b)*s
inh(d*x + c)^3 + 8*(3*(a + 5*b)*d*x + 8*b)*cosh(d*x + c) + (5*(a + b)*cosh(
d*x + c)^4 - 3*(7*a + 15*b)*cosh(d*x + c)^2 - 8*a - 80*b)*sinh(d*x + c))/(d
*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\sinh^4(c+dx)\,dx$$

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(67) = 134.

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.11

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right) + \frac{1}{64}\,b\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15\,e^{(-2\,dx-2\,c)}+144\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}\right)$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*
(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.95

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{24\,(dx+c)(a+5\,b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-16\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4\,c}\right.}{64\,d}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*(a + 5*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*
e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 90*b*e^(4*
d*x + 4*c) - 8*a*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) + a + b)*e^(-4*d*x
- 4*c) + 128*b/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)+\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)}{64\,d} + \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a+2\,b)}{8\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+2\,b)}{8\,d}$$

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 + (15*b)/8) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 4*c - 4*
d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x)*(a + b))/(64*d) + (exp(- 2*c - 2*d
*x)*(a + 2*b))/(8*d) - (exp(2*c + 2*d*x)*(a + 2*b))/(8*d)
```

3.2 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+2b)\cosh(c+dx)}{d}+\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] -(a+2*b)*cosh(d*x+c)/d+1/3*(a+b)*cosh(d*x+c)^3/d-b*sech(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3745, 459}

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{(a+2b)\cosh(c+dx)}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cosh[c + d*x])/d) + ((a + b)*Cosh[c + d*x]^3)/(3*d) - (b*Sech[
c + d*x])/d
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)}{x^4}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-b+\frac{-a-b}{x^4}+\frac{a+2b}{x^2}\right)\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{(a+2b)\cosh(c+dx)}{d}+\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{3a\cosh(c+dx)}{4d}-\frac{7b\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\cosh(3(c+dx))}{12d}-\frac{b\text{sech}(c+dx)}{d}$$

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) - (7*b*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*x
)])/(12*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (b*Sech[c + d*x])/d
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)}{d}$
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3e^{dx+c}a}{8d}-\frac{7e^{dx+c}b}{8d}-\frac{3e^{-dx-c}a}{8d}-\frac{7e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}+\frac{e^{-3dx-3c}b}{24d}-\frac{2}{d(e^{2}}$

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^4/cosh(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a+b)\cosh(dx+c)^4+(a+b)\sinh(dx+c)^4-4(2a+5b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2\right.}{24\,d\cosh(dx+c)}$$

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] 1/24*((a + b)*cosh(d*x + c)^4 + (a + b)*sinh(d*x + c)^4 - 4*(2*a + 5*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 4*a - 10*b)*sinh(d*x + c)^2 - 9*a - 45*b)/(d*cosh(d*x + c))

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2(c+dx)\right)\sinh^3(c+dx)\,dx$$

[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)

[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**3, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(45) = 90.

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{1}{24}\,b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right) + \frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69
*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)-24\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{}{e^{(dx+c)}}}{24\,d}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 + b*(e^(d*x + c) + e^(-d*x - c))^3 -
 12*a*(e^(d*x + c) + e^(-d*x - c)) - 24*b*(e^(d*x + c) + e^(-d*x - c)) - 48
*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\mathrm{e}^{-3\,c-3\,d\,x}\,(a+b)}{24\,d}+\frac{\mathrm{e}^{3\,c+3\,d\,x}\,(a+b)}{24\,d}-\frac{\mathrm{e}^{c+d\,x}\,(3\,a+7\,b)}{8\,d}-\frac{\mathrm{e}^{-c-d\,x}\,(3\,a+7\,b)}{8\,d}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (exp(3*c + 3*d*x)*(a + b))/(24*d) - (
exp(c + d*x)*(3*a + 7*b))/(8*d) - (exp(- c - d*x)*(3*a + 7*b))/(8*d) - (2*b
*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.3 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{1}{2}(a+3b)x+\frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] -1/2*(a+3*b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d+b*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3744, 466, 396, 212}

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{1}{2}x(a+3b)+\frac{b\tanh(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((a + 3*b)*x) + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b*Tanh[
c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b+2bx^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}\\
&= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d}\\
&\quad - \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}\\
&= -\frac{1}{2}(a+3b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-2(a+3b)(c+dx)+(a+b)\sinh(2(c+dx))+4b\tanh(c+dx)}{4d}$$

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] (-2*(a + 3*b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)] + 4*b*Tanh[c + d*x])/(4
*d)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
risch	$-\frac{ax}{2}-\frac{3bx}{2}+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-2dx-2c}b}{8d}-\frac{2b}{d\left(e^{2dx+2c}+1\right)}$	89

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^3/cos
h(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\sinh(dx+c)^3-4\left((a+3b)dx+2b\right)\cosh(dx+c)+\left(3(a+b)\cosh(dx+c)^2+a+9b\right)\sinh(dx+}{8d\cosh(dx+c)}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/8*((a + b)*sinh(d*x + c)^3 - 4*((a + 3*b)*d*x + 2*b)*cosh(d*x + c) + (3*(
a + b)*cosh(d*x + c)^2 + a + 9*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \int \left(a+b\tanh^2{(c+dx)}\right)\sinh^2{(c+dx)}\, dx$$

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.

Time $= 0.22$ (sec) , antiderivative size $= 101$, normalized size of antiderivative $= 2.30$

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = -\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{8}\,b\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(12*(d*x + c)
/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(40) = 80$.

Time $= 0.29$ (sec) , antiderivative size $= 107$, normalized size of antiderivative $= 2.43$

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = -\frac{4\,(dx+c)(a+3\,b)-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-\frac{ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-14\,be^{(2\,dx+2\,c)}-a-b}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -1/8*(4*(d*x + c)*(a + 3*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (a*e^
(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 14*b*e^(2*d*x + 2*c) - a - b)/(e^(4*d
*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{e^{2c+2dx}\,(a+b)}{8\,d}-\frac{2\,b}{d\,\left(e^{2c+2dx}+1\right)}-\frac{e^{-2c-2dx}\,(a+b)}{8\,d}-x\left(\frac{a}{2}+\frac{3\,b}{2}\right)$$

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp(
- 2*c - 2*d*x)*(a + b))/(8*d) - x*(a/2 + (3*b)/2)
```

3.4 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] (a+b)*cosh(d*x+c)/d+b*sech(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3745, 14}

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
```

```
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a+b-bx^2}{x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-b+\frac{a+b}{x^2}\right)\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\cosh(c)\cosh(dx)}{d}+\frac{b\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*Cosh[c]*Cosh[d*x])/d + (b*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d + (a*Si
nh[c]*Sinh[d*x])/d
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result	size
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
risch	$\frac{e^{dx+c}a}{2d}+\frac{e^{dx+c}b}{2d}+\frac{e^{-dx-c}a}{2d}+\frac{e^{-dx-c}b}{2d}+\frac{2b\,e^{dx+c}}{d\left(e^{2dx+2c}+1\right)}$	81

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\cosh\left(dx+c\right)^2+(a+b)\sinh\left(dx+c\right)^2+a+3\,b}{2\,d\cosh\left(dx+c\right)}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a + 3*b)/(d*cosh(d
*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}\,b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right) + \frac{a\cosh\left(dx+c\right)}{d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*d
*x - 3*c)))) + a*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)+b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{4b}{e^{(dx+c)}+e^{(-dx-c)}}}{2d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*(e^(d*x + c) + e^(-d*x - c)) + b*(e^(d*x + c) + e^(-d*x - c)) + 4*b/
(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b}{d\cosh(c+d\,x)}+\frac{\cosh(c+d\,x)\,(a+b)}{d}$$

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] b/(d*cosh(c + d*x)) + (cosh(c + d*x)*(a + b))/d
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] -a*arctanh(cosh(d*x+c))/d-b*sech(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3745, 396, 213}

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Sech[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b-bx^2}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{b\text{sech}(c+dx)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{a\text{arctanh}(\cosh(c+dx))}{d} - \frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \text{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\text{sech}(c+dx)}{d}$$

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d - (b*Sec
h[c + d*x])/d
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
default	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
risch	$-\frac{2b\,\mathrm{e}^{dx+c}}{d\left(\mathrm{e}^{2dx+2c}+1\right)}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	56

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))-b/cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 6.42

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\,b\cosh\left(dx+c\right)+\left(a\cosh\left(dx+c\right)^2+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^2+a\right)\log\left(\cosh\left(\right.\right.}{d\cosh}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -(2*b*cosh(d*x + c) + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c)
+ a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d
*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log(co
sh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 +
2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] a*log(tanh(1/2*d*x + 1/2*c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=-\frac{a\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-a\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -1/2*(a*log(e^(d*x + c) + e^(-d*x - c) + 2) - a*log(e^(d*x + c) + e^(-d*x -
 c) - 2) + 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x),x)
[Out] - (2*atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-
d^2)^(1/2) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] -a*coth(d*x+c)/d+b*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3744, 14}

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{b\tanh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
```

```
+ 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{b\tanh(dx+c)-\frac{a}{\tanh(dx+c)}}{d}$	25
default	$\frac{b\tanh(dx+c)-\frac{a}{\tanh(dx+c)}}{d}$	25
risch	$-\frac{2\left(\mathrm{e}^{2dx+2c}a+b\,\mathrm{e}^{2dx+2c}+a-b\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)\left(\mathrm{e}^{2dx+2c}+1\right)}$	59

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*tanh(d*x+c)-a/tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(24) = 48.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{4\,(a\cosh(dx+c)+b\sinh(dx+c))}{d\cosh(dx+c)^3+3\,d\cosh(dx+c)\sinh(dx+c)^2+d\sinh(dx+c)^3-d\cosh(dx+c)+\left(3\,d\cosh(dx+c)^2+d\right)\sinh(dx+c)}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4*(a*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x +
c)^2 + d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\,b}{d(e^{(-2\,dx-2\,c)}+1)} + \frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{2\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right)}{d(e^{(4\,dx+4\,c)}-1)}$$

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] -2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/(d*(e^(4*d*x + 4*c) - 1)
)

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{\frac{2\,(a-b)}{d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$

[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^2,x)

[Out] -((2*(a - b))/d + (2*exp(2*c + 2*d*x)*(a + b))/d)/(exp(4*c + 4*d*x) - 1)

3.7 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-2b)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] 1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d+b*sech(d
*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3745, 466, 396, 213}

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-2b)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d) + (b*Sech[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(-1+x^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= -\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{\text{Subst}\left(\int \frac{-a+2bx^2}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2d}\\
&= -\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{b\text{sech}(c+dx)}{d}-\frac{(a-2b)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2d}\\
&= \frac{(a-2b)\text{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{b\text{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.41

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d + (a*Log[Cosh[(c + d*x)/2]])/(2*d) - (b*Log[
Cosh[(c + d*x)/2]])/d - (a*Log[Sinh[(c + d*x)/2]])/(2*d) + (b*Log[Sinh[(c +
 d*x)/2]])/d - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x])/d
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{4dx+4c}-2b\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a-2b\right)}{d\left(e^{2dx+2c}-1\right)^2\left(e^{2dx+2c}+1\right)}-\frac{a\ln\left(e^{dx+c}-1\right)}{2d}+\frac{\ln\left(e^{dx+c}-1\right)b}{d}+\frac{a\ln(e^{dx+c}+\ldots}{2d}$

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/cosh(d*x+c)-
2*arctanh(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 924, normalized size of antiderivative = 18.12

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - 2*b)*cosh(d*x + c)^5 + 10*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c
)^4 + 2*(a - 2*b)*sinh(d*x + c)^5 + 4*(a + 2*b)*cosh(d*x + c)^3 + 4*(5*(a -
 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d*x
+ c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a - 2*b)*cosh(d*x
+ c) - ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)
^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a - 2*b)*
cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x + c)^3
 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c)^2 + (
15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(
d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*x + c)^3 -
 (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x + c) + sinh
(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sin
h(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*
(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(
d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x
+ c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a +
2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*
x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x +
 c) + sinh(d*x + c) - 1) + 2*(5*(a - 2*b)*cosh(d*x + c)^4 + 6*(a + 2*b)*cos
h(d*x + c)^2 + a - 2*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x +
c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*si
nh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x +
c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 -
d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^3\left(c+dx\right) dx$$

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(47) = 94$.

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.98

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
+ e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(l
og(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2
*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-2\,b\left(e^{(dx+}\right.\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4\,e^{(dx+}}}{4\,d}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x +
 c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b*(e^(d*x
 + c) + e^(-d*x - c))^2 + 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*x +
 c) - 4*e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a\sqrt{-d^2}-2\,b\sqrt{-d^2}\right)}{d\sqrt{a^2-4\,a\,b+4\,b^2}}\right)\sqrt{a^2-4\,a\,b+4\,b^2}}{\sqrt{-d^2}} - \frac{a\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} + \frac{2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{2\,a\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*b
 + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*
x))/(d*(exp(2*c + 2*d*x) - 1)) + (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) +
1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] (a-b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d-b*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3744, 459}

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x])/d
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^2)}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-b+\frac{a}{x^4}+\frac{-a+b}{x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2a\coth(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c +
 d*x]^2)/(3*d) - (b*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	55
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	55
risch	$-\frac{4\left(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+2\,e^{2dx+2c}a-6b\,e^{2dx+2c}-a+3b\right)}{3d\left(e^{2dx+2c}-1\right)^3\left(e^{2dx+2c}+1\right)}$	87

[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.55

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{}{3\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6-2\,d\cosh\left(dx+c\right)^4+\left(15\,d\cos\right.\right.}$$

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] -8/3*((a + 3*b)*cosh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 + a - 3*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^4\left(c+dx\right)dx$$

[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)

[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**4, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.57

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+\frac{4\,b}{d(e^{(-4\,dx-4\,c)}-1)}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1}-\frac{3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}\right)}{3\,d}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c)
 - 6*b*e^(2*d*x + 2*c) - 2*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.93

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\frac{2\,(2\,a-b)}{3\,d}+\frac{2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1} - \frac{2\,b}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{\frac{2\,b}{3\,d}+\frac{2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a-b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}$$

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

```
[Out] (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - ((2*(2*a - b))/(3*d) + (2*b*exp(2*c + 2*
d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (2*b)/(3*d*(exp(
2*c + 2*d*x) - 1)) - ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) + (4*exp(2
*c + 2*d*x)*(2*a - b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) - 1)
```

3.9 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{8}\left(3a^2+30ab+35b^2\right)x - \frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2+10ab+13b^2\right)\tanh(c+dx)}{4d} + \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] 1/8*(3*a^2+30*a*b+35*b^2)*x-1/8*(a+b)*(a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/4
*(a^2+10*a*b+13*b^2)*tanh(d*x+c)/d+1/4*(a+b)^2*sinh(d*x+c)^4*tanh(d*x+c)/d-
1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {3744, 474, 466, 1167, 212}

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a^2+10ab+13b^2)\tanh(c+dx)}{4d} + \frac{1}{8}x\left(3a^2+30ab+35b^2\right) + \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{(a+b)(a+9b)\sinh(c+dx)\cosh(c+dx)}{8d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*x)/8 - ((a + b)*(a + 9*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*d) - ((a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x])/(4*d) + ((a + b)^2*S
inh[c + d*x]^4*Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^2}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{x^4(a^2+10ab+5b^2+4b^2x^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}\\
&= -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-((a+b)(a+9b))-2(a+b)(a+9b)x^2-8b^2x^4}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}\\
&= -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d}\\
&\quad - \frac{\text{Subst}\left(\int \left(2(a^2+10ab+13b^2)+8b^2x^2+\frac{-3a^2-30ab-35b^2}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{8d}\\
&= -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{(a^2+10ab+13b^2)\tanh(c+dx)}{4d}\\
&\quad + \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{b^2\tanh^3(c+dx)}{3d}\\
&\quad + \frac{(3a^2+30ab+35b^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}\\
&= \frac{1}{8}(3a^2+30ab+35b^2)\,x - \frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d}\\
&\quad - \frac{(a^2+10ab+13b^2)\tanh(c+dx)}{4d}\\
&\quad + \frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{b^2\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{12(3a^2+30ab+35b^2)(c+dx)-24(a^2+4ab+3b^2)\sinh(2(c+dx))+3(a+b)^2\sinh(4(c+dx))+32b(-6}{96d}$$

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (12*(3*a^2 + 30*a*b + 35*b^2)*(c + d*x) - 24*(a^2 + 4*a*b + 3*b^2)*Sinh[2*(
c + d*x)] + 3*(a + b)^2*Sinh[4*(c + d*x)] + 32*b*(-6*a - 10*b + b*Sech[c +
d*x]^2)*Tanh[c + d*x])/(96*d)
```

Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
risch	$\frac{3a^2x}{8}+\frac{15abx}{4}+\frac{35b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}ab}{2d}-\frac{3e^{2dx+2c}b^2}{8d}+$

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+1
5/8*c-15/8*tanh(d*x+c))+b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh(d*x+c
)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.34

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^7+3\left(21\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^2-5\,a^2-26\,ab-21\,b^2\right)\sinh\left(dx+c\right)}{}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^7 + 3*(21*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 - 5*a^2 - 26*a*b - 21*b^2)*sinh(d*x + c)^5 + 8*(3*(3*a^2 + 30
*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)^3 + 24*(3*(3*a^2 + 30*a
*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (105*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 30*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x +
 c)^2 - 63*a^2 - 654*a*b - 847*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 + 30*a*b
 + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c) + 3*(7*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^6 - 5*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x + c)^4 - (63*a^2 + 6
54*a*b + 847*b^2)*cosh(d*x + c)^2 - 15*a^2 - 190*a*b - 175*b^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x +
 c))
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh^4\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(108) = 216$.

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.50

$$\begin{aligned}&\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)\\&\quad+\frac{1}{192}b^2\left(\frac{840(dx+c)}{d}+\frac{3\left(24e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{63e^{(-2dx-2c)}+1487e^{(-4dx-4c)}+2517e^{(-6dx-}}{d(e^{(-4dx-4c)}+3e^{(-6dx-6c)}+3e^{(-8dx-}}\right.\\&\quad+\frac{1}{32}ab\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d(e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)\end{aligned}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/192*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x -
 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c)
+ 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) +
```

```
3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 1/32*a*b
*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*
d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(108) = 216$.

Time = 0.40 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.48

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\,a^2e^{(4\,dx+4\,c)}+6\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}-24\,a^2e^{(2\,dx+2\,c)}-96\,abe^{(2\,dx+2\,c)}-72\,b^2e^{(2\,dx+2\,c)}+24\,(3\,a^2+3}{}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*e^(4*d*x + 4*c) + 6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c
) - 24*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) - 72*b^2*e^(2*d*x + 2*c
) + 24*(3*a^2 + 30*a*b + 35*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) + 18
0*a*b*e^(4*d*x + 4*c) + 210*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 3
2*a*b*e^(2*d*x + 2*c) - 24*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d
*x - 4*c) + 256*(3*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2
*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) + 3*a*b + 5*b^2)/(e^(2*d*x + 2*c) + 1)^
3)/d
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.48

$$\begin{aligned}\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\frac{4\,(b^2+a\,b)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,b^2+a\,b)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+x\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)\\ &+\frac{\frac{4\,(2\,b^2+a\,b)}{3\,d}+\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+a\,b)}{3\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(2\,b^2+a\,b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}\\ &+\frac{4\,(2\,b^2+a\,b)}{3\,d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a^2+4\,a\,b+3\,b^2)}{8\,d}\\ &-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a^2+4\,a\,b+3\,b^2)}{8\,d}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)^2}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)^2}{64\,d}\end{aligned}$$

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b + 2*b^2))/(3*d))/(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/
8) + ((4*(a*b + 2*b^2))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d) + (4
*exp(4*c + 4*d*x)*(a*b + 2*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1) + (4*(a*b + 2*b^2))/(3*d*(exp(2*c + 2*d*x) +
1)) + (exp(- 2*c - 2*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(2*c + 2*d*x)*
(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)/(64*d) + (exp
(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

3.10 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}+\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] -(a+b)*(a+3*b)*cosh(d*x+c)/d+1/3*(a+b)^2*cosh(d*x+c)^3/d-b*(2*a+3*b)*sech(d
*x+c)/d+1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3745, 459}

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)*(a + 3*b)*Cosh[c + d*x])/d) + ((a + b)^2*Cosh[c + d*x]^3)/(3*d)
- (b*(2*a + 3*b)*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)^2}{x^4}\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b(2a+3b)-\frac{(a+b)^2}{x^4}+\frac{(a+b)(a+3b)}{x^2}+b^2x^2\right)\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= -\frac{(a+b)(a+3b)\cosh(c+dx)}{d}+\frac{(a+b)^2\cosh^3(c+dx)}{3d}\\
&\quad -\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-3(3a^2+14ab+11b^2)\cosh(c+dx)+(a+b)^2\cosh(3(c+dx))+4b\text{sech}(c+dx)\left(-6a-9b+b\text{sech}^2(c+\right.}{12d}$$

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (-3*(3*a^2 + 14*a*b + 11*b^2)*Cosh[c + d*x] + (a + b)^2*Cosh[3*(c + d*x)] +
 4*b*Sech[c + d*x]*(-6*a - 9*b + b*Sech[c + d*x]^2))/(12*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(73) = 146$.

Time = 1.97 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^{4}}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^{2}}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+b^{2}\left(\frac{\sinh(dx+c)^{6}}{3\cosh(dx+c)^{3}}-\frac{2\sinh(dx+c)^{4}}{\cosh(dx+c)^{3}}-\right.}{d}$
default	$\frac{a^{2}\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^{4}}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^{2}}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+b^{2}\left(\frac{\sinh(dx+c)^{6}}{3\cosh(dx+c)^{3}}-\frac{2\sinh(dx+c)^{4}}{\cosh(dx+c)^{3}}-\right.}{d}$
risch	$\frac{e^{3dx+3c}a^{2}}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^{2}}{24d}-\frac{3\,e^{dx+c}a^{2}}{8d}-\frac{7\,e^{dx+c}ab}{4d}-\frac{11b^{2}e^{dx+c}}{8d}-\frac{3\,e^{-dx-c}a^{2}}{8d}-\frac{7\,e^{-dx-c}ab}{4d}-$

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^4/cosh
(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c))+b^2*(1/3*sinh(d*x+c)
^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8*sinh(d*x+c)^2/cosh(d*x+c)^
3-16/3/cosh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(73) = 146$.

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.36

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)\cosh\left(dx+c\right)^6+(a^2+2\,ab+b^2)\sinh\left(dx+c\right)^6-6\left(a^2+6\,ab+5\,b^2\right)\cosh\left(dx+c\right)^4+3\,(5$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^6 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 - 2*a^2 - 12*a*b - 10*b^2)*sinh(d*x + c)^4 - 3*(11*a^2 + 86*
a*b + 91*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - 11*a^2 - 86*a*b - 91*b^2)*sinh(d
*x + c)^2 - 26*a^2 - 220*a*b - 210*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2 \sinh^3{(c+dx)} \, dx$$

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(73) = 146$.

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.44

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{1}{24} b^2 \left(\frac{33 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{30 e^{(-2dx-2c)} + 240 e^{(-4dx-4c)} + 322 e^{(-6dx-6c)} + 177 e^{(-8dx-8c)} - 1}{d\left(e^{(-3dx-3c)} + 3e^{(-5dx-5c)} + 3e^{(-7dx-7c)} + e^{(-9dx-9c)}\right)}\right) - \frac{1}{12} ab \left(\frac{21 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{20 e^{(-2dx-2c)} + 69 e^{(-4dx-4c)} - 1}{d\left(e^{(-3dx-3c)} + e^{(-5dx-5c)}\right)}\right) + \frac{1}{24} a^2 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9 e^{(dx+c)}}{d} - \frac{9 e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right)$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/24*b^2*((33*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) +
240*e^(-4*d*x - 4*c) + 322*e^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*
(e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9
*c)))) - 1/12*a*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x -
 2*c) + 69*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c))))
 + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3
*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(73) = 146$.

Time = 0.37 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.66

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 + 2ab\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 + b^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 - 12a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) - \cdots}{24d \cdots}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) + e^(-d*x - c
))^3 + b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x -
 c)) - 48*a*b*(e^(d*x + c) + e^(-d*x - c)) - 36*b^2*(e^(d*x + c) + e^(-d*x
- c)) - 16*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 9*b^2*(e^(d*x + c) + e^(
-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.79

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{e^{-3c-3dx}(a+b)^2}{24d} - \frac{e^{c+dx}(3a^2+14ab+11b^2)}{8d} + \frac{e^{3c+3dx}(a+b)^2}{24d} - \frac{e^{-c-dx}(3a^2+14ab+11b^2)}{8d} - \frac{8b^2 e^{c+dx}}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)} - \frac{2e^{c+dx}(3b^2+2ab)}{d\left(e^{2c+2dx}+1\right)} + \frac{8b^2 e^{c+dx}}{3d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(14*a*b + 3*a^2 + 11*
b^2))/(8*d) + (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- c - d*x)*(14*a*b
 + 3*a^2 + 11*b^2))/(8*d) - (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) +
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b + 3*b
^2))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1))
```

3.11 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{1}{2}(a+b)(a+5b)x+\frac{(a+b)(a+5b)\tanh(c+dx)}{2d}+\frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -1/2*(a+b)*(a+5*b)*x+1/2*(a+b)*(a+5*b)*tanh(d*x+c)/d+1/2*(a+b)^2*sinh(d*x+c
)^2*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3744, 474, 470, 327, 212}

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)(a+5b)\tanh(c+dx)}{2d}+\frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d}-\frac{1}{2}x(a+b)(a+5b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)*(a + 5*b)*x) + ((a + b)*(a + 5*b)*Tanh[c + d*x])/(2*d) + ((a
+ b)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x^2(a^2+6ab+3b^2+2b^2x^2)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$
\begin{aligned}
&= \frac{(a+b)^2 \sinh^2(c+dx) \tanh(c+dx)}{2d} + \frac{b^2 \tanh^3(c+dx)}{3d} \\
&\quad - \frac{((a+b)(a+5b))\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2 \sinh^2(c+dx)\tanh(c+dx)}{2d} \\
&\quad + \frac{b^2 \tanh^3(c+dx)}{3d} - \frac{((a+b)(a+5b))\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d} \\
&= -\frac{1}{2}(a+b)(a+5b)x + \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} \\
&\quad + \frac{(a+b)^2 \sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b^2 \tanh^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$
\begin{aligned}
&\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx \\
&= \frac{-6(a^2+6ab+5b^2)(c+dx) + 3(a+b)^2 \sinh(2(c+dx)) + 4b\left(6a+7b-b\,\mathrm{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}
\end{aligned}
$$

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-6*(a^2 + 6*a*b + 5*b^2)*(c + d*x) + 3*(a + b)^2*Sinh[2*(c + d*x)] + 4*b*(
6*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\cdots\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\cdots\right.}{d}$
risch	$-\frac{a^2x}{2}-3abx-\frac{5b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\cdots$

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c)
^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^5/cosh(d
*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.68

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^5-4\left(3\left(a^2+6\,ab+5\,b^2\right)dx+12\,ab+14\,b^2\right)\cosh\left(dx+c\right)^3-12\left(3\left(a^2+6\,a\right.\right.}{}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^5 - 4*(3*(a^2 + 6*a*b + 5*b^2)*d*
x + 12*a*b + 14*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x + 12
*a*b + 14*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + 9*a^2 + 66*a*b + 65*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b
 + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c) + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + (9*a^2 + 66*a*b + 65*b^2)*cosh(d*x + c)^2 + 2*a^2 + 20*a*b
 + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x +
 c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(71) = 142$.

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.75

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= -\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)\\&-\frac{1}{24}\,b^2\left(\frac{60\,(dx+c)}{d}+\frac{3\,e^{(-2\,dx-2\,c)}}{d}-\frac{121\,e^{(-2\,dx-2\,c)}+201\,e^{(-4\,dx-4\,c)}+147\,e^{(-6\,dx-6\,c)}+3}{d(e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)})}\right)\\&-\frac{1}{4}\,ab\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)\end{aligned}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^2*(60*(d*x
 + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*
c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*
d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x -
 4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(71) = 142$.

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.70

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\,a^2e^{(2\,dx+2\,c)}+6\,abe^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}-12\,(a^2+6\,ab+5\,b^2)(dx+c)+3\,\left(2\,a^2e^{(2\,dx+2\,c)}+12\,abe^{(2\,dx}\right.}{}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*e^(2*d*x + 2*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c)
 - 12*(a^2 + 6*a*b + 5*b^2)*(d*x + c) + 3*(2*a^2*e^(2*d*x + 2*c) + 12*a*b*e
^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) - a^2 - 2*a*b - b^2)*e^(-2*d*x - 2*
c) - 16*(6*a*b*e^(4*d*x + 4*c) + 9*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x +
2*c) + 12*b^2*e^(2*d*x + 2*c) + 6*a*b + 7*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.14

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{e^{2\,c+2\,d\,x}\,(a+b)^2}{8\,d}-x\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right) \\ &\quad-\frac{\frac{2\,(3\,b^2+2\,a\,b)}{3\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(3\,b^2+2\,a\,b)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{2\,(3\,b^2+2\,a\,b)}{3\,d\,(e^{2\,c+2\,d\,x}+1)} \\ &\quad-\frac{e^{-2\,c-2\,d\,x}\,(a+b)^2}{8\,d}-\frac{\frac{2\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(3\,b^2+2\,a\,b)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}\end{aligned}$$

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - x*(3*a*b + a^2/2 + (5*b^2)/2) - ((2*(2
*a*b + 3*b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(4*
```

```
c + 4*d*x)*(2*a*b + 3*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1) - (2*(2*a*b + 3*b^2))/(3*d*(exp(2*c + 2*d*x) + 1))
 - (exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - ((2*(2*a*b + b^2))/(3*d) + (2*exp
(2*c + 2*d*x)*(2*a*b + 3*b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x
) + 1)
```

3.12 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] (a+b)^2*cosh(d*x+c)/d+2*b*(a+b)*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3745, 276}

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Cosh[c + d*x])/d + (2*b*(a + b)*Sech[c + d*x])/d - (b^2*Sech[c +
 d*x]^3)/(3*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^2}{x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3(a+b)^2\cosh(c+dx)+b\text{sech}(c+dx)\left(6(a+b)-b\text{sech}^2(c+dx)\right)}{3d}$$

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (3*(a + b)^2*Cosh[c + d*x] + b*Sech[c + d*x]*(6*(a + b) - b*Sech[c + d*x]^2
))/(3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(47) = 94.

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
default	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{b^2e^{dx+c}}{2d}+\frac{e^{-dx-c}a^2}{2d}+\frac{e^{-dx-c}ab}{d}+\frac{e^{-dx-c}b^2}{2d}+\frac{4b\,e^{dx+c}(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+6\,e}{3d(e^{2dx+2c}}$

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(cosh(d*x+c)*a^2+2*a*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+b^2*(s
inh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d*x+c)^3)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.41

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^4+3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^4+12\left(a^2+4\,ab+3\,b^2\right)\cosh\left(dx+c\right)^2}{6\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\right.}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 12*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + 6*(3*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^2 + 2*a^2 + 8*a*b + 6*b^2)*sinh(d*x + c)^2 + 9*a^2 + 42*a*
b + 25*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*co
sh(d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.49

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{6}\,b^2\left(\frac{3\,e^{(-dx-c)}}{d}+\frac{33\,e^{(-2\,dx-2\,c)}+41\,e^{(-4\,dx-4\,c)}+27\,e^{(-6\,dx-6\,c)}+3}{d(e^{(-dx-c)}+3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)$$
$$+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right)+\frac{a^2\cosh\left(dx+c\right)}{d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*(3*e^(-d*x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27
*e^(-6*d*x - 6*c) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x
- 5*c) + e^(-7*d*x - 7*c)))) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) +
1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^2*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{8\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx}\cdots\right.}{\left(e^{(dx+c)}+e^{(-dx-c}\cdots\right.}}{6\,d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) + 6*a*b*(e^(d*x + c) + e^(-d*x - c)
) + 3*b^2*(e^(d*x + c) + e^(-d*x - c)) + 8*(3*a*b*(e^(d*x + c) + e^(-d*x -
c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b^2)/(e^(d*x + c) + e^(-d*
x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.14

$$\int \sinh(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{e^{c+dx}\,(a+b)^2}{2\,d} + \frac{e^{-c-dx}\,(a+b)^2}{2\,d} + \frac{8\,b^2\,e^{c+dx}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} + \frac{4\,e^{c+dx}\,(b^2+a\,b)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{8\,b^2\,e^{c+dx}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a + b)^2)/(2*d) + (8*b^2*
exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d
*x) + 1)) + (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b^
2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] -a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3745, 398, 213}

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Sech[c + d*x])/d + (b^2*Se
ch[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^2}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-b(2a+b)+b^2x^2+\frac{a^2}{-1+x^2}\right)\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{b(2a+b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}+\frac{a^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{a^2\text{arctanh}(\cosh(c+dx))}{d}-\frac{b(2a+b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \text{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3a^2\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)-3b(2a+b)\text{sech}(c+dx)+b^2\text{sech}^3(c+dx)}{3d}$$

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (3*a^2*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]) - 3*b*(2*a + b)*S
ech[c + d*x] + b^2*Sech[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	63
default	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	63
risch	$-\frac{2b\,e^{dx+c}\left(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}+12\,e^{2dx+2c}a+2b\,e^{2dx+2c}+6a+3b\right)}{3d\left(e^{2dx+2c}+1\right)^3}+\frac{a^2\ln\left(e^{dx+c}-1\right)}{d}-\frac{a^2\ln\left(e^{dx+c}+1\right)}{d}$	115

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))-2*a*b/cosh(d*x+c)+b^2*(-sinh(d*x+c)^2/cosh(
d*x+c)^3-2/3/cosh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 890, normalized size of antiderivative = 17.45

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + b^2)*cosh(d*x +
c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + b^2)*sinh(d*x + c)^3 +
 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + b^2)*cosh(d*x + c))*sinh(d*
x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*c
osh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4
+ 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 +
 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2
*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(
a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(a^2*cosh(d*x + c)^6 + 6*a^
2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)
^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^
2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*
a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 +
6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*
x + c)^4 + 2*(6*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d
```

```
*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 +
3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2
+ 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*
cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{csch}\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(49) = 98$.

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.84

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx= -\frac{2}{3}b^2\left(\frac{3\,e^{(-dx-c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right. + \frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^2*log(tanh(1/2*d*x +
1/2*c))/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(49) = 98$.

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.41

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{3\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}+e^{(-d\cdots}\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6\,d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*a^2*log(e^(d*x + c) + e
^(-d*x - c) - 2) + 4*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x
+ c) + e^(-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.14

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} - \frac{2\,\mathrm{e}^{c+d\,x}\,\left(b^2+2\,a\,b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x),x)
```

```
[Out] (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8
*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1))
- (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/
(-d^2)^(1/2)
```

3.14 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -a^2*coth(d*x+c)/d+2*a*b*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 276}

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x]^3)/
(3*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-3a^2\coth(c+dx)+b\left(6a+b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (-3*a^2*Coth[c + d*x] + b*(6*a + b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*d
)
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{-a^{2}\coth(dx+c)+2ab\tanh(dx+c)+b^{2}\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^{3}}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{-a^{2}\coth(dx+c)+2ab\tanh(dx+c)+b^{2}\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^{3}}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
risch	$-\frac{2(3a^{2}e^{6dx+6c}+6ab\,e^{6dx+6c}+3b^{2}e^{6dx+6c}+9a^{2}e^{4dx+4c}+6ab\,e^{4dx+4c}-3\,e^{4dx+4c}b^{2}+9a^{2}e^{2dx+2c}-6ab\,e^{2dx+2c}+e^{2dx+2}}{3d\left(e^{2dx+2c}+1\right)^{3}\left(e^{2dx+2c}-1\right)}$

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*tanh(d*x+c)+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3
+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(44) = 88$.

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 5.74

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(\left(3\,a^2+b^2\right)\cosh\left(dx+c\right)^3+3\left(3\,a^2+b^2\right)\cosh\left(dx+c\right)\sin\right.}{3\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^5+d\cosh\left(dx+c\right)^3+\left(10\,d\cosh\right.\right.}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/3*((3*a^2 + b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + (9*a^2 - b^2)*cosh(d*x + c) +
2*(3*(3*a*b + b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + d*cosh(d
*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^3 + (10*d*cosh(d*x +
 c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)^2\operatorname{csch}^2{(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(44)=88$.

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{2}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{4ab}{d(e^{(-2dx-2c)}+1)}+\frac{2a^2}{d(e^{(-2dx-2c)}-1)}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a^2/(d*(e^(-2*d
*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=-\frac{2\left(\frac{3a^2}{e^{(2dx+2c)}-1}+\frac{6abe^{(4dx+4c)}+3b^2e^{(4dx+4c)}+12abe^{(2dx+2c)}+6ab+b^2}{\left(e^{(2dx+2c)}+1\right)^3}\right)}{3d}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] -2/3*(3*a^2/(e^(2*d*x + 2*c) - 1) + (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x
 + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.54

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2(2ab-b^2)}{3d}+\frac{2e^{2c+2dx}(b^2+2ab)}{3d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{\frac{2(b^2+2ab)}{3d}+\frac{2e^{4c+4dx}(b^2+2ab)}{3d}+\frac{4e^{2c+2dx}(2ab-b^2)}{3d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{2a^2}{d\left(e^{2c+2dx}-1\right)}-\frac{2(b^2+2ab)}{3d\left(e^{2c+2dx}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

```
[Out] - ((2*(2*a*b - b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(2*a*b + b^2))/(3*d) + (2*exp
(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*
d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a
^2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(3*d*(exp(2*c + 2*d*x) +
 1))
```

3.15 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a(a-4b)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a(a-4b)\operatorname{sech}(c+dx)}{2d}-\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] 1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d-1/2*a*(a-4*b)*sech(d*x+c)/d-1/2*a^2*cs
ch(d*x+c)^2*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3745, 474, 470, 327, 213}

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{a(a-4b)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a(a-4b)\operatorname{sech}(c+dx)}{2d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*(a - 4*b)*Sech[c + d*x])/(2
*d) - (a^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)^2}{(-1+x^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= -\frac{a^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x^2(3a^2-2(a+b)^2+2b^2x^2)}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2d}$$

$$= -\frac{a^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{2d} - \frac{b^2\text{sech}^3(c+dx)}{3d} - \frac{(a(a-4b))\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2d}$$

$$= -\frac{a(a-4b)\text{sech}(c+dx)}{2d} - \frac{a^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{2d} - \frac{b^2\text{sech}^3(c+dx)}{3d} - \frac{(a(a-4b))\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2d}$$

$$= \frac{a(a-4b)\text{arctanh}(\cosh(c+dx))}{2d} - \frac{a(a-4b)\text{sech}(c+dx)}{2d} - \frac{a^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{2d} - \frac{b^2\text{sech}^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

$$\int \text{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{3a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right) - 12a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 48ab\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2\log\left(\sinh\left(\frac{1}{2}(c+\right.\right.}{24d}$$

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/24*(3*a^2*Csch[(c + d*x)/2]^2 - 12*a^2*Log[Cosh[(c + d*x)/2]] + 48*a*b*L
og[Cosh[(c + d*x)/2]] + 12*a^2*Log[Sinh[(c + d*x)/2]] - 48*a*b*Log[Sinh[(c
+ d*x)/2]] + 3*a^2*Sech[(c + d*x)/2]^2 - 48*a*b*Sech[c + d*x] + 8*b^2*Sech[
c + d*x]^3)/d
```

Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^2\left(-\frac{\text{csch}(dx+c)\coth(dx+c)}{2}+\text{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\,\text{arctanh}\left(e^{dx+c}\right)\right)-\frac{b^2}{3\cosh(dx+c)^3}}{d}$
default	$\frac{a^2\left(-\frac{\text{csch}(dx+c)\coth(dx+c)}{2}+\text{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\,\text{arctanh}\left(e^{dx+c}\right)\right)-\frac{b^2}{3\cosh(dx+c)^3}}{d}$
risch	$-\frac{e^{dx+c}\left(3a^2e^{8dx+8c}-12ab\,e^{8dx+8c}+12a^2e^{6dx+6c}+8b^2e^{6dx+6c}+18a^2e^{4dx+4c}+24ab\,e^{4dx+4c}-16\,e^{4dx+4c}b^2+12a^2e^{2dx+2}\right.}{3d\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)^2}$

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/cosh(d
*x+c)-2*arctanh(exp(d*x+c)))-1/3*b^2/cosh(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2462 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 2462, normalized size of antiderivative = 30.02

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 54*(a^2 - 4*a*b)*cosh(d*x + c)*sinh
(d*x + c)^8 + 6*(a^2 - 4*a*b)*sinh(d*x + c)^9 + 8*(3*a^2 + 2*b^2)*cosh(d*x
+ c)^7 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)
^7 + 56*(9*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (3*a^2 + 2*b^2)*cosh(d*x + c))*s
inh(d*x + c)^6 + 4*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 -
 4*a*b)*cosh(d*x + c)^4 + 42*(3*a^2 + 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a
*b - 8*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 70*(3*
a^2 + 2*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c))*si
nh(d*x + c)^4 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 8*(63*(a^2 - 4*a*b)*cos
h(d*x + c)^6 + 35*(3*a^2 + 2*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 12*a*b - 8*b
^2)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 - 4*a*b)*
cosh(d*x + c)^7 + 21*(3*a^2 + 2*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 12*a*b -
8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 +
 6*(a^2 - 4*a*b)*cosh(d*x + c) - 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^
2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 +
 (a^2 - 4*a*b)*cosh(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 -
4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b
)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105
*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4
*a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a
*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a
^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^
2 - 4*a*b)*cosh(d*x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b
)*sinh(d*x + c)^4 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*c
osh(d*x + c)^5 - 5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x +
 c))*sinh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*co
sh(d*x + c)^8 + 28*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*
x + c)^4 - 12*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2
+ a^2 - 4*a*b + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d
*x + c)^7 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)
```

```
^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^2 - 4*a*b)*cosh(d*
x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 + (a^2 - 4*a*b)*cos
h(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x +
c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105*(a^2 - 4*a*b)*cos
h(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c
)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^
3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d
*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^2 - 4*a*b)*cosh(d*
x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^4
+ 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 5
*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28
*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 12*(a^2
 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2 + a^2 - 4*a*b + 2*
(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d*x + c)^7 - 6*(a^2
 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)
*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(
27*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28*(3*a^2 + 2*b^2)*cosh(d*x + c)^6 + 10*
(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 2*b^2)*cosh(d*x + c)
^2 + 3*a^2 - 12*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)^8 + (45*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x + c)^4 + 14*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 14*d*cosh(d*x
+ c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^4 + 2*(105*
d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 - 15*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 - 5*d*cosh(d*x +
c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (45*d*cosh(d*
x + c)^8 + 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 - 6*
d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) +
d)
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)^2\operatorname{csch}^3{(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(74) = 148$.

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$
$$-\frac{8\,b^2}{3\,d(e^{(dx+c)}+e^{(-dx-c)})^3}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c
) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*
a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*
(e^(-2*d*x - 2*c) + 1))) - 8/3*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(74) = 148$.

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{\frac{12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}-3\,(a^2-4\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+3\,(a^2-4\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}-\right.}{12\,d}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
- 4) - 3*(a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(a^2 - 4*a*b
)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 16*(3*a*b*(e^(d*x + c) + e^(-d*x -
c))^2 - 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.18

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(a^2\sqrt{-d^2}-4\,a\,b\sqrt{-d^2}\right)}{d\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}\right)\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}{\sqrt{-d^2}} + \frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} - \frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{2\,a^2\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} - \frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} + \frac{4\,a\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 - 8
*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(1/
2) + (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) - (a^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*
a^2*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*b^2*
exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (4*a*b*ex
p(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.16 $\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(2a-b)b\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-(2*a-b)*b*tanh(d*x+c)/d-1/3
*b^2*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 459}

$$\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b(2a-b)\tanh(c+dx)}{d}+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^2}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b(-2a+b) + \frac{a^2}{x^4} - \frac{a(a-2b)}{x^2} - b^2x^2\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a(a-2b)\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{(2a-b)b\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{-a\coth(c+dx)\left(-2a+6b+a\operatorname{csch}^2(c+dx)\right) + b\left(-6a+2b+b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (-(a*Coth[c + d*x]*(-2*a + 6*b + a*Csch[c + d*x]^2)) + b*(-6*a + 2*b + b*Se
ch[c + d*x]^2)*Tanh[c + d*x])/(3*d)
```

Maple [A] (verified)

Time = 8.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
risch	$-\frac{4(3a^2e^{8dx+8c}+6ab\,e^{8dx+8c}+3b^2e^{8dx+8c}+8a^2e^{6dx+6c}-8b^2e^{6dx+6c}+6a^2e^{4dx+4c}-12ab\,e^{4dx+4c}+6\,e^{4dx+4c}b^2-a^2+6a}{3d(e^{2dx+2c}+1)^3(e^{2dx+2c}-1)^3}$

[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 5.46

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{8\left(\left(a^2+6\,ab+b^2\right)\cosh\left(dx+c\right)^4+8\left(a^2+b^2\right)\cosh\left(dx+\right.\right.}{3\left(d\cosh\left(dx+c\right)^8+56\,d\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^5+28\,d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^6+8\,d\cosh\left(d\right.\right.}$$

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -8/3*((a^2 + 6*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 6*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 6*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*b^2)*sinh(d*x + c)^2 + 3*a^2 - 6*a*b + 3*b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 6*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + 3*d)

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2\operatorname{csch}^4{(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(68) = 136$.

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.92

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{4}{3}\,b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+\frac{8\,ab}{d(e^{(-4\,dx-4\,c)}-1)}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 8*a*b/(d*(e^(-4*d*x - 4*c) -
1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(68) = 136$.

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{4\left(3\,a^2e^{(8\,dx+8\,c)}+6\,abe^{(8\,dx+8\,c)}+3\,b^2e^{(8\,dx+8\,c)}+8\,a^2e^{(6\,dx+6\,c)}-8\,b^2e^{(6\,dx+6\,c)}+6\,a^2e^{(4\,dx+4\,c)}-12\,abe^{(4\,dx}\right.}{3\,d(e^{(4\,dx+4\,c)}-1)^3}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*a^2*e^(8*d*x + 8*c) + 6*a*b*e^(8*d*x + 8*c) + 3*b^2*e^(8*d*x + 8*c)
 + 8*a^2*e^(6*d*x + 6*c) - 8*b^2*e^(6*d*x + 6*c) + 6*a^2*e^(4*d*x + 4*c) -
12*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) - a^2 + 6*a*b - b^2)/(d*(e^(
4*d*x + 4*c) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(6\,a\,b-a^2-b^2+6\,a^2\,e^{4\,c+4\,d\,x}+8\,a^2\,e^{6\,c+6\,d\,x}+3\,a^2\,e^{8\,c+8\,d\,x}+6\,b^2\,e^{4\,c+4\,d\,x}-8\,b^2\,e^{6\,c+6\,d\,x}+3\,b^2\,e^{8\,c+8\,\cdots}\right.}{3\,d\left(e^{4\,c+4\,d\,x}-1\right)^3}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

```
[Out] -(4*(6*a*b - a^2 - b^2 + 6*a^2*exp(4*c + 4*d*x) + 8*a^2*exp(6*c + 6*d*x) +
3*a^2*exp(8*c + 8*d*x) + 6*b^2*exp(4*c + 4*d*x) - 8*b^2*exp(6*c + 6*d*x) +
3*b^2*exp(8*c + 8*d*x) - 12*a*b*exp(4*c + 4*d*x) + 6*a*b*exp(8*c + 8*d*x)))
/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

3.17 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\
&=\frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x-\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d}\\
&\quad-\frac{b(6a^2+35ab+21b^2)\tanh^3(c+dx)}{8d}-\frac{3b^2(5a+21b)\tanh^5(c+dx)}{40d}\\
&\quad-\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d}\\
&\quad+\frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d}
\end{aligned}$$

```
[Out] 3/8*(a+b)*(a^2+14*a*b+21*b^2)*x-3/8*(a+b)*(a^2+14*a*b+21*b^2)*tanh(d*x+c)/d
-1/8*b*(6*a^2+35*a*b+21*b^2)*tanh(d*x+c)^3/d-3/40*b^2*(5*a+21*b)*tanh(d*x+c
)^5/d-3/8*(a+3*b)*sinh(d*x+c)^2*tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2/d+1/4*cos
h(d*x+c)*sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {3744, 478, 591, 584, 212}

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= -\frac{b(6a^2+35ab+21b^2)\tanh^3(c+dx)}{8d} - \frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d}$$
$$+\frac{3}{8}x(a+b)\left(a^2+14ab+21b^2\right) - \frac{3b^2(5a+21b)\tanh^5(c+dx)}{40d}$$
$$-\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d}$$
$$+\frac{\sinh^3(c+dx)\cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d}$$

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + b)*(a^2 + 14*a*b + 21*b^2)*x)/8 - (3*(a + b)*(a^2 + 14*a*b + 21*b^2
)*Tanh[c + d*x])/(8*d) - (b*(6*a^2 + 35*a*b + 21*b^2)*Tanh[c + d*x]^3)/(8*d
) - (3*b^2*(5*a + 21*b)*Tanh[c + d*x]^5)/(40*d) - (3*(a + 3*b)*Sinh[c + d*x
]^2*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c
+ d*x]^3*(a + b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2(3a+9bx^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= -\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)(-3a(a+9b)-3b(5a+21b)x^2)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= -\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int \left(3(a+b)(a^2+14ab+21b^2)+3b(6a^2+35ab+21b^2)x^2+3b^2(5a+21b)x^4-\frac{3(a^3+15a^2b}{1}\right.\right.}{8d}$$

$$= -\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d} - \frac{b(6a^2+35ab+21b^2)\tanh^3(c+dx)}{8d} - \frac{3b^2(5a+21b)\tanh^5(c+dx)}{40d} - \frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d} + \frac{(3(a+b)\left(a^2+14ab+21b^2\right))\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{8d}$$

$$= \frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x - \frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d} - \frac{b(6a^2+35ab+21b^2)\tanh^3(c+dx)}{8d} - \frac{3b^2(5a+21b)\tanh^5(c+dx)}{40d} - \frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} + \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d}$$

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{60(a^3+15a^2b+35ab^2+21b^3)(c+dx)-40(a+b)^2(a+4b)\sinh(2(c+dx))+5(a+b)^3\sinh(4(c+dx))}{160d}$$

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (60*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(c + d*x) - 40*(a + b)^2*(a + 4*b)
*Sinh[2*(c + d*x)] + 5*(a + b)^3*Sinh[4*(c + d*x)] - 32*b*(15*a^2 + 50*a*b
+ 36*b^2 - b*(5*a + 7*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*
x])/(160*d)
```

Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right.}{}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right.}{}$
risch	$\frac{3a^3x}{8}+\frac{45b\,a^2x}{8}+\frac{105a\,b^2x}{8}+\frac{63b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3\,e^{4dx+4c}a^2b}{64d}+\frac{3\,e^{4dx+4c}a\,b^2}{64d}+\frac{e^{4dx+4c}b^3}{64d}-\frac{e^{2dx+2c}a^3}{8d}$

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c))+3*a*b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh
(d*x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^
3)+b^3*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^5+63/
8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x+c)^3-63/40*tanh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(170) = 340.

Time = 0.27 (sec) , antiderivative size = 879, normalized size of antiderivative = 4.83

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 - 15*(a^3 + 11*a^2
*b + 19*a*b^2 + 9*b^3 - 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^7 + 8*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b
+ 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288
*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)*sinh(d*x
+ c)^4 + (630*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 150*a^3 - 2
010*a^2*b - 4850*a*b^2 - 3054*b^3 - 315*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288*b^3 + 1
5*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 5*(84*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 105*(a^3 + 11*a^2*b + 19*a*b^2 +
 9*b^3)*cosh(d*x + c)^4 - 62*a^3 - 978*a^2*b - 2282*a*b^2 - 1302*b^3 - 4*(7
5*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
3 + 40*(2*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2
+ 21*b^3)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a
```

```
^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 80
*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)
*d*x)*cosh(d*x + c) + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
- 21*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)*cosh(d*x + c)^6 - 2*(75*a^3 + 1005
*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^4 - 36*a^3 - 612*a^2*b - 1372
*a*b^2 - 924*b^3 - 6*(31*a^3 + 489*a^2*b + 1141*a*b^2 + 651*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^
4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2(c+dx)\right)^3\sinh^4(c+dx)\,dx$$

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(170) = 340$.

Time = 0.20 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.64

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&=\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad+\frac{1}{320}b^3\left(\frac{2520(dx+c)}{d}+\frac{5\left(32e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{135e^{(-2dx-2c)}+5358e^{(-4dx-4c)}+18190e^{(-\cdots}}{d(e^{(-4dx-4c)}+5e^{(-6dx-6c)}+10\cdots}\right. \\
&\quad+\frac{1}{64}ab^2\left(\frac{840(dx+c)}{d}+\frac{3\left(24e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{63e^{(-2dx-2c)}+1487e^{(-4dx-4c)}+2517e^{(-6dx-\cdots}}{d(e^{(-4dx-4c)}+3e^{(-6dx-6c)}+3e^{(-8dx-\cdots}}\right. \\
&\quad+\frac{3}{64}a^2b\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d(e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)
\end{aligned}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^3*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c
) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10
*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c
```

```
) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c)))) + 1/64*a*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x - 2*c)
- e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c) + 2517
*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) + 3*e^(
-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 3/64*a^2*b*(12
0*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x
- 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(170) = 340$.

Time = 0.55 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.77

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{5\,a^3e^{(4\,dx+4\,c)}+15\,a^2be^{(4\,dx+4\,c)}+15\,ab^2e^{(4\,dx+4\,c)}+5\,b^3e^{(4\,dx+4\,c)}-40\,a^3e^{(2\,dx+2\,c)}-240\,a^2be^{(2\,dx+2\,c)}-360\,a}{}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/320*(5*a^3*e^(4*d*x + 4*c) + 15*a^2*b*e^(4*d*x + 4*c) + 15*a*b^2*e^(4*d*x
 + 4*c) + 5*b^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 240*a^2*b*e^(2*d
*x + 2*c) - 360*a*b^2*e^(2*d*x + 2*c) - 160*b^3*e^(2*d*x + 2*c) + 120*(a^3
+ 15*a^2*b + 35*a*b^2 + 21*b^3)*(d*x + c) - 5*(18*a^3*e^(4*d*x + 4*c) + 270
*a^2*b*e^(4*d*x + 4*c) + 630*a*b^2*e^(4*d*x + 4*c) + 378*b^3*e^(4*d*x + 4*c
) - 8*a^3*e^(2*d*x + 2*c) - 48*a^2*b*e^(2*d*x + 2*c) - 72*a*b^2*e^(2*d*x +
2*c) - 32*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x -
4*c) + 128*(15*a^2*b*e^(8*d*x + 8*c) + 60*a*b^2*e^(8*d*x + 8*c) + 50*b^3*e^
(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 210*a*b^2*e^(6*d*x + 6*c) + 150*
b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 290*a*b^2*e^(4*d*x + 4*c)
+ 210*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 190*a*b^2*e^(2*d*x +
 2*c) + 130*b^3*e^(2*d*x + 2*c) + 15*a^2*b + 50*a*b^2 + 36*b^3)/(e^(2*d*x +
 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 730, normalized size of antiderivative = 4.01

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\frac{2(3a^2b+12ab^2+10b^3)}{5d}+\frac{8e^{2c+2dx}(3a^2b+9ab^2+5b^3)}{5d}+\frac{12e^{4c+4dx}(3a^2b+8ab^2+6b^3)}{5d}+\frac{8e^{6c+6dx}(3a^2b+9ab^2+5b^3)}{5d}+\frac{2e^{8c+}}{}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$+\frac{\frac{2(3a^2b+9ab^2+5b^3)}{5d}+\frac{2e^{2c+2dx}(3a^2b+12ab^2+10b^3)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}+x\left(\frac{3a^3}{8}+\frac{45a^2b}{8}+\frac{105ab^2}{8}+\frac{63b^3}{8}\right)$$
$$+\frac{\frac{2(3a^2b+9ab^2+5b^3)}{5d}+\frac{6e^{2c+2dx}(3a^2b+8ab^2+6b^3)}{5d}+\frac{6e^{4c+4dx}(3a^2b+9ab^2+5b^3)}{5d}+\frac{2e^{6c+6dx}(3a^2b+12ab^2+10b^3)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$+\frac{\frac{2(3a^2b+8ab^2+6b^3)}{5d}+\frac{4e^{2c+2dx}(3a^2b+9ab^2+5b^3)}{5d}+\frac{2e^{4c+4dx}(3a^2b+12ab^2+10b^3)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$+\frac{2(3a^2b+12ab^2+10b^3)}{5d\,(e^{2c+2dx}+1)}-\frac{e^{-4c-4dx}(a+b)^3}{64d}+\frac{e^{4c+4dx}(a+b)^3}{64d}$$
$$+\frac{e^{-2c-2dx}(a+b)^2(a+4b)}{8d}-\frac{e^{2c+2dx}(a+b)^2(a+4b)}{8d}$$

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(9*a*b^2 + 3
*a^2*b + 5*b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b + 6*b^3))/
(5*d) + (8*exp(6*c + 6*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(8*c
 + 8*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*ex
p(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d
*x) + 1) + ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(12
*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
 1) + x*((105*a*b^2)/8 + (45*a^2*b)/8 + (3*a^3)/8 + (63*b^3)/8) + ((2*(9*a*
b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a^2*b + 6*
b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*e
xp(6*c + 6*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(4*exp(2*c + 2*d*x) +
 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(8*a
*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5
*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(3
*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (2*(12*a*b
^2 + 3*a^2*b + 10*b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) - (exp(- 4*c - 4*d*x)*
(a + b)^3)/(64*d) + (exp(4*c + 4*d*x)*(a + b)^3)/(64*d) + (exp(- 2*c - 2*d*
x)*(a + b)^2*(a + 4*b))/(8*d) - (exp(2*c + 2*d*x)*(a + b)^2*(a + 4*b))/(8*d
)
```

3.18 $\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = -\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d} + \frac{(a+b)^3\cosh^3(c+dx)}{3d} - \frac{3b(a+b)(a+2b)\operatorname{sech}(c+dx)}{d} + \frac{b^2(3a+4b)\operatorname{sech}^3(c+dx)}{3d} - \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -(a+b)^2*(a+4*b)*cosh(d*x+c)/d+1/3*(a+b)^3*cosh(d*x+c)^3/d-3*b*(a+b)*(a+2*b
)*sech(d*x+c)/d+1/3*b^2*(3*a+4*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3745, 459}

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{b^2(3a+4b)\operatorname{sech}^3(c+dx)}{3d} + \frac{(a+b)^3\cosh^3(c+dx)}{3d} - \frac{(a+b)^2(a+4b)\cosh(c+dx)}{d} - \frac{3b(a+b)(a+2b)\operatorname{sech}(c+dx)}{d} - \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^2*(a + 4*b)*Cosh[c + d*x])/d) + ((a + b)^3*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(a + 2*b)*Sech[c + d*x])/d + (b^2*(3*a + 4*b)*Sech[c + d*x
]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)^3}{x^4}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(3(-a-2b)b(a+b) - \frac{(a+b)^3}{x^4} + \frac{(a+b)^2(a+4b)}{x^2} + b^2(3a+4b)x^2 - b^3x^4\right)\,dx, x, \text{sech}(c+dx)}{d}$$

$$= -\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d} + \frac{(a+b)^3\cosh^3(c+dx)}{3d} - \frac{3b(a+b)(a+2b)\text{sech}(c+dx)}{d} + \frac{b^2(3a+4b)\text{sech}^3(c+dx)}{3d} - \frac{b^3\text{sech}^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 11.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-45(a+b)^2(a+5b)\cosh(c+dx) + 5(a+b)^3\cosh(3(c+dx)) - 180b(a+b)(a+2b)\text{sech}(c+dx) + 20b^2(}{60d}$$

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-45*(a + b)^2*(a + 5*b)*Cosh[c + d*x] + 5*(a + b)^3*Cosh[3*(c + d*x)] - 18
0*b*(a + b)*(a + 2*b)*Sech[c + d*x] + 20*b^2*(3*a + 4*b)*Sech[c + d*x]^3 -
12*b^3*Sech[c + d*x]^5)/(60*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(99) = 198.

Time = 5.00 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.28

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+c}{\cosh(dx+c)}\right.}{d}$
default	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+c}{\cosh(dx+c)}\right.}{d}$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}a\,b^2}{8d}+\frac{e^{3dx+3c}b^3}{24d}-\frac{3\,e^{dx+c}a^3}{8d}-\frac{21\,e^{dx+c}a^2b}{8d}-\frac{33\,e^{dx+c}a\,b^2}{8d}-\frac{15b^3e^{dx+c}}{8d}$

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^4/co
sh(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c))+3*a*b^2*(1/3*sinh(
d*x+c)^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8*sinh(d*x+c)^2/cosh(d
*x+c)^3-16/3/cosh(d*x+c)^3)+b^3*(1/3*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3*sinh(d
*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-64/3*sinh(d*x+c)^2/cos
h(d*x+c)^5-128/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(99) = 198.

Time = 0.25 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.14

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{5\left(a^3+3a^2b+3ab^2+b^3\right)\cosh\left(dx+c\right)^8+5\left(a^3+3a^2b+3ab^2+b^3\right)\sinh\left(dx+c\right)^8-20\left(a^3+12a^2b+21\right.}{}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^8 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)
*cosh(d*x + c)^6 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3 - 7*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 20*(11*a^3 + 123*a^2*
b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^4 + 10*(35*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^4 - 22*a^3 - 246*a^2*b - 498*a*b^2 - 274*b^3 - 30*(a^3
 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 425*a^3
 - 5235*a^2*b - 10395*a*b^2 - 5649*b^3 - 20*(31*a^3 + 372*a^2*b + 747*a*b^2
 + 390*b^3)*cosh(d*x + c)^2 + 20*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^6 - 15*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^4 - 31*a^3
```

```
 - 372*a^2*b - 747*a*b^2 - 390*b^3 - 6*(11*a^3 + 123*a^2*b + 249*a*b^2 + 13
7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
+ c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \sinh^3\left(c+dx\right) dx$$

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(99) = 198$.

Time = 0.21 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.18

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx =$$
$$-\frac{1}{120} b^3 \left(\frac{5\left(45 e^{(-dx-c)} - e^{(-3dx-3c)}\right)}{d} + \frac{200 e^{(-2dx-2c)} + 2515 e^{(-4dx-4c)} + 6680 e^{(-6dx-6c)} + 9073 e^{(-8dx-8c)} + \cdots}{d\left(e^{(-3dx-3c)} + 5 e^{(-5dx-5c)} + 10 e^{(-7dx-7c)} + 10 e^{(-9dx-9c)} + \cdots\right)}\right)$$
$$-\frac{1}{8} ab^2 \left(\frac{33 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{30 e^{(-2dx-2c)} + 240 e^{(-4dx-4c)} + 322 e^{(-6dx-6c)} + 177 e^{(-8dx-8c)} - 1}{d\left(e^{(-3dx-3c)} + 3 e^{(-5dx-5c)} + 3 e^{(-7dx-7c)} + e^{(-9dx-9c)}\right)}\right)$$
$$-\frac{1}{8} a^2 b \left(\frac{21 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{20 e^{(-2dx-2c)} + 69 e^{(-4dx-4c)} - 1}{d\left(e^{(-3dx-3c)} + e^{(-5dx-5c)}\right)}\right)$$
$$+\frac{1}{24} a^3 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9 e^{(dx+c)}}{d} - \frac{9 e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right)$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*b^3*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c
) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) +
 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c
) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-
11*d*x - 11*c) + e^(-13*d*x - 13*c)))) - 1/8*a*b^2*((33*e^(-d*x - c) - e^(-
3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) + 240*e^(-4*d*x - 4*c) + 322*e^(-6*d
*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*(e^(-3*d*x - 3*c) + 3*e^(-5*d*x -
5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9*c)))) - 1/8*a^2*b*((21*e^(-d*x -
c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69*e^(-4*d*x - 4*c) - 1)/
(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(99) = 198$.

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{5a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+5b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right.}{}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 15*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^3 + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 5*b^3*(e^(d*x + c) +
 e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x - c)) - 360*a^2*b*(e^(d*x
+ c) + e^(-d*x - c)) - 540*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 240*b^3*(e^
(d*x + c) + e^(-d*x - c)) - 16*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 1
35*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 90*b^3*(e^(d*x + c) + e^(-d*x - c
))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 80*b^3*(e^(d*x + c) + e^(-
d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.44

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{e^{-3c-3dx}(a+b)^3}{24d}+\frac{e^{3c+3dx}(a+b)^3}{24d}+\frac{8e^{c+dx}(4b^3+3ab^2)}{3d\,(2e^{2c+2dx}+e^{4c+4dx}+1)}$$
$$+\frac{64b^3e^{c+dx}}{5d\,(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1)}$$
$$-\frac{8e^{c+dx}(32b^3+15ab^2)}{15d\,(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}$$
$$-\frac{32b^3e^{c+dx}}{5d\,(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1)}$$
$$-\frac{3e^{c+dx}(a+b)^2(a+5b)}{8d}-\frac{6e^{c+dx}(a^2b+3ab^2+2b^3)}{d\,(e^{2c+2dx}+1)}-\frac{3e^{-c-dx}(a+b)^2(a+5b)}{8d}$$

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*(a + b)^3)/(24*d)
 + (8*exp(c + d*x)*(3*a*b^2 + 4*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c +
4*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c +
```

```
 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15
*a*b^2 + 32*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*
c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
+ 1)) - (3*exp(c + d*x)*(a + b)^2*(a + 5*b))/(8*d) - (6*exp(c + d*x)*(3*a*b
^2 + a^2*b + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x)*(a + b)
^2*(a + 5*b))/(8*d)
```

3.19 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{1}{2}(a+b)^2(a+7b)x+\frac{(a+b)^3}{4d(1-\tanh(c+dx))}+\frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}-\frac{(a+b)^3}{4d(1+\tanh(c+dx))}$$

```
[Out] -1/2*(a+b)^2*(a+7*b)*x+1/4*(a+b)^3/d/(1-tanh(d*x+c))+3*b*(a+b)^2*tanh(d*x+c
)/d+1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d-1/4*(a+b)^3/d
/(1+tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3744, 478, 542, 396, 212}

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{b(81a^2+190ab+105b^2)\tanh(c+dx)}{30d}+\frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d}+\frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d}+\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d}-\frac{1}{2}x(a+b)^2(a+7b)$$

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((a + b)^2*(a + 7*b)*x) + (b*(81*a^2 + 190*a*b + 105*b^2)*Tanh[c + d*x
])/(30*d) + (b*(33*a + 35*b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(30*d)
+ (7*b*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(10*d) + (Cosh[c + d*x]*Sin
h[c + d*x]*(a + b*Tanh[c + d*x]^2)^3)/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^3}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2(a+7bx^2)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d} + \frac{\text{Subst}\left(\int \frac{(a+bx^2)(-a(5a+7b)-b(33a+35b)x^2)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{10d}$$

$$= \frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d} + \frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{a(15a^2+54ab+35b^2)+b(81a^2+190ab+105b^2)x^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{30d}$$

$$= \frac{b(81a^2+190ab+105b^2)\tanh(c+dx)}{30d} + \frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d} + \frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d} - \frac{((a+b)^2(a+7b))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= -\frac{1}{2}(a+b)^2(a+7b)x + \frac{b(81a^2+190ab+105b^2)\tanh(c+dx)}{30d} + \frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d} + \frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d}$$

Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.78

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-30(a+b)^2(a+7b)(c+dx)+15(a+b)^3\sinh(2(c+dx))+4b\big(45a^2+105ab+58b^2-b(15a+16b)\mathrm{sech}^2}{60d}$$

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-30*(a + b)^2*(a + 7*b)*(c + d*x) + 15*(a + b)^3*Sinh[2*(c + d*x)] + 4*b*(
45*a^2 + 105*a*b + 58*b^2 - b*(15*a + 16*b)*Sech[c + d*x]^2 + 3*b^2*Sech[c
+ d*x]^4)*Tanh[c + d*x])/(60*d)
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tan}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tan}{}\right.}{d}$
risch	$-\frac{a^3x}{2}-\frac{9b\,a^2x}{2}-\frac{15a\,b^2x}{2}-\frac{7b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3\,e^{2dx+2c}a^2b}{8d}+\frac{3\,e^{2dx+2c}a\,b^2}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}}{8d}$

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x+
c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^5/
cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3)+b^3*(1/2*sin
h(d*x+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/
10*tanh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(110) = 220$.

Time = 0.27 (sec) , antiderivative size = 725, normalized size of antiderivative = 5.94

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\sinh\left(dx+c\right)^7-4\left(90a^2b+210ab^2+116b^3+15\left(a^3+9a^2b+15ab^2+7b^3\right)dx\right.}{}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 - 4*(90*a^2*b + 2
10*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x +
c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 +
7*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (75*a^3 + 585*a^2*b + 1065*a*b^
2 + 539*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2
 + 7*b^3)*d*x)*cosh(d*x + c)^3 + 5*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^4 + 27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3 + 2*(75*a^3 + 585*a
^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 20*(2*(90*a
^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cos
h(d*x + c)^3 + 3*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 40*(90*a^2*b + 210*a*b^
2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c) + 5*
(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + (75*a^3 + 585*a^2*b +
 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^4 + 15*a^3 + 189*a^2*b + 285*a*b^2 + 1
75*b^3 + 3*(27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cos
h(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2
+ 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh^2\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(110) = 220$.

Time = 0.21 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.09

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{120}b^3\left(\frac{420(dx+c)}{d}+\frac{15e^{(-2dx-2c)}}{d}-\frac{1003e^{(-2dx-2c)}+3350e^{(-4dx-4c)}+5590e^{(-6dx-6c)}+3915e^{(-\cdots}}{d(e^{(-2dx-2c)}+5e^{(-4dx-4c)}+10e^{(-6dx-6c)}+10e^{(-8dx-8c)}+\cdots}\right) - \frac{1}{8}ab^2\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right) - \frac{3}{8}a^2b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^3*(420*(d
*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*x
 - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x -
 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1/
8*a*b^2*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 20
1*e^(-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 3/8*a^2*b*(12*(d
*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x -
2*c) + e^(-4*d*x - 4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(110) = 220$.

Time = 0.48 (sec) , antiderivative size = 393, normalized size of antiderivative = 3.22

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{15a^3e^{(2dx+2c)}+45a^2be^{(2dx+2c)}+45ab^2e^{(2dx+2c)}+15b^3e^{(2dx+2c)}-60(a^3+9a^2b+15ab^2+7b^3)(dx+c\cdots}{\cdots}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^3*e^(2*d*x + 2*c) + 45*a^2*b*e^(2*d*x + 2*c) + 45*a*b^2*e^(2*d*
x + 2*c) + 15*b^3*e^(2*d*x + 2*c) - 60*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*(
d*x + c) + 15*(2*a^3*e^(2*d*x + 2*c) + 18*a^2*b*e^(2*d*x + 2*c) + 30*a*b^2*
e^(2*d*x + 2*c) + 14*b^3*e^(2*d*x + 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^3)*e
```

```
^(-2*d*x - 2*c) - 16*(45*a^2*b*e^(8*d*x + 8*c) + 135*a*b^2*e^(8*d*x + 8*c)
+ 90*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6*d*x +
 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 600*a*b^2*e^(
4*d*x + 4*c) + 340*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 390*a*
b^2*e^(2*d*x + 2*c) + 200*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 105*a*b^2 + 58*b
^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.48

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{e^{2c+2dx}(a+b)^3}{8d} - \frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d}+\frac{6e^{2c+2dx}(a^2b+3ab^2+2b^3)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$-\frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d}+\frac{6e^{6c+6dx}(a^2b+3ab^2+2b^3)}{5d}+\frac{6e^{4c+4dx}(3a^2b+6ab^2+2b^3)}{5d}+\frac{2e^{2c+2dx}(9a^2b+15ab^2+10b^3)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{\frac{2(9a^2b+15ab^2+10b^3)}{15d}+\frac{6e^{4c+4dx}(a^2b+3ab^2+2b^3)}{5d}+\frac{4e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{6(a^2b+3ab^2+2b^3)}{5d(e^{2c+2dx}+1)}-\frac{e^{-2c-2dx}(a+b)^3}{8d}$$
$$-\frac{\frac{6(a^2b+3ab^2+2b^3)}{5d}+\frac{8e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d}+\frac{6e^{8c+8dx}(a^2b+3ab^2+2b^3)}{5d}+\frac{8e^{6c+6dx}(3a^2b+6ab^2+2b^3)}{5d}+\frac{4e^{4c+4dx}(\cdots}{}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{x(a+b)^2(a+7b)}{2}$$

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d)
 + (6*exp(2*c + 2*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d))/(2*exp(2*c + 2*d*x
) + exp(4*c + 4*d*x) + 1) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*exp
(6*c + 6*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(6*a*b
^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(15*a*b^2 + 9*a^2*b + 10
*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x)
 + exp(8*c + 8*d*x) + 1) - ((2*(15*a*b^2 + 9*a^2*b + 10*b^3))/(15*d) + (6*e
xp(4*c + 4*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(6*a
*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) + 1) - (6*(3*a*b^2 + a^2*b + 2*b^3))/(5*d*(exp(2*c + 2*d*x
) + 1)) - (exp(- 2*c - 2*d*x)*(a + b)^3)/(8*d) - ((6*(3*a*b^2 + a^2*b + 2*b
^3))/(5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*ex
p(8*c + 8*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(6*a*
b^2 + 3*a^2*b + 2*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(15*a*b^2 + 9*a^2*b + 1
0*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
```

```
*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (x*(a + b)^2*(a + 7*b)
)/2
```

3.20 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\mathrm{sech}(c+dx)}{d}-\frac{b^2(a+b)\mathrm{sech}^3(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

```
[Out] (a+b)^3*cosh(d*x+c)/d+3*b*(a+b)^2*sech(d*x+c)/d-b^2*(a+b)*sech(d*x+c)^3/d+1
/5*b^3*sech(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3745, 276}

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{b^2(a+b)\mathrm{sech}^3(c+dx)}{d}+\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\mathrm{sech}(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d + (3*b*(a + b)^2*Sech[c + d*x])/d - (b^2*(a + b
)*Sech[c + d*x]^3)/d + (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^3}{x^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-3b(a+b)^2+\frac{(a+b)^3}{x^2}+3b^2(a+b)x^2-b^3x^4\right)\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= \frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}-\frac{b^2(a+b)\text{sech}^3(c+dx)}{d}+\frac{b^3\text{sech}^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{(a+b)^3\cosh(c+dx)+3b(a+b)^2\text{sech}(c+dx)-b^2(a+b)\text{sech}^3(c+dx)+\frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
[Out] ((a + b)^3*Cosh[c + d*x] + 3*b*(a + b)^2*Sech[c + d*x] - b^2*(a + b)*Sech[c
 + d*x]^3 + (b^3*Sech[c + d*x]^5)/5)/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(68) = 136.

Time = 1.81 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)+b^3\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\right.}{d}$
default	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)+b^3\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\right.}{d}$
risch	$\frac{e^{dx+c}a^3}{2d}+\frac{3\,e^{dx+c}a^2b}{2d}+\frac{3\,e^{dx+c}a\,b^2}{2d}+\frac{b^3e^{dx+c}}{2d}+\frac{e^{-dx-c}a^3}{2d}+\frac{3\,e^{-dx-c}a^2b}{2d}+\frac{3\,e^{-dx-c}a\,b^2}{2d}+\frac{e^{-dx-c}b^3}{2d}+$

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+3*a*
b^2*(sinh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d*x
+c)^3)+b^3*(sinh(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sin
h(d*x+c)^2/cosh(d*x+c)^5+16/5/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 383, normalized size of antiderivative = 5.47

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{5\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\cosh\left(dx+c\right)^6+5\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sinh\left(dx+c\right)^6+30\left(a^3+5\,a^2b+7\,ab\right.}{}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sinh(d*x + c)^6 + 30*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cos
h(d*x + c)^4 + 15*(2*a^3 + 10*a^2*b + 14*a*b^2 + 6*b^3 + 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 50*a^3 + 330*a^2*b + 430
*a*b^2 + 182*b^3 + 5*(15*a^3 + 93*a^2*b + 125*a*b^2 + 47*b^3)*cosh(d*x + c)
^2 + 5*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*a^3 + 93*a^
2*b + 125*a*b^2 + 47*b^3 + 36*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)
^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3\sinh(c+dx)\,dx$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(68) = 136$.

Time = 0.20 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.59

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{10}b^3\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c)} + 5}{d\left(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx-11\,c)}\right)}\right)$$
$$+ \frac{1}{2}ab^2\left(\frac{3\,e^{(-dx-c)}}{d} + \frac{33\,e^{(-2\,dx-2\,c)} + 41\,e^{(-4\,dx-4\,c)} + 27\,e^{(-6\,dx-6\,c)} + 3}{d\left(e^{(-dx-c)} + 3\,e^{(-3\,dx-3\,c)} + 3\,e^{(-5\,dx-5\,c)} + e^{(-7\,dx-7\,c)}\right)}\right)$$
$$+ \frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d} + \frac{5\,e^{(-2\,dx-2\,c)} + 1}{d\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}\right) + \frac{a^3\cosh(dx+c)}{d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/10*b^3*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*c) +
314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) + 5)/(d
*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d*x -
7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + 1/2*a*b^2*(3*e^(-d*x -
c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27*e^(-6*d*x - 6*c) + 3
)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + e^(-7*d*x -
7*c)))) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x
- c) + e^(-3*d*x - 3*c)))) + a^3*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.37

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{5\,a^3\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 15\,a^2b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 15\,ab^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 5\,b^3\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) + 15*a^2*b*(e^(d*x + c) + e^(-d*x
- c)) + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 5*b^3*(e^(d*x + c) + e^(-d*
x - c)) + 4*(15*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 30*a*b^2*(e^(d*x + c
) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 - 20*a*b^2*(e^(
d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3
)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.40

$$\int \sinh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{e^{c+dx}\,(a+b)^3}{2\,d} + \frac{e^{-c-dx}\,(a+b)^3}{2\,d} + \frac{6\,e^{c+dx}\,(a^2\,b+2\,a\,b^2+b^3)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{64\,b^3\,e^{c+dx}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)} + \frac{8\,e^{c+dx}\,(9\,b^3+5\,a\,b^2)}{5\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} + \frac{32\,b^3\,e^{c+dx}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)} - \frac{8\,e^{c+dx}\,(b^3+a\,b^2)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a + b)^3)/(2*d) + (6*exp(
c + d*x)*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (64*b^3*exp(
c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x
) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 + 9*b^3))/(5*d*(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c
 + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (8*exp(c + d*x)*(a*b^2
 + b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*(3*a+
2*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3745, 398, 213}

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x])
/d + (b^2*(3*a + 2*b)*Sech[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^3}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-b(3a^2+3ab+b^2)+b^2(3a+2b)x^2-b^3x^4+\frac{a^3}{-1+x^2}\right)\,dx, x, \text{sech}(c+dx)\right)}{d} \\
&= -\frac{b(3a^2+3ab+b^2)\,\text{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\text{sech}^3(c+dx)}{3d} \\
&\quad -\frac{b^3\text{sech}^5(c+dx)}{5d}+\frac{a^3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{d} \\
&= -\frac{a^3\text{arctanh}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\,\text{sech}(c+dx)}{d} \\
&\quad +\frac{b^2(3a+2b)\text{sech}^3(c+dx)}{3d}-\frac{b^3\text{sech}^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{15a^3\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)-15b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)+5b^2(3a+2}{15d}$$

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (15*a^3*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]) - 15*b*(3*a^2 +
3*a*b + b^2)*Sech[c + d*x] + 5*b^2*(3*a + 2*b)*Sech[c + d*x]^3 - 3*b^3*Sech
[c + d*x]^5)/(15*d)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-2a^3\operatorname{arctanh}(e^{dx+c})-\frac{3a^2b}{\cosh(dx+c)}+3ab^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(d}\right.}{d}$
default	$\frac{-2a^3\operatorname{arctanh}(e^{dx+c})-\frac{3a^2b}{\cosh(dx+c)}+3ab^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(d}\right.}{d}$
risch	$-\frac{2b\,e^{dx+c}(45a^2e^{8dx+8c}+45ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+180a^2e^{6dx+6c}+120ab\,e^{6dx+6c}+20b^2e^{6dx+6c}+270a^2e^{4dx+4c}+15}{15d\left(e^{2dx+2c}+1\right)^5}$

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))-3*a^2*b/cosh(d*x+c)+3*a*b^2*(-sinh(d*x+c)^2
/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3)+b^3*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/3*sin
h(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 2277, normalized size of antiderivative = 27.11

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(
```

d*x + c)^9 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b + 6
*a*b^2 + b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 6*a*b^2 +
b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 75*a*b^2 + 29*b^3)*co
sh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 135*a^2*
b + 75*a*b^2 + 29*b^3 + 210*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(9*a^2
*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (135*a^2*b + 75*a*b^2 + 29*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 +
40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b + 6*a*b^2 +
b^3)*cosh(d*x + c)^4 + 9*a^2*b + 6*a*b^2 + b^3 + (135*a^2*b + 75*a*b^2 + 29
*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^7 + 21*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^5 + (135*a^2*b
+ 75*a*b^2 + 29*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c) + 15*(a
^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x +
c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6 + 5*(9*a^3*cosh(d*x
+ c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
c))*sinh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^4 + 10*(21*a^3*cosh(d*x + c)^4
+ 14*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 4*(63*a^3*cosh(d*x + c)^5
+ 70*a^3*cosh(d*x + c)^3 + 15*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^3*c
osh(d*x + c)^2 + 10*(21*a^3*cosh(d*x + c)^6 + 35*a^3*cosh(d*x + c)^4 + 15*a
^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 40*(3*a^3*cosh(d*x + c)^7 + 7*a
^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x +
c)^3 + a^3 + 5*(9*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c)^6 + 30*a^3*cos
h(d*x + c)^4 + 12*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 10*(a^3*cosh
(d*x + c)^9 + 4*a^3*cosh(d*x + c)^7 + 6*a^3*cosh(d*x + c)^5 + 4*a^3*cosh(d*
x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
c) + 1) - 15*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9
+ a^3*sinh(d*x + c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6 + 5
*(9*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x + c)^3
+ a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^4 + 10*(21*a^3*
cosh(d*x + c)^4 + 14*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 4*(63*a^3
*cosh(d*x + c)^5 + 70*a^3*cosh(d*x + c)^3 + 15*a^3*cosh(d*x + c))*sinh(d*x
+ c)^5 + 5*a^3*cosh(d*x + c)^2 + 10*(21*a^3*cosh(d*x + c)^6 + 35*a^3*cosh(d
*x + c)^4 + 15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 40*(3*a^3*cosh(
d*x + c)^7 + 7*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
c))*sinh(d*x + c)^3 + a^3 + 5*(9*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c
)^6 + 30*a^3*cosh(d*x + c)^4 + 12*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^
2 + 10*(a^3*cosh(d*x + c)^9 + 4*a^3*cosh(d*x + c)^7 + 6*a^3*cosh(d*x + c)^5
+ 4*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x +
c) + sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
+ 28*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 2*(135*a^2*b + 75*a*b^2 +
29*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b + 6*a*b^
2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*

```
x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d*x + c)^8 + 5*(9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 + d*cosh(d*x
 + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 +
14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 70*d*
cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*d*cosh(d*x + c)^
4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2
+ d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 +
5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 + 4*d*cosh(d*
x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{csch}\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(80) = 160.

Time = 0.21 (sec) , antiderivative size = 560, normalized size of antiderivative = 6.67

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{2}{15}\,b^3\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c}}\right.$$
$$-2\,ab^2\left(\frac{3\,e^{(-dx-c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$
$$+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{6\,a^2b}{d(e^{(dx+c)}+e^{(-dx-c)})}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -2/15*b^3*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(
-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
```

```
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*lo
g(tanh(1/2*d*x + 1/2*c))/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(80) = 160$.

Time = 0.42 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{15\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+45\,ab^2\left(e^{(dx+c} \cdots\right.\right.}{\cdots}}{30\,d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*a^3*log(e^(d*x + c)
+ e^(-d*x - c) - 2) + 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b^2
*(e^(d*x + c) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 - 6
0*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 40*b^3*(e^(d*x + c) + e^(-d*x - c)
)^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.77

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{8\,e^{c+d\,x}\,(2\,b^3+3\,a\,b^2)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}} - \frac{2\,e^{c+d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{64\,b^3\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)} - \frac{8\,e^{c+d\,x}\,(22\,b^3+15\,a\,b^2)}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} - \frac{32\,b^3\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1)) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^
6)^(1/2))/(-d^2)^(1/2) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp
(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d
*x)*(15*a*b^2 + 22*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10
*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1))
```

3.22 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] -a^3*coth(d*x+c)/d+3*a^2*b*tanh(d*x+c)/d+a*b^2*tanh(d*x+c)^3/d+1/5*b^3*tanh
(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 276}

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x])/d + (a*b^2*Tanh[c + d*x]
^3)/d + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x^2}+3ab^2x^2+b^3x^4\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-5a^3\coth(c+dx)+b\left(15a^2+5ab+b^2-b(5a+2b)\operatorname{sech}^2(c+dx)+b^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{5d}$$

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-5*a^3*Coth[c + d*x] + b*(15*a^2 + 5*a*b + b^2 - b*(5*a + 2*b)*Sech[c + d*
x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.

Time = 5.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{-a^3\coth(dx+c)+3a^2b\tanh(dx+c)+3ab^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh}{8\cosh}\right.}{d}$
default	$\frac{-a^3\coth(dx+c)+3a^2b\tanh(dx+c)+3ab^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh}{8\cosh}\right.}{d}$
risch	$-\frac{2(5a^3e^{10dx+10c}+15a^2b\,e^{10dx+10c}+15a\,b^2e^{10dx+10c}+5b^3e^{10dx+10c}+25a^3e^{8dx+8c}+45a^2b\,e^{8dx+8c}+15a\,b^2e^{8dx+8c}-5b^3}{}$

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(-1/2*sinh(d*x+c)/cosh(d*
x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^3/cos
h(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*s
ech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 572, normalized size of antiderivative = 8.94

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{4\left((5a^3+5ab^2+2b^3)\cosh(dx+c)^5+5(5a^3+5ab^2+2b^3)\cosh(dx+c)\sinh(dx+c)^4+(15a^2b+10\right.}{5\left(d\cosh(dx+c)^7+7d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+3d\cosh(dx\right.}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 5*a*b^2 + 2*b^
3)*cosh(d*x + c)*sinh(d*x + c)^4 + (15*a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x +
 c)^5 + (25*a^3 + 5*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + (45*a^2*b + 10*a*b^2 -
 3*b^3 + 10*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (10*(5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 5*a*b^2 - 2*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 - a*b^2)*cosh(d*x + c) + (5*
(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 10*b^3 + 3*(45*a
^2*b + 10*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c)
^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d*x
 + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2(c+dx)\right)^3\operatorname{csch}^2(c+dx)\,dx$$

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(62) = 124$.

Time = 0.20 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.44

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{2}{5}b^3\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$+2\,ab^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$
$$+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}-1)}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 2/5*b^3*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^
(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2
*c) + 1)) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(62) = 124$.

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.16

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{2\left(\frac{5a^3}{e^{(2dx+2c)}-1}+\frac{15a^2be^{(8dx+8c)}+15ab^2e^{(8dx+8c)}+5b^3e^{(8dx+8c)}+60a^2be^{(6dx+6c)}+30ab^2e^{(6dx+6c)}+90a^2be^{(4dx+4c)}+20ab^2e^{(4dx+4c)}\cdots}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{5d}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(8*d*x + 8*c) + 15*a*b^2*e^
(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 30*a*b^2
*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 20*a*b^2*e^(4*d*x + 4*c) + 10
*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 10*a*b^2*e^(2*d*x + 2*c)
+ 15*a^2*b + 5*a*b^2 + b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 590, normalized size of antiderivative = 9.22

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{\frac{2\,(3\,a^2\,b-b^3)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}$$
$$-\frac{\frac{2\,(3\,a^2\,b-a\,b^2+b^3)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$
$$-\frac{\frac{2\,(3\,a^2\,b-b^3)}{5\,d}+\frac{6\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b-a\,b^2+b^3)}{5\,d}+\frac{2\,e^{6\,c+6\,d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{2\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{12\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b-a\,b^2+b^3)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}\cdots}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$
$$-\frac{2\,a^3}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

```
[Out] - ((2*(3*a^2*b - b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + b^3
))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - a*b^
2 + b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (4
*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a^2*b - b^3))/(5*d) + (6*exp(2*c +
 2*d*x)*(3*a^2*b - a*b^2 + b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2 + 3*a
^2*b + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4*exp(2*c
 + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
```

```
 - ((2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(3*a^2*b - a
*b^2 + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) +
 (8*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(3*a^2*b
- b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (2*a^3)/(d*(exp(2*c +
2*d*x) - 1)) - (2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.23 $\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{a^2(a-6b)\operatorname{arctanh}(\cosh(c+dx))}{2d}+\frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d}+\frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)}{30d}+\frac{7b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2}{10d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^3}{2d}$$

```
[Out] 1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+1/30*b*(81*a^2-28*a*b-4*b^2)*sech(d*
x+c)/d+1/30*(33*a-2*b)*b*sech(d*x+c)*(a+b-b*sech(d*x+c)^2)/d+7/10*b*sech(d*
x+c)*(a+b-b*sech(d*x+c)^2)^2/d-1/2*coth(d*x+c)*csch(d*x+c)*(a+b-b*sech(d*x+
c)^2)^3/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {3745, 478, 542, 396, 213}

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{a^2(a-6b)\operatorname{arctanh}(\cosh(c+dx))}{2d}+\frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d}$$
$$+\frac{7b\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}{10d}$$
$$+\frac{b(33a-2b)\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}{30d}$$
$$-\frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^3}{2d}$$

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(81*a^2 - 28*a*b - 4*b^2)
*Sech[c + d*x])/(30*d) + ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c +
d*x]^2))/(30*d) + (7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/(10*d)
- (Coth[c + d*x]*Csch[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
```

```
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)^3}{(-1+x^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)\text{csch}(c+dx)\left(a+b-b\text{sech}^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+b-7bx^2)(a+b-bx^2)^2}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2d} \\
&= \frac{7b\text{sech}(c+dx)\left(a+b-b\text{sech}^2(c+dx)\right)^2}{10d} - \frac{\coth(c+dx)\text{csch}(c+dx)\left(a+b-b\text{sech}^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)((5a-2b)(a+b)-(33a-2b)bx^2)}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{10d} \\
&= \frac{(33a-2b)b\text{sech}(c+dx)\left(a+b-b\text{sech}^2(c+dx)\right)}{30d} + \frac{7b\text{sech}(c+dx)\left(a+b-b\text{sech}^2(c+dx)\right)^2}{10d} - \frac{\coth(c+dx)\text{csch}(c+dx)\left(a+b-b\text{sech}^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+b)(15a^2-24ab-4b^2)-b(81a^2-28ab-4b^2)x^2}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{30d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d} \\
&\quad + \frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)}{30d} \\
&\quad + \frac{7b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2}{10d} \\
&\quad - \frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^3}{2d} \\
&\quad - \frac{(a^2(a-6b))\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{2d} \\
&= \frac{a^2(a-6b)\operatorname{arctanh}(\cosh(c+dx))}{2d} + \frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d} \\
&\quad + \frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)}{30d} \\
&\quad + \frac{7b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2}{10d} \\
&\quad - \frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^3}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 17.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= -\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} \\
&\quad + \frac{(a^3-6a^2b)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} \\
&\quad + \frac{(-a^3+6a^2b)\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} \\
&\quad - \frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{3a^2b\operatorname{sech}(c+dx)}{d} \\
&\quad - \frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d} + \frac{b^3\operatorname{sech}^5(c+dx)}{5d}
\end{aligned}
$$

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*(a^3*Csch[(c + d*x)/2]^2)/d + ((a^3 - 6*a^2*b)*Log[Cosh[(c + d*x)/2]])
/(2*d) + ((-a^3 + 6*a^2*b)*Log[Sinh[(c + d*x)/2]])/(2*d) - (a^3*Sech[(c + d
*x)/2]^2)/(8*d) + (3*a^2*b*Sech[c + d*x])/d - (b^2*(3*a + b)*Sech[c + d*x]^
3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 11.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a^{3}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^{2}b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{a\,b^{2}}{\cosh(dx+c)^{3}}+b^{3}\left(-\frac{\sinh(dx+c)^{2}}{3\cosh(dx+c)^{5}}\right.}{d}$
default	$\frac{a^{3}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^{2}b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{a\,b^{2}}{\cosh(dx+c)^{3}}+b^{3}\left(-\frac{\sinh(dx+c)^{2}}{3\cosh(dx+c)^{5}}\right.}{d}$
risch	$-\frac{e^{dx+c}\left(15a^{3}e^{12dx+12c}-90a^{2}b\,e^{12dx+12c}+90a^{3}e^{10dx+10c}-180a^{2}b\,e^{10dx+10c}+120a\,b^{2}e^{10dx+10c}+40b^{3}e^{10dx+10c}+225a^{3}\right.}{}$

[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))-a*b^2/cosh(d*x+c)^3+b^3*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^5-2/15/cosh(d*x+c)^5))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5037 vs. 2(151) = 302.

Time = 0.30 (sec) , antiderivative size = 5037, normalized size of antiderivative = 33.14

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{csch}^3\left(c+dx\right)dx$$

[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**3, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(151) = 302$.

Time = 0.21 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.65

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$
$$-\frac{8}{15}b^3\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c}}\right.$$
$$-\frac{8\,ab^2}{d(e^{(dx+c)}+e^{(-dx-c)})^3}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c
) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*
a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(
d*(e^(-2*d*x - 2*c) + 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - 8*
a*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{\frac{60\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4} - 15\,(a^3-6\,a^2b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) + 15\,(a^3-6\,a^2b)\log\left(e^{(dx+c)}+e^{(-dx-c}\right.}{60\,d}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
- 4) - 15*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 15*(a^3 - 6
*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 8*(45*a^2*b*(e^(d*x + c) + e^
(-d*x - c))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x +
c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.71

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^c\,\left(a^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}\right)\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}{\sqrt{-d^2}} - \frac{64\,b^3\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)} + \frac{8\,e^{c+d\,x}\,\left(17\,b^3+15\,a\,b^2\right)}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} + \frac{32\,b^3\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)} - \frac{a^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{8\,e^{c+d\,x}\,\left(b^3+3\,a\,b^2\right)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{2\,a^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} + \frac{6\,a^2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6 -
 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^2)
^(1/2) - (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x
) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(15*a*b^2
 + 17*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) + 1)) + (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*
d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
- (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (8*exp(c + d*x)*(3*a*b^2
+ b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*a^3*exp(c +
d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (6*a^2*b*exp(c + d*
x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.24 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh(c+dx)}{d}-\frac{(3a-b)b^2\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*(a-3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-3*a*(a-b)*b*tanh(d*x+c)/d
-1/3*(3*a-b)*b^2*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 459}

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{b^2(3a-b)\tanh^3(c+dx)}{3d}-\frac{3ab(a-b)\tanh(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x])/d - ((3*a - b)*b^2*Tanh[c + d*x]^3)/(3*d) - (b^3*Tanh[c
+ d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a(a-b)b + \frac{a^3}{x^4} - \frac{a^2(a-3b)}{x^2} - (3a-b)b^2x^2 - b^3x^4\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2(a-3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3a(a-b)b\tanh(c+dx)}{d} - \frac{(3a-b)b^2\tanh^3(c+dx)}{3d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{-5a^2\coth(c+dx)\left(-2a+9b+a\operatorname{csch}^2(c+dx)\right) + b\left(-45a^2+30ab+2b^2+b(15a+b)\operatorname{sech}^2(c+dx) - 3b^2\mathrm{s}\right.}{15d}$$

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-5*a^2*Coth[c + d*x]*(-2*a + 9*b + a*Csch[c + d*x]^2) + b*(-45*a^2 + 30*a*
b + 2*b^2 + b*(15*a + b)*Sech[c + d*x]^2 - 3*b^2*Sech[c + d*x]^4)*Tanh[c +
d*x])/(15*d)
```

Maple [A] (verified)

Time = 27.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3a\,b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3a\,b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
risch	$-\frac{4(-15a\,b^2+15a^3e^{12dx+12c}+15b^3e^{12dx+12c}+70a^3e^{10dx+10c}-5a^3-30a\,b^2e^{10dx+10c}+17\,e^{4dx+4c}b^3+45a^2b-30\,e^{2dx+2}}{}$

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/sinh(d*x+c)/cosh(d
*x+c)-2*tanh(d*x+c))+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(-1/4*
sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*t
anh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(92) = 184.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 9.44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/15*((5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
15*a*b^2 + 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 + 45*a^2*b + 15*a*
b^2 + 7*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 + 45*a^2*b - 15*a*b^2 - 13*b^3)*co
sh(d*x + c)^4 + (30*a^3 + 90*a^2*b - 30*a*b^2 - 26*b^3 + 15*(5*a^3 + 45*a^2
*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*
a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 - 3*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 50*a^3 - 90*a^2*b + 30*a*b^2 - 22*b^3 + (75*a^3 - 45*a^2*b - 15*a*
b^2 + 41*b^3)*cosh(d*x + c)^2 + (15*(5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*c
osh(d*x + c)^4 + 75*a^3 - 45*a^2*b - 15*a*b^2 + 41*b^3 + 12*(15*a^3 + 45*a^
2*b - 15*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5*a^3 + 1
5*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 8*(5*a^3 - 3*b^3)*cosh(d*x + c)^3 + (25*
a^3 - 45*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10
+ 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*cosh(d*x +
c)^8 + (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)
```

```
^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 + (210*d*cosh
(d*x + c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d
*x + c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 8*d
*cosh(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4 - 45*d*co
sh(d*x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 + 28*d*cosh(
d*x + c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*d
*cosh(d*x + c)^2 + (45*d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^6 - 45*d*cosh
(d*x + c)^4 - 48*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x
 + c)^9 + 8*d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 -
 2*d*cosh(d*x + c))*sinh(d*x + c) + 6*d)
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{csch}^4\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(92) = 184$.

Time = 0.20 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.03

$$\begin{aligned}
&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&=\frac{4}{15}b^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right. \\
&\quad+4\,ab^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right) \\
&\quad+\frac{4}{3}\,a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right) \\
&\quad+\frac{12\,a^2b}{d(e^{(-4\,dx-4\,c)}-1)}
\end{aligned}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c
```

)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 12*a^2*b/(d*(e^(-4*d*x - 4*c) - 1))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(92) = 184$.

Time = 0.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{2\left(\frac{5\left(9a^2be^{(4dx+4c)}+6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}-2a^3+9a^2b\right)}{\left(e^{(2dx+2c)}-1\right)^3}-\frac{45a^2be^{(8dx+8c)}+180a^2be^{(6dx+6c)}-90ab^2e^{(6dx+6c)}-30b^3e^{(6dx}}{}\right.}{15}$$

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] -2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) - 2*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) - 210*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) - 150*a*b^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + 45*a^2*b - 30*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 622, normalized size of antiderivative = 6.35

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{\frac{2(9a^2b-12ab^2+4b^3)}{15d} - \frac{4e^{2c+2dx}(-3a^2b+3ab^2+b^3)}{5d} + \frac{6a^2be^{4c+4dx}}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$- \frac{\frac{2(-3a^2b+3ab^2+b^3)}{5d} + \frac{6e^{4c+4dx}(-3a^2b+3ab^2+b^3)}{5d} - \frac{2e^{2c+2dx}(9a^2b-12ab^2+4b^3)}{5d} - \frac{6a^2be^{6c+6dx}}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$+ \frac{\frac{6a^2b}{5d} - \frac{8e^{6c+6dx}(-3a^2b+3ab^2+b^3)}{5d} - \frac{8e^{2c+2dx}(-3a^2b+3ab^2+b^3)}{5d} + \frac{4e^{4c+4dx}(9a^2b-12ab^2+4b^3)}{5d} + \frac{6a^2be^{8c+8dx}}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$- \frac{\frac{2(-3a^2b+3ab^2+b^3)}{5d} - \frac{6a^2be^{2c+2dx}}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1} - \frac{4a^3}{d\,(e^{4c+4dx}-2e^{2c+2dx}+1)}$$
$$- \frac{8a^3}{3d\,(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1)} - \frac{6a^2b}{d\,(e^{2c+2dx}-1)} + \frac{6a^2b}{5d\,(e^{2c+2dx}+1)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

```
[Out] ((2*(9*a^2*b - 12*a*b^2 + 4*b^3))/(15*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 - 3
*a^2*b + b^3))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 - 3*a^2*b + b
^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*exp(
2*c + 2*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) - (6*a^2*b*exp(6*c + 6*d*x
))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1) + ((6*a^2*b)/(5*d) - (8*exp(6*c + 6*d*x)*(3*a*b^2 - 3*
a^2*b + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d)
+ (4*exp(4*c + 4*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) + (6*a^2*b*exp(8*
c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(3*a*b^2 - 3*a
^2*b + b^3))/(5*d) - (6*a^2*b*exp(2*c + 2*d*x))/(5*d))/(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1)) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) - 1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) - 1)) + (6*a^2*b)/(5*d*(exp(2
*c + 2*d*x) + 1))
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3d} - \frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

```
[Out] 1/8*(3*a^2-6*a*b-b^2)*x/(a+b)^3-1/8*(5*a+b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2
/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)
/a^(1/2))*b^(1/2)/(a+b)^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3744, 481, 541, 536, 212, 211}

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^3} + \frac{x(3a^2-6ab-b^2)}{8(a+b)^3} + \frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)} - \frac{(5a+b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2}$$

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*x)/(8*(a + b)^3) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tanh[c + d*x])/Sqrt[a]])/((a + b)^3*d) - ((5*a + b)*Cosh[c + d*x]*Sinh[c +
d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
```

```
+ 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} - \frac{\text{Subst}\left(\int \frac{a+(4a+b)x^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$
$$= -\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} - \frac{\text{Subst}\left(\int \frac{-a(3a-b)+b(5a+b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^2d}$$
$$= -\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d} + \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^3d}$$
$$= \frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3d} - \frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{4(3a^2-6ab-b^2)(c+dx) + 32a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8a(a+b)\sinh(2(c+dx)) + (a+b)^2\sinh(}{32(a+b)^3d}$$

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (4*(3*a^2 - 6*a*b - b^2)*(c + d*x) + 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
h[c + d*x])/Sqrt[a]] - 8*a*(a + b)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(c
+ d*x)])/(32*(a + b)^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(104) = 208$.

Time = 11.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

method	result
risch	$\frac{3a^2x}{8(a+b)^3} - \frac{3axb}{4(a+b)^3} - \frac{x\,b^2}{8(a+b)^3} + \frac{e^{4dx+4c}}{64d(a+b)} - \frac{a\,e^{2dx+2c}}{8(a+b)^2 d} + \frac{a\,e^{-2dx-2c}}{8(a^2+2ab+b^2)d} - \frac{e^{-4dx-4c}}{64d(a+b)} + \frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}\right.}{2(a+b}$
derivativedivides	$\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4} + \frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{a-3b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{3a-b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{(-3a}{}$
default	$\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4} + \frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{a-3b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{3a-b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{(-3a}{}$

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x/(a+b)^3-3/4*a*x/(a+b)^3*b-1/8*x/(a+b)^3*b^2+1/64/d/(a+b)*exp(4*d*
x+4*c)-1/8*a/(a+b)^2/d*exp(2*d*x+2*c)+1/8*a/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*
c)-1/64/d/(a+b)*exp(-4*d*x-4*c)+1/2*(-a*b)^(1/2)*a/(a+b)^3/d*ln(exp(2*d*x+2
*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))-1/2*(-a*b)^(1/2)*a/(a+b)^3/d*ln(exp(2*d*x+2
*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 2024, normalized size of antiderivative = 17.15

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 - 6*
a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 - 6*a*b - b^2)*d
```

```
*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c) - 20*(a^2
+ a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4
*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 - 6*a*b - b^2)*d*x*cosh
(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)
^2 + 32*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*sinh(d*x + c) + 6*a*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
+ c)^4)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*co
sh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cos
h(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(
d*x + c)^8 + 8*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*co
sh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*s
inh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*c
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
 4*(3*a^2 - 6*a*b - b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*
x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2
+ a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a
^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*
a^2 + 2*a*b)*sinh(d*x + c)^2 + 64*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*
sinh(d*x + c) + 6*a*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(a*b)/(a*b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(
d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4)]
```

Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(104) = 208$.

Time = 0.34 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.36

$$\begin{aligned}\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= -\frac{(ab-b^2)(dx+c)}{2(a^3+3a^2b+3ab^2+b^3)d} + \frac{(8be^{(-2dx-2c)}+a+b)e^{(4dx+4c)}}{64(a^2+2ab+b^2)d} \\ &\quad - \frac{b\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4(a^2+2ab+b^2)d} \\ &\quad + \frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4(a^2+2ab+b^2)d} \\ &\quad + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{4(a^2+2ab+b^2)\sqrt{ab}d} \\ &\quad - \frac{(a^2b-6ab^2+b^3)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}d} \\ &\quad - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4(a^2+2ab+b^2)\sqrt{ab}d} \\ &\quad - \frac{3b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}(a+b)d} \\ &\quad - \frac{8be^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}}{64(a^2+2ab+b^2)d} \\ &\quad + \frac{3(dx+c)}{8(a+b)d} - \frac{e^{(2dx+2c)}}{8(a+b)d} + \frac{e^{(-2dx-2c)}}{8(a+b)d}\end{aligned}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b*
e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) - 1/4*b*l
og((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a
```

```
*b + b^2)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/8
*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) - 1/4*(a*b - b^2)*arct
an(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*s
qrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) - 1/8*e^(2*d*x +
2*c)/((a + b)*d) + 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^4}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.12

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{4c+4dx}}{64d(a+b)} - \frac{e^{-4c-4dx}}{64d(a+b)} - \frac{x(-3a^2+6ab+b^2)}{8(a+b)^3} + \frac{a\,e^{-2c-2dx}}{8d(a+b)^2} - \frac{a\,e^{2c+2dx}}{8d(a+b)^2} + \frac{(-a)^{3/2}\sqrt{b}\ln\left((-a)^{3/2}b^{3/2}\left(e^{2c+2dx}-1\right)-2a^2b\,e^{2c+2dx}+(-a)^{5/2}\sqrt{b}\left(e^{2c+2dx}+1\right)\right)}{2d(a+b)^3} - \frac{(-a)^{3/2}\sqrt{b}\ln\left(2a^2b\,e^{2c+2dx}+(-a)^{3/2}b^{3/2}\left(e^{2c+2dx}-1\right)+(-a)^{5/2}\sqrt{b}\left(e^{2c+2dx}+1\right)\right)}{2d(a+b)^3}$$

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(4*c + 4*d*x)/(64*d*(a + b)) - exp(- 4*c - 4*d*x)/(64*d*(a + b)) - (x*(6
*a*b - 3*a^2 + b^2))/(8*(a + b)^3) + (a*exp(- 2*c - 2*d*x))/(8*d*(a + b)^2)
 - (a*exp(2*c + 2*d*x))/(8*d*(a + b)^2) + ((-a)^(3/2)*b^(1/2)*log((-a)^(3/2
)*b^(3/2)*(exp(2*c + 2*d*x) - 1) - 2*a^2*b*exp(2*c + 2*d*x) + (-a)^(5/2)*b^
(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3) - ((-a)^(3/2)*b^(1/2)*log(2*
a^2*b*exp(2*c + 2*d*x) + (-a)^(3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) + (-a)^(
5/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d}-\frac{a\cosh(c+dx)}{(a+b)^2d}+\frac{\cosh^3(c+dx)}{3(a+b)d}$$

```
[Out] -a*cosh(d*x+c)/(a+b)^2/d+1/3*cosh(d*x+c)^3/(a+b)/d+a*arctanh(sech(d*x+c)*b^
(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3745, 464, 331, 214}

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}+\frac{\cosh^3(c+dx)}{3d(a+b)}-\frac{a\cosh(c+dx)}{d(a+b)^2}$$

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d)
- (a*Cosh[c + d*x])/((a + b)^2*d) + Cosh[c + d*x]^3/(3*(a + b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)d}\\
&= -\frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{(ab)\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)^2d}\\
&= \frac{a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} - \frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{12ia\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)-3(3a-b)\sqrt{a+b}\cosh(c+}{12(a+b)^{5/2}d}$$

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((12*I)*a*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]])/Sqrt[b]]) -
3*(3*a - b)*Sqrt[a + b]*Cosh[c + d*x] + (a + b)^(3/2)*Cosh[3*(c + d*x)])/(1
2*(a + b)^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(65) = 130.

Time = 3.90 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.69

method	result
derivativedivides	$\frac{-\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{ab\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2}{4\sqrt{ab+b^2}}\right.}{(a+b)^2\sqrt{ab+b^2}}}{d}$
default	$\frac{-\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{ab\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2}{4\sqrt{ab+b^2}}\right.}{(a+b)^2\sqrt{ab+b^2}}}{d}$
risch	$\frac{e^{3dx+3c}}{24d(a+b)}-\frac{3e^{dx+c}a}{8(a+b)^2d}+\frac{e^{dx+c}b}{8(a+b)^2d}-\frac{3e^{-dx-c}a}{8(a+b)^2d}+\frac{e^{-dx-c}b}{8(a+b)^2d}+\frac{e^{-3dx-3c}}{24d(a+b)}+\frac{\sqrt{(a+b)b}\,a\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e}{a+b}\right.}{2(a+b)^3d}$

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(16*a+16*b)/(1+tanh(1/2*d*x+1/2*c))^2+16/3/(1+tanh(1/2*d*x+1/2*c))^
3/(16*a+16*b)-1/2*(a-b)/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))+a*b/(a+b)^2/(a*b+b^
2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1
6/3/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c
)-1)^2-1/2/(a+b)^2*(-a+b)/(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 1367, normalized size of antiderivative = 18.23

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 +
(a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d
*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*(3*
a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c)^2 + 3*(5*
(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x +
 c)^2 + 12*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cos
h(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(b/(a + b))*log(((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
 + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a +
b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + 6*((a + b)*cosh(d*x + c)^5 - 2*(3*a - b)*cosh(d*x
+ c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^
2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 + 2*a*b +
b^2)*d*sinh(d*x + c)^3), 1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)
^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)
*cosh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)
*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)
^2 - 3*a + b)*sinh(d*x + c)^2 + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2
*sinh(d*x + c) + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqr
t(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(
a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 24*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh(d*x + c)*s
inh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(-b/(a + b))*arctan(1/2*((a + b)*co
sh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) + 6*((a + b)*cosh(
d*x + c)^5 - 2*(3*a - b)*cosh(d*x + c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*
x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b
```

```
^2)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^3{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^3}{b\tanh{(dx+c)}^2+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) - 3*(3*a*e^(4*c) - b*e^(4*c))*e^(4*
d*x) - 3*(3*a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-3*d*x)/(a^2*d*e^(
3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) - 1/8*integrate(16*(a*b*e^(3*d*x +
3*c) - a*b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) + 3*a
^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) +
a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^3}{b\tanh{(dx+c)}^2+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 955, normalized size of antiderivative = 12.73

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{-3c-3dx}}{24d\,(a+b)} + \frac{e^{3c+3dx}}{24d\,(a+b)} - \frac{\sqrt{a^2 b}\left(2\,\mathrm{atan}\left(\frac{\left(e^{dx}\,e^{c}\left(\frac{4\left(2a^2b^3d\sqrt{a^2b}+4a^3b^2d\sqrt{a^2b}+2a^4bd\sqrt{a^2b}\right)}{a\,(a+b)\sqrt{-d^2(a+b)^5}\left(a^2+2ab+b^2\right)\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-a^5d^2-5a^4bd^2-10a^3b^2d^2-10a^2b^3d^2-5ab^4d^2-b^5}}\right.\right.}{}\right.\right.}{} - \frac{e^{-c-dx}\,(3a-b)}{8d\,(a+b)^2} - \frac{e^{c+dx}\,(3a-b)}{8d\,(a+b)^2}$$

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(- 3*c - 3*d*x)/(24*d*(a + b)) + exp(3*c + 3*d*x)/(24*d*(a + b)) - ((a^2
*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((4*(2*a^2*b^3*d*(a^2*b)^(1/2) + 4*a^3*
b^2*d*(a^2*b)^(1/2) + 2*a^4*b*d*(a^2*b)^(1/2)))/(a*(a + b)*(-d^2*(a + b)^5)
^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(- a^5*d^2 - b^5
*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2))
+ (2*a^3*b)/(d*(a + b)^3*(a^2*b)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2
*b + a^3 + b^3))) + (2*a^3*b*exp(3*c)*exp(3*d*x))/(d*(a + b)^3*(a^2*b)^(1/2
)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(a^6*(- a^5*d^2 - b
^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2)
 + b^6*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 -
10*a^3*b^2*d^2)^(1/2) + 15*a^2*b^4*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a
^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 20*a^3*b^3*(- a^5*d^2 -
 b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/
2) + 15*a^4*b^2*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b
^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6*a*b^5*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2
 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6*a^5*b*(- a^5*d^
2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^
(1/2)))/(4*a^2*b)) - 2*atan((a*exp(d*x)*exp(c)*(-d^2*(a + b)^5)^(1/2))/(2*d
*(a + b)^2*(a^2*b)^(1/2)))))/(2*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*
b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2)) - (exp(- c - d*x)*(3*a - b)
)/(8*d*(a + b)^2) - (exp(c + d*x)*(3*a - b))/(8*d*(a + b)^2)
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

```
[Out] -1/2*(a-b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d-arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3744, 482, 536, 212, 211}

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)} - \frac{x(a-b)}{2(a+b)^2}$$

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((a - b)*x)/(a + b)^2 - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/((a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{a-bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^2d}\\
&\quad - \frac{(ab)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d}
\end{aligned}$$

$$= -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-2(a-b)(c+dx) - 4\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + (a+b)\sinh(2(c+dx))}{4(a+b)^2 d}$$

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-2*(a - b)*(c + d*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sq
rt[a]] + (a + b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(66) = 132$.

Time = 1.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{ax}{2(a+b)^2}+\frac{xb}{2(a+b)^2}+\frac{e^{2dx+2c}}{8d(a+b)}-\frac{e^{-2dx-2c}}{8d(a+b)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2(a+b)^2 d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\cdots}{a+\cdots}\right)}{2(a+b)^2 d}$
derivativedivides	$-\frac{4}{(8a+8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2}+\frac{2a^2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a\cdots}\right.}}\right.}{\cdots}$
default	$-\frac{4}{(8a+8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2}+\frac{2a^2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a\cdots}\right.}}\right.}{\cdots}$

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*x/(a+b)^2+1/2*x/(a+b)^2*b+1/8/d/(a+b)*exp(2*d*x+2*c)-1/8/d/(a+b)*exp
(-2*d*x-2*c)+1/2*(-a*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a
```

```
+b)/(a+b))-1/2*(-a*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b
)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(66) = 132$.

Time = 0.29 (sec) , antiderivative size = 916, normalized size of antiderivative = 11.74

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*x
- 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(-a*b)*(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d
*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
+ c) + a + b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cosh(d*x + c)^3)*
sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*
a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x +
 c)^2), -1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a -
b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*sqrt(a*b)*(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*arctan(1/2*((a +
 b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(
d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a
+ b)*cosh(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a
*b + b^2)*d*sinh(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(66) = 132.

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.05

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &\quad -\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &\quad -\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &\quad +\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &\quad +\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}(a+b)d} \\ &\quad -\frac{dx+c}{2\,(a+b)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((
a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d
) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x -
 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/2*(d*x + c)/((a + b)*d)
 + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.54

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{e^{2c+2dx}}{8d\,(a+b)} - \frac{e^{-2c-2dx}}{8d\,(a+b)} - \frac{x\,(a-b)}{2\,(a+b)^2} \\ &\quad - \frac{\sqrt{-a}\,\sqrt{b}\,\ln\left(\sqrt{-a}\,b^{3/2}\,\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\,\sqrt{b}\,\left(e^{2c+2dx}+1\right)-2\,a\,b\,e^{2c+2dx}\right)}{2\,d\,(a+b)^2} \\ &\quad + \frac{\sqrt{-a}\,\sqrt{b}\,\ln\left(\sqrt{-a}\,b^{3/2}\,\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\,\sqrt{b}\,\left(e^{2c+2dx}+1\right)+2\,a\,b\,e^{2c+2dx}\right)}{2\,d\,(a+b)^2}\end{aligned}$$

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
[Out] exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) - (x*(a -
 b))/(2*(a + b)^2) - ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2*c +
2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) - 2*a*b*exp(2*c + 2
*d*x)))/(2*d*(a + b)^2) + ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2
*c + 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) + 2*a*b*exp(2*
c + 2*d*x)))/(2*d*(a + b)^2)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}+\frac{\cosh(c+dx)}{(a+b)d}$$

```
[Out] cosh(d*x+c)/(a+b)/d-arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^
(3/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3745, 331, 214}

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\cosh(c+dx)}{d(a+b)}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d))
 + Cosh[c + d*x]/((a + b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{\cosh(c+dx)}{(a+b)d} - \frac{b\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)d}\\ &= -\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{(a+b)d}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)+\sqrt{a+b}\cosh(c+dx)}{(a+b)^{3/2}d}$$

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]

[Out] ((-I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b
]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]) + Sqrt
[a + b]*Cosh[c + d*x])/((a + b)^(3/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{-\frac{b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}}+\frac{4}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
default	$\frac{-\frac{b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}}+\frac{4}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
risch	$\frac{e^{dx+c}}{2d(a+b)}+\frac{e^{-dx-c}}{2d(a+b)}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)^2 d}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)^2 d}$

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*
b)/(a*b+b^2)^(1/2))+4/(4*a+4*b)/(1+tanh(1/2*d*x+1/2*c))-4/(4*a+4*b)/(tanh(1
/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(45) = 90$.

Time = 0.31 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{\frac{b}{a+b}}(\cosh(dx+c)+\sinh(dx+c))\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a+3\,b)\,\cdots}{(a+b)\,\mathrm{co}\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\sqrt{-\frac{b}{a+b}}(\cosh(dx+c)+\sinh(dx+c))\arctan\left(\frac{\left((a+b)\cosh(dx+c)^3+3\,(a+b)\cosh(dx+c)\sinh(dx+c)^2+(a+b)\sinh(dx+c)\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*log(((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*si
```

```
nh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x +
c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*
x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d
*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)), -1/2*(2*sqr
t(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x
+ c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3
+ (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x
+ c))*sqrt(-b/(a + b))/b) - 2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x +
c))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a +
 b))/b) - cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) - sinh(d*x + c)^2
 - 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\sinh{\left(c+dx \right)}}{a+b\tanh^{2}{\left(c+dx \right)}}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\sinh\left(dx+c\right)}{b\tanh\left(dx+c\right)^{2}+a}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b
*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b
*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)
), x)
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 520, normalized size of antiderivative = 9.81

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{c+dx}}{2d\,(a+b)} + \frac{e^{-c-dx}}{2d\,(a+b)} - \frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\sqrt{-d^2\,(a+b)^3}}{2\sqrt{b}\,d\,(a+b)}\right) - 2\,\mathrm{atan}\left(\frac{\left(e^{dx}\,e^{c}\left(\frac{2a\sqrt{b}}{d\,(a+b)^2\,(a^3+3a^2b+3ab^2+b^3)} + \frac{4\,(2a^2b^{3/2}d+2ab^{5}\cdots}{(a+b)\sqrt{-d^2\,(a+b)^3}\sqrt{-a^3d^2-3a^2bd^2-3ab^2\cdots}}\cdots\right)\cdots\right)}{\cdots}\right)\cdots\right)}{\cdots}$$

[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2),x)

[Out] exp(c + d*x)/(2*d*(a + b)) + exp(- c - d*x)/(2*d*(a + b)) - (b^(1/2)*(2*atan((exp(d*x)*exp(c)*(-d^2*(a + b)^3)^(1/2))/(2*b^(1/2)*d*(a + b))) - 2*atan(((exp(d*x)*exp(c)*((2*a*b^(1/2))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (4*(2*a^2*b^(3/2)*d + 2*a*b^(5/2)*d))/((a + b)*(-d^2*(a + b)^3)^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))) + (2*a*b^(1/2)*exp(3*c)*exp(3*d*x))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(a^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + b^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4*a*b^3*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4*a^3*b*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 6*a^2*b^2*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*a*b))))/(2*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))

3.29 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/
a/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3745, 400, 213, 214}

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}$$

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/
Sqrt[a + b]])/(a*Sqrt[a + b]*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{ad} + \frac{b\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{ad}\\ &= -\frac{\text{arctanh}(\cosh(c+dx))}{ad} + \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45

$$\int \frac{\text{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{i\sqrt{b}\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}} + \frac{i\sqrt{b}\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}} - \log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sinh\left(\frac{1}{2}(c\right.\right.}{ad}$$

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] ((I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])
/Sqrt[a + b] + (I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)
/2])/Sqrt[b]])/Sqrt[a + b] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]
])/(a*d)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{a\sqrt{ab+b^{2}}}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{a\sqrt{ab+b^{2}}}}{d}$
risch	$-\frac{\ln(e^{dx+c}+1)}{da}+\frac{\ln(e^{dx+c}-1)}{da}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)da}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)}}{a+}\right.}{2(a+b)da}$

[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))+b/a/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 587, normalized size of antiderivative = 10.67

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a+3\,b)\cosh(dx+c)^2+2\left(3\,(a+b)\cosh(dx+c)^2+a\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx}\right.}{}\right.$$

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*(sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d),

```
(sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) -
sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))
*sqrt(-b/(a + b))/b) - log(cosh(d*x + c) + sinh(d*x + c) + 1) + log(cosh(d*
x + c) + sinh(d*x + c) - 1))/(a*d)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c)
 + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.16

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(9\,b^4\,\sqrt{-a^2\,d^2}+16\,a^2\,b^2\,\sqrt{-a^2\,d^2}+24\,a\,b^3\,\sqrt{-a^2\,d^2}\right)}{16\,d\,a^3\,b^2+24\,d\,a^2\,b^3+9\,d\,a\,b^4}\right)}{\sqrt{-a^2\,d^2}} - \frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}\,\sqrt{-a^2\,d^2\,(a+b)}+e^{3\,c}\,e^{3\,d\,x}\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}\,\sqrt{-a^2\,d^2\,(a+b)}+4\,a^2\,b\,d^2\,e^{d\,x}\,e^{c}}{2\,a\,\sqrt{b}\,d\,\sqrt{-a^2\,d^2\,(a+b)}}\right) - 2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}}{}\right.\right.}{2\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}}$$

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] - (2*atan((exp(d*x)*exp(c)*(9*b^4*(-a^2*d^2)^(1/2) + 16*a^2*b^2*(-a^2*d^2)^
(1/2) + 24*a*b^3*(-a^2*d^2)^(1/2)))/(24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^4*
d)))/(-a^2*d^2)^(1/2) - (b^(1/2)*(2*atan((exp(d*x)*exp(c)*(- a^3*d^2 - a^2*
b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(- a^3*d^2 - a^
2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + 4*a^2*b*d^2*exp(d*x)*exp(c))/(2*a
*b^(1/2)*d*(-a^2*d^2*(a + b))^(1/2))) - 2*atan((exp(d*x)*exp(c)*(-a^2*d^2*(
a + b))^(1/2))/(2*a*b^(1/2)*d))))/(2*(- a^3*d^2 - a^2*b*d^2)^(1/2))
```

3.30 $\int \frac{\mathrm{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \frac{\mathrm{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

```
[Out] -coth(d*x+c)/a/d-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3744, 331, 211}

$$\int \frac{\mathrm{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{\coth(c+dx)}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{ad}\\ &= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(40) = 80$.

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.33

method	result
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a/(exp(2*d*x+2*c)-1)+1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b
)^(1/2)-a+b)/(a+b))-1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2
)+a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 618, normalized size of antiderivative = 12.88

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{\left(\cosh\left(dx+c\right)^2+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2-1\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(dx+c)^4+4\,(a^2\right.}{}\right.}{}\right.$$

$$-\frac{\left(\cosh\left(dx+c\right)^2+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2-1\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2\,(a+b)\right.}{}\right.}{ad\cosh\left(dx+c\right)^2+2\,ad\cosh\left(dx+c\right)\sinh\left(dx+c\right)+ad\sinh\left(dx+\right.}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2
*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cos
h(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh
(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b
)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4)/(a*
d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2
 - a*d), -((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
+ c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 2)/(a*
d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2
 - a*d)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}ad} + \frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
+ 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2}{a\,d-a\,d\,e^{2\,c+2\,d\,x}} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d}-\frac{\sqrt{b}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}+\frac{e^{2\,c}\,e^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d}+\frac{\sqrt{b}\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}\right)}{\sqrt{a^3\,d^2}}$$

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
[Out] 2/(a*d - a*d*exp(2*c + 2*d*x)) - (b^(1/2)*atan((a^3*d^2)^(1/2)/(2*a*b^(1/2)
*d) - (b^(1/2)*(a^3*d^2)^(1/2))/(2*a^2*d) + (exp(2*c)*exp(2*d*x)*(a^3*d^2)^
(1/2))/(2*a*b^(1/2)*d) + (b^(1/2)*exp(2*c)*exp(2*d*x)*(a^3*d^2)^(1/2))/(2*a
^2*d)))/(a^3*d^2)^(1/2)
```

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+2b)\operatorname{arctanh}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[Out] 1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d-arct
anh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 482, 536, 213, 214}

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+2b)\operatorname{arctanh}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Sqrt[b]*Sqrt[a + b]*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*
x])/(2*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{2ad}\\
&= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{(b(a+b))\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{a^2d}\\
&\quad - \frac{(a+2b)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2a^2d}
\end{aligned}$$

$$= \frac{(a+2b)\text{arctanh}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{\text{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{8i\sqrt{b}\sqrt{a+b}\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + 8i\sqrt{b}\sqrt{a+b}\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + a\text{csch}^2(}{}$$

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/8*((8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c
+ d*x)/2])/Sqrt[b]] + (8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] +
Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + a*Csch[(c + d*x)/2]^2 - 4*a*Log[Cosh[
(c + d*x)/2]] - 8*b*Log[Cosh[(c + d*x)/2]] + 4*a*Log[Sinh[(c + d*x)/2]] + 8
*b*Log[Sinh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^2*d)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a} - \frac{(a+b)b\,\text{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{a^2\sqrt{ab+b^2}} - \frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a} - \frac{(a+b)b\,\text{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{a^2\sqrt{ab+b^2}} - \frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}}{d}$
risch	$-\frac{e^{dx+c}(e^{2dx+2c}+1)}{da(e^{2dx+2c}-1)^2} + \frac{\ln(e^{dx+c}+1)}{2da} + \frac{\ln(e^{dx+c}+1)b}{d\,a^2} - \frac{\ln(e^{dx+c}-1)}{2da} - \frac{\ln(e^{dx+c}-1)b}{d\,a^2} + \frac{\sqrt{ab+b^2}\,\ln\left(e^{2dx+2c}\right.}{2}$

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-(a+b)*b/a^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2
*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/8/a/tanh(1/2*d*x+1/2*c
)^2+1/4/a^2*(-2*a-4*b)*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(73) = 146$.

Time = 0.30 (sec) , antiderivative size = 1790, normalized size of antiderivative = 21.06

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
 c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(
cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b + b^2)*log(((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d
*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c
))*sqrt(a*b + b^2) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*a*cosh(d*x +
c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2
*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cos
h(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d
*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(
a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(
d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 -
 a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*s
inh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^
2 + 2*a*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a
*b - b^2)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a +
b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-a*b - b^2)/(a*b + b^2))
- 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a*b - b^2)*arctan(1/2*sqr
```

```
t(-a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/b) + 2*a*cosh(d*x + c) - ((a
+ 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2
*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x
 + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*
b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c
) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cos
h(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^
4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*s
inh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a +
2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*((a*b*e^(3*c)
+ b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3
*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*
d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{b\tanh\left(dx+c\right)^2+a}\,dx$$

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 787, normalized size of antiderivative = 9.26

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(18\,b^7\,\sqrt{-a^4\,d^2}+48\,a^2\,b^5\,\sqrt{-a^4\,d^2}+27\,a^3\,b^4\,\sqrt{-a^4\,d^2}+8\,a^4\,b^3\,\sqrt{-a^4\,d^2}+a^5\,b^2\,\sqrt{-a^4\,d^2}+45\,a\,b^6\,\sqrt{-a^4\,d^2}\right)}{9\,a^2\,b^6\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+18\,a^3\,b^5\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+15\,a^4\,b^4\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+6\,a^5\,b^3\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+a^6\,b^2\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}}\right)\,\sqrt{\cdots}}{\sqrt{-a^4\,d^2}}$$
$$-\frac{\left(2\,\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,(a+b)\,\sqrt{-a^4\,d^2}}{2\,a^2\,d\,\sqrt{b\,(a+b)}}\right)+2\,\operatorname{atan}\left(\frac{\left(e^{dx}\,e^{c}\,\left(\frac{64\,\left(2\,a^4\,b\,d\,\sqrt{b^2+a\,b}+6\,a^2\,b^3\,d\,\sqrt{b^2+a\,b}+6\,a^3\,b^2\,d\,\sqrt{b^2+a\,b}\right)}{a^9\,d^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}-\frac{32\,\left(3\,b^4\,\sqrt{-a^4\,d^2}+4\,a^2\,b^2\cdots\right.}{a^7\,d\,(a+b)\,\sqrt{\cdots}}\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}$$
$$-\frac{e^{c+d\,x}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,e^{c+d\,x}}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)

[Out] (atan((exp(d*x)*exp(c)*(18*b^7*(-a^4*d^2)^(1/2) + 48*a^2*b^5*(-a^4*d^2)^(1/2) + 27*a^3*b^4*(-a^4*d^2)^(1/2) + 8*a^4*b^3*(-a^4*d^2)^(1/2) + a^5*b^2*(-a^4*d^2)^(1/2) + 45*a*b^6*(-a^4*d^2)^(1/2)))/(9*a^2*b^6*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 18*a^3*b^5*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 15*a^4*b^4*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 6*a^5*b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + a^6*b^2*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a^4*d^2)^(1/2) - ((2*atan((exp(d*x)*exp(c)*(a + b)*(-a^4*d^2)^(1/2))/(2*a^2*d*(b*(a + b))^(1/2))) + 2*atan(((exp(d*x)*exp(c)*((64*(2*a^4*b*d*(a*b + b^2)^(1/2) + 6*a^2*b^3*d*(a*b + b^2)^(1/2) + 6*a^3*b^2*d*(a*b + b^2)^(1/2)))/(a^9*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2))) - (32*exp(3*c)*exp(3*d*x)*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2)))*(a^8*(-a^4*d^2)^(1/2) + a^5*b^3*(-

```
a^4*d^2)^(1/2) + 3*a^6*b^2*(-a^4*d^2)^(1/2) + 3*a^7*b*(-a^4*d^2)^(1/2)))/(1
92*a*b^2 + 64*a^2*b + 192*b^3)))*(a*b + b^2)^(1/2))/(2*(-a^4*d^2)^(1/2)) -
exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(4*c
+ 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.32 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

```
[Out] (a+b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d+(a+b)*arctan(b^(1/2)*tanh(d*x
+c)/a^(1/2))*b^(1/2)/a^(5/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3744, 464, 331, 211}

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((a
 + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth^3(c+dx)}{3ad} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{(b(a+b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{a^2d}\\
&= \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{3\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\coth(c+dx)\left(2a+3b-a\operatorname{csch}^2(c+dx)\right)}{3a^{5/2}d}$$

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (3*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Coth[c
 + d*x]*(2*a + 3*b - a*Csch[c + d*x]^2))/(3*a^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(60) = 120.

Time = 1.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.51

method	result
risch	$-\frac{2\left(-3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+6b\,\mathrm{e}^{2dx+2c}-2a-3b\right)}{3d\,a^2\left(\mathrm{e}^{2dx+2c}-1\right)^3}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2d}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}}{a+}\right.}{2a^3d}$
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{2b(a+b)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+\right.}{2a}\right.}{a}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{2b(a+b)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+\right.}{2a}\right.}{a}}{d}$

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a+6*b*exp(2*d*x+2*c)-2*a-3*b)/d/
a^2/(exp(2*d*x+2*c)-1)^3+1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)
^(1/2)+a-b)/(a+b))+1/2/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)
+a-b)/(a+b))*b-1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b
)/(a+b))-1/2/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b
))*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(60) = 120.

Time = 0.28 (sec) , antiderivative size = 1628, normalized size of antiderivative = 23.26

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sinh
(d*x + c)^4 - 24*(a + b)*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 - a - b)
*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(
a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c
)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2
+ 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 + (a + b)
*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c)
)*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
+ a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
c))*sinh(d*x + c) + a + b)) + 48*(b*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c)
)*sinh(d*x + c) + 8*a + 12*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c
)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2
*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 +
4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d
 + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x +
 c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x
 + c))*sinh(d*x + c)), 1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 6*b*sinh(d*x + c)^4 - 12*(a + b)*cosh(d*x + c)^2 + 12*(3*b*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a +
b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh
(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*
(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a +
 b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c
)^2 + a + b)*sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(
d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(b/a)*arctan
(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a
```

```
+ b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 24*(b*cosh(d*x + c)^3 - (a + b
)*cosh(d*x + c))*sinh(d*x + c) + 4*a + 6*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*
d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x
+ c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh
(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x
+ c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2
*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 +
a^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}^4\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{2\left(6\,(a+b)e^{(-2\,dx-2\,c)}-3\,be^{(-4\,dx-4\,c)}-2\,a-3\,b\right)}{3\left(3\,a^2e^{(-2\,dx-2\,c)}-3\,a^2e^{(-4\,dx-4\,c)}+a^2e^{(-6\,dx-6\,c)}-a^2\right)d}-\frac{(ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2d}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 2/3*(6*(a + b)*e^(-2*d*x - 2*c) - 3*b*e^(-4*d*x - 4*c) - 2*a - 3*b)/((3*a^2
*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*d)
 - (a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sq
rt(a*b)*a^2*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^4}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.63

$$\begin{aligned}
&\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx \\
&= \frac{2b}{a^2 d\,(e^{2c+2dx}-1)} - \frac{8}{3ad\,(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1)} \\
&\quad - \frac{4}{ad\,(e^{4c+4dx}-2e^{2c+2dx}+1)} \\
&\quad + \frac{\sqrt{-b}\,\ln\left(-\frac{4b\,e^{2c+2dx}}{a^2}-\frac{2\sqrt{-b}\,(ad+bd+ad\,e^{2c+2dx}-bd\,e^{2c+2dx})}{a^{5/2}\,d}\right)(a+b)}{2a^{5/2}\,d} \\
&\quad - \frac{\sqrt{-b}\,\ln\left(\frac{2\sqrt{-b}\,(ad+bd+ad\,e^{2c+2dx}-bd\,e^{2c+2dx})}{a^{5/2}\,d}-\frac{4b\,e^{2c+2dx}}{a^2}\right)(a+b)}{2a^{5/2}\,d}
\end{aligned}$$

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
[Out] (2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp(2
*c + 2*d*x) + 1)) + ((-b)^(1/2)*log(- (4*b*exp(2*c + 2*d*x))/a^2 - (2*(-b)^
(1/2)*(a*d + b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/2)*d
))*(a + b))/(2*a^(5/2)*d) - ((-b)^(1/2)*log((2*(-b)^(1/2)*(a*d + b*d + a*d*
exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/2)*d) - (4*b*exp(2*c + 2*d*
x))/a^2)*(a + b))/(2*a^(5/2)*d)
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3(a^2-6ab+b^2)\,x}{8(a+b)^4}+\frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4d} - \frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 3/8*(a^2-6*a*b+b^2)*x/(a+b)^4+3/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))
*a^(1/2)*b^(1/2)/(a+b)^4/d-1/8*(5*a-b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d/(a
+b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)
+3/8*(3*a-b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

= {3744, 481, 541, 536, 212, 211}

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3x(a^2-6ab+b^2)}{8(a+b)^4} + \frac{3\sqrt{a}\sqrt{b}(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2d(a+b)^4} + \frac{3b(3a-b)\tanh(c+dx)}{8d(a+b)^3\left(a+b\tanh^2(c+dx)\right)} + \frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{(5a-b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}$$

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*x)/(8*(a + b)^4) + (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a + b)^4*d) - ((5*a - b)*Cosh[c + d*
x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]^
3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)) + (3*(3*a - b)*b*Tan
h[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(4a-b)x^2}{(1-x^2)^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}\\
&= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-3a(a-b)+3(5a-b)bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^2d}\\
&= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{6a^2(a-3b)-6a(3a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{16a(a+b)^3d}
\end{aligned}
$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{(3a(a-b)b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^4d} + \frac{(3(a^2-6ab+b^2))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^4d}$$

$$= \frac{3(a^2-6ab+b^2)\,x}{8(a+b)^4} + \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4d} - \frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$= \frac{12(a^2-6ab+b^2)(c+dx) + 48\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8(a-b)(a+b)\sinh(2(c+dx)) + \cdots}{32(a+b)^4d}$$

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (12*(a^2 - 6*a*b + b^2)*(c + d*x) + 48*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sinh[2*(c + d*x)] + (16*a*b
*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + (a + b)^2
*Sinh[4*(c + d*x)])/(32*(a + b)^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(174) = 348.

Time = 69.26 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.67

method	result
derivativedivides	$-\frac{2ab\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}+\frac{(3a-3b)a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\cdots\right.}{2}\right)}{(a+b)^{4}}$
default	$-\frac{2ab\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}+\frac{(3a-3b)a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\cdots\right.}{2}\right)}{(a+b)^{4}}$
risch	$\frac{3x\,a^{2}}{8(a+b)^{2}(a^{2}+2ab+b^{2})}-\frac{9xab}{4(a+b)^{2}(a^{2}+2ab+b^{2})}+\frac{3x\,b^{2}}{8(a+b)^{2}(a^{2}+2ab+b^{2})}+\frac{e^{4dx+4c}}{64d(a^{2}+2ab+b^{2})}-\frac{e^{2dx+2c}a}{8(a+b)(a^{2}+2ab+b^{2}}$

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*b/(a+b)^4*(((-1/2*b-1/2*a)*tanh(1/2*d*x+1/2*c)^3+(-1/2*b-1/2*a)*t
anh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)+1/2*(3*a-3*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b
)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b
)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+1/4/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^4
+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a-7*b)/(a+b)^3/(tanh(1/2*d*x+1/
2*c)-1)^2-1/8*(3*a-5*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/8/(a+b)^4*(-3*a^2
+18*a*b-3*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/(a+b)^2/(1+tanh(1/2*d*x+1/2*c)
)^4+1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^3-1/8*(-a+7*b)/(a+b)^3/(1+tanh(1/2*
d*x+1/2*c))^2-1/8*(3*a-5*b)/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))+1/8/(a+b)^4*(3*
a^2-18*a*b+3*b^2)*ln(1+tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3531 vs. 2(174) = 348.

Time = 0.38 (sec) , antiderivative size = 7366, normalized size of antiderivative = 38.36

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh^4\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. 2(174) = 348.

Time = 0.48 (sec) , antiderivative size = 1690, normalized size of antiderivative = 8.80

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/4*(a*b - 2*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/2*b*log((a + b
)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d) + 1/4*(a*b - 2*b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b
)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)
 + 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/32*(3*a^3*b - 33*a^2*b^2 + 13*a*b^
3 + b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*a*b^
2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 33*a^2*b^2 + 13*a
```

```
*b^3 + b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) - 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 3/16*(3*a*b + b^2)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sq
rt(a*b)*d) - 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b^4 + (a^3*b - 15*a^2*b^2
+ 15*a*b^3 - b^4)*e^(2*d*x + 2*c))/((a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^
3 + 5*a^2*b^4 + a*b^5 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^
4 + a*b^5)*e^(4*d*x + 4*c) + 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a
^2*b^4 - a*b^5)*e^(2*d*x + 2*c))*d) + 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b
^4 + (a^3*b - 15*a^2*b^2 + 15*a*b^3 - b^4)*e^(-2*d*x - 2*c))/((a^6 + 5*a^5*
b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + 2*(a^6 + 3*a^5*b + 2*a^4*
b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*e^(-2*d*x - 2*c) + (a^6 + 5*a^5*b + 10
*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2*
b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e
^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(2*d*x + 2*c))*d)
+ 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d
*x - 4*c))*d) + 3/8*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c
) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) + 3/8*(d*x + c
)/((a^2 + 2*a*b + b^2)*d) + 1/64*((a + b)*e^(4*d*x + 4*c) + 16*b*e^(2*d*x +
 2*c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/64*(16*b*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*e^(2*d*
x + 2*c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2
)*d)
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

3.34 $\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{(3a-2b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} - \frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

```
[Out] -(a-b)*cosh(d*x+c)/(a+b)^3/d+1/3*cosh(d*x+c)^3/(a+b)^2/d+1/2*a*b*sech(d*x+c
)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)+1/2*(3*a-2*b)*arctanh(sech(d*x+c)*b^(1/2)
/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3745, 467, 1275, 214}

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}} + \frac{\cosh^3(c+dx)}{3d(a+b)^2} - \frac{(a-b)\cosh(c+dx)}{d(a+b)^3} + \frac{ab\text{sech}(c+dx)}{2d(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)}$$

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a +
b)^(7/2)*d) - ((a - b)*Cosh[c + d*x])/((a + b)^3*d) + Cosh[c + d*x]^3/(3*(a
 + b)^2*d) + (a*b*Sech[c + d*x])/(2*(a + b)^3*d*(a + b - b*Sech[c + d*x]^2)
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\operatorname{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{b\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}+\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4(a+b-bx^2)}\,dx, x, \operatorname{sech}(c+dx)\right)}{2d}$$

$$= \frac{ab\text{sech}(c+dx)}{2(a+b)^3 d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{b\text{Subst}\left(\int \left(-\frac{2}{b(a+b)^2x^4}+\frac{2(a-b)}{b(a+b)^3x^2}+\frac{3a-2b}{(a+b)^3(a+b-bx^2)}\right)dx, x, \text{sech}(c+dx)\right)}{2d}$$

$$= -\frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{((3a-2b)b)\text{Subst}\left(\int \frac{1}{a+b-bx^2}dx, x, \text{sech}(c+dx)\right)}{2(a+b)^3d}$$

$$= \frac{(3a-2b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} - \frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$= \frac{\frac{6i(3a-2b)\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}} + \frac{3\cosh(c+dx)\left(5b+a\left(-3+\frac{4b}{a-b+(a+b)\cosh(2(c+dx))}\right)\right)}{(a+b)^3} \cdots}{12d}$$

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((6*I)*(3*a - 2*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d
*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]]))/(a + b)^(7/2) + (3*Cosh[c + d*x]*(5*b + a*(-3 + (4*b)/(a - b + (a
+ b)*Cosh[2*(c + d*x)]))))/(a + b)^3 + Cosh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(110) = 220.

Time = 14.74 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-3b}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}\right.}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right.}{}}{}$
default	$\frac{\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-3b}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}\right.}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right.}{}}{}$
risch	$\frac{e^{3dx+3c}}{24d(a^2+2ab+b^2)}-\frac{3\,e^{dx+c}a}{8(a+b)(a^2+2ab+b^2)d}+\frac{5\,e^{dx+c}b}{8(a+b)(a^2+2ab+b^2)d}-\frac{3\,e^{-dx-c}a}{8(a^3+3a^2b+3a\,b^2+b^3)d}+\frac{5\,e^{-dx-c}}{8(a^3+3a^2b+3a}$

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^3-1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*
c))^2-1/2*(a-3*b)/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))-4*b/(a+b)^3*(((-1/4*a-1/2
*b)*tanh(1/2*d*x+1/2*c)^2-1/4*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8*(3*a-2*b)/(a*b+b^2)^(1/2)*arctanh
(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^2/(tan
h(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^3*(-a
+3*b)/(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. $2(113) = 226$.

Time = 0.34 (sec) , antiderivative size = 5025, normalized size of antiderivative = 40.52

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh^3{(c+dx)}}{(a+b\tanh^2{(c+dx)})^2}\,dx$$

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh{(dx+c)}^3}{(b\tanh{(dx+c)}^2+a)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(a^2 + 2*a*b + b^2 + (a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c))*e^
(10*d*x) - (7*a^2*e^(8*c) - 6*a*b*e^(8*c) - 13*b^2*e^(8*c))*e^(8*d*x) - 2*(
13*a^2*e^(6*c) - 40*a*b*e^(6*c) + 7*b^2*e^(6*c))*e^(6*d*x) - 2*(13*a^2*e^(4
*c) - 40*a*b*e^(4*c) + 7*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c) - 6*a*b*e^
(2*c) - 13*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(7*c) + 4*a^3*b*d*e^(7*c) + 6*
a^2*b^2*d*e^(7*c) + 4*a*b^3*d*e^(7*c) + b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^4*d
*e^(5*c) + 2*a^3*b*d*e^(5*c) - 2*a*b^3*d*e^(5*c) - b^4*d*e^(5*c))*e^(5*d*x)
 + (a^4*d*e^(3*c) + 4*a^3*b*d*e^(3*c) + 6*a^2*b^2*d*e^(3*c) + 4*a*b^3*d*e^(
3*c) + b^4*d*e^(3*c))*e^(3*d*x)) - 1/8*integrate(8*((3*a*b*e^(3*c) - 2*b^2*
e^(3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c)
+ 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*
c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh{(dx+c)}^3}{(b\tanh{(dx+c)}^2+a)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.35 $\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx = -\frac{(a-3b)x}{2(a+b)^3} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*(a-3*b)*x/(a+b)^3-1/2*(3*a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1
/2)/(a+b)^3/d/a^(1/2)+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^
2)-b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3744, 482, 541, 536, 212, 211}

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx = -\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^3} - \frac{b\tanh(c+dx)}{d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{x(a-3b)}{2(a+b)^3}$$

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a - 3*b)*x)/(a + b)^3 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(2*Sqrt[a]*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
(a + b)*d*(a + b*Tanh[c + d*x]^2)) - (b*Tanh[c + d*x])/((a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-3bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}\\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\ &\quad+ \frac{\text{Subst}\left(\int \frac{-2a(a-b)+4abx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)^2d}\\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\ &\quad- \frac{(a-3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^3d}\\ &\quad- \frac{((3a-b)b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^3d}\\ &= -\frac{(a-3b)x}{2(a+b)^3} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^3d}\\ &\quad+ \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{-2(a-3b)(c+dx) + \frac{2\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + (a+b)\sinh(2(c+dx)) - \frac{2b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{4(a+b)^3d}$$

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-2*(a - 3*b)*(c + d*x) + (2*Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a]])/Sqrt[a] + (a + b)*Sinh[2*(c + d*x)] - (2*b*(a + b)*Sinh[2*(c
+ d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(4*(a + b)^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(118) = 236$.

Time = 4.50 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.89

method	result
risch	$-\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{3xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2dx+2c}}{8d(a^2+2ab+b^2)}-\frac{e^{-2dx-2c}}{8d(a^2+2ab+b^2)}+\frac{b(e^{2dx+2}}{d(a+b)^3(a\,e^{4dx+4c}+b\,e^{4c}}$
derivativedivides	$-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\right.}\right.}{}$
default	$-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\right.}\right.}{}$

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(a+b)/(a^2+2*a*b+b^2)*a+3/2*x/(a+b)/(a^2+2*a*b+b^2)*b+1/8/d/(a^2+2*a
*b+b^2)*exp(2*d*x+2*c)-1/8/d/(a^2+2*a*b+b^2)*exp(-2*d*x-2*c)+b*(exp(2*d*x+2
*c)*a-b*exp(2*d*x+2*c)+a+b)/d/(a+b)^3/(a*exp(4*d*x+4*c)+b*exp(4*d*x+4*c)+2*
exp(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)+3/4*(-a*b)^(1/2)/(a+b)^3/d*ln(exp(
2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))-1/4/a*(-a*b)^(1/2)/(a+b)^3/d*ln(exp(
2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))*b-3/4*(-a*b)^(1/2)/(a+b)^3/d*ln(exp(
2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))+1/4/a*(-a*b)^(1/2)/(a+b)^3/d*ln(exp(
2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. 2(118) = 236.

Time = 0.34 (sec) , antiderivative size = 3918, normalized size of antiderivative = 29.68

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*
a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*
d*x - 14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 +
 4*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x
 - a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x
 - a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
 4*(a^2 - 4*a*b + 3*b^2)*d*x - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)
*cosh(d*x + c)^2 + 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 - 5*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
c)^3 - 4*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^3 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^
2 + 2*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)
*d*x - a^2 + b^2)*cosh(d*x + c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2
- 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*si
nh(d*x + c)^2 - 2*((3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b
 - b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)
^6 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*co
sh(d*x + c)^2 + 6*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*
b - b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*
cosh(d*x + c)^4 + 12*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*
a^2 - 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)) - a^2 - 2*a*b - b^2 + 4*(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a
```

```
^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*
b^2)*d*x - a*b + b^2)*cosh(d*x + c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2
- 4*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*sinh(d*x + c)^6 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x
 + c)^4 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2
 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4)*d*cosh(d*x + c)^4 + 12*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x +
 c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*
(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 +
 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2
)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - 14*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c))
*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^
4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 4*(a^2 - 4*a*b + 3*b^2)*d*x
 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*a*b - 4
*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 5*(2*(a^
2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^3 - 4*((a^2 - 4*a*b + 3*b
^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 - 2*a*b - 3
*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^2 + 2*(14*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
 c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2 - 4*a*b + 3*b^2)*d*x - a*b +
 b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*sinh(d*x + c)^2 - 4*((3*a^2 +
2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 2*(3*a^2 - 4*a*b + b^2)
*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + 6*a^2 - 8*a*
b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^3 + 2
*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 2*a*b - b^
2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^4 + 12*(3*a^2
- 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 2*(
3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + b^2)*cosh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arct
an(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 4*(2*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b
^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x +
c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c))*s
```

```
inh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^
6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^6 + 2*
(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + (15*(a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b
^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh
(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x +
 c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 12*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(
d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(118) = 236$.

Time = 0.40 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.36

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$-\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$-\frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d}$$
$$+\frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d}$$
$$+\frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d}$$
$$+\frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(2\,dx+2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)e^{(4\,dx+4\,c)}+2\,(a^5+2\,a^4b-2\,a}$$
$$-\frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(-2\,dx-2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+2\,(a^5+2\,a^4b-2\,a^2b^3-ab^4)e^{(-2\,dx-2\,c)}+(a^5+4\,a^4b+6\,a^3b^2+}$$
$$-\frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4}}$$
$$-\frac{dx+c}{2\,(a^2+2\,ab+b^2)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a^2+2\,ab+b^2)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a^2+2\,ab+b^2)d}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a
 + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*(
3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/4*(3*a*b + b^2)*arctan(1/2
*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqr
t(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a
^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 +
 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x
 - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b
 - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x
 - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a
*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*
```

```
c))*d) - 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2*d*x + 2*c)/((a^2
+ 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{3\cosh(c+dx)}{2(a+b)^2d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

```
[Out] 3/2*cosh(d*x+c)/(a+b)^2/d-1/2*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)-3/2
*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3745, 296, 331, 214}

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}} + \frac{3\cosh(c+dx)}{2d(a+b)^2} - \frac{\cosh(c+dx)}{2d(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}$$

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*
d) + (3*Cosh[c + d*x])/(2*(a + b)^2*d) - Cosh[c + d*x]/(2*(a + b)*d*(a + b
- b*Sech[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{2(a+b)d}$$

$$= \frac{3\cosh(c+dx)}{2(a+b)^2d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{2(a+b)^2d}$$

$$= -\frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{3\cosh(c+dx)}{2(a+b)^2d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{3i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{5/2}}+\frac{2\cosh(c+dx)\left(1-\frac{b}{a-b+(a+b)\cosh(2(c+dx))}\right)}{(a+b)^2}}{2d}$$

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((-3*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqr
t[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(a
 + b)^(5/2) + (2*Cosh[c + d*x]*(1 - b/(a - b + (a + b)*Cosh[2*(c + d*x)])))
/(a + b)^2)/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 1.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{2b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{1}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}-\frac{3\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{\frac{2b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{1}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}-\frac{3\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$\frac{e^{dx+c}}{2d(a^2+2ab+b^2)}+\frac{e^{-dx-c}}{2d(a^2+2ab+b^2)}-\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{3\sqrt{(a+b)b}}{}$

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a+b)^2*((-1/2*(a+2*b)/a*tanh(1/2*d*x+1/2*c)^2-1/2)/(tanh(1/2*d*x+
1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-3/4/(a*b+
b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))
)-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/(a+b)^2/(1+tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(81) = 162.

Time = 0.31 (sec) , antiderivative size = 2252, normalized size of antiderivative = 24.48

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*cosh(d*x + c)^6 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5
+ 2*(a + b)*sinh(d*x + c)^6 + 6*(a - b)*cosh(d*x + c)^4 + 6*(5*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 8*(5*(a + b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)^2 + 6*(5*(a +
 b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a +
 b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cos
h(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x +
 c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b))*l
og(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a +
b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d
*x + c))*sinh(d*x + c) + a + b)) + 12*((a + b)*cosh(d*x + c)^5 + 2*(a - b)*
cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 2*b)/((a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*si
nh(d*x + c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)
*d)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c) + 2*(
5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^
2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*cosh(d*x + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + (
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)*sinh(d*x + c)), 1/2*((a + b)*cosh(d*x + c
)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 + 3
*(a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x +
 c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x +
c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 3*((a + b)*cosh(d*x + c)^5 + 5*
```

```
(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)
*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 +
2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (
a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)
^2 + a + b)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x +
c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 +
(a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x +
c))*sqrt(-b/(a + b))/b) + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 +
 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(
d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x +
c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d
*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-b/(a + b))/b) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a - b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)*sinh(d*x + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x +
 c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh
(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c) + 2*(5*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*co
sh(d*x + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d)*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh{(dx+c)}}{\left(b\tanh{(dx+c)}^2+a\right)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(a*e^(4*c) - b*e^(4*c))*e^(4*d*x
) + 3*(a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)/((a^3*d*e^(5*c) + 3*a^2*b*
```

```
d*e^(5*c) + 3*a*b^2*d*e^(5*c) + b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^3*d*e^(3*c)
 + a^2*b*d*e^(3*c) - a*b^2*d*e^(3*c) - b^3*d*e^(3*c))*e^(3*d*x) + (a^3*d*e^
c + 3*a^2*b*d*e^c + 3*a*b^2*d*e^c + b^3*d*e^c)*e^(d*x)) + 1/2*integrate(6*(
b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^
(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3
*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)}{(b\tanh(dx+c)^2+a)^2}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh(c+d\,x)}{(b\tanh(c+d\,x)^2+a)^2}\,dx$$

```
[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.37 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=-\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^2d}+\frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d}+\frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

```
[Out] -arctanh(cosh(d*x+c))/a^2/d+1/2*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech(d*x+c)^
2)+1/2*(3*a+2*b)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b)
^(3/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3745, 425, 536, 213, 214}

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^2d}+\frac{b\operatorname{sech}(c+dx)}{2ad(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}$$

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*S
ech[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Sech[c + d*x])/(2*
a*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{b\text{sech}(c+dx)}{2a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b+bx^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{b\text{sech}(c+dx)}{2a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{a^2d} + \frac{(b(3a+2b))\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{2a^2(a+b)d}$$

$$= -\frac{\text{arctanh}(\cosh(c+dx))}{a^2d} + \frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\text{sech}(c+dx)}{2a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.83

$$\int \frac{\text{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{i\sqrt{b}(3a+2b)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}(3a+2b)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{2ab\cosh(c+dx)}{(a+b)(a-b+(a+b)\cosh(2(c+dx)))}}{2a^2d}$$

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2
])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b
] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (2*a*b*Cosh[c + d*
x])/((a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])) - 2*Log[Cosh[(c + d*x)/2]
] + 2*Log[Sinh[(c + d*x)/2]])/(2*a^2*d)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a+b)}-\frac{a}{4(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{(3a+2b)\,\text{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right)}{8(a+b)\sqrt{ab+b^2}}\right.}{a^2}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a+b)}-\frac{a}{4(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{(3a+2b)\,\text{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right)}{8(a+b)\sqrt{ab+b^2}}\right.}{a^2}}{d}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{a(a+b)d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{dx+c}-1\right)}{a^2d}-\frac{\ln\left(e^{dx+c}+1\right)}{a^2d}+\frac{3\sqrt{(a+b)b}\,\ln\left(e^{2dx}\right.}{4}$

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))-4/a^2*b*((-1/4*(a+2*b)/(a+b)*tanh(1/2*d*
x+1/2*c)^2-1/4*a/(a+b))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8*(3*a+2*b)/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/
4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(94) = 188$.

Time = 0.35 (sec) , antiderivative size = 2614, normalized size of antiderivative = 25.38

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*
sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x +
 c)^4 + 4*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 +
5*a*b + 2*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 +
2*(3*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*
x + c)^2 + 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)
^3 + (3*a^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*lo
g(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*c
osh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d
*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) +
 (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b
```

```
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) - 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*(
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*cosh(d*x + c)^2 + a*
b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^
2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (
a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3
 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c
)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 + 2*a*b*c
osh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 5*a*b
+ 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2*b^2)*sinh(d*x +
 c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 5*a*b + 2*b
^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + 5*a*b
+ 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a^2 - a*b - 2*b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)
^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d
*x + c))*sqrt(-b/(a + b))/b) - ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4
*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2
*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a
^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2
+ 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a
^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2
*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*((
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 -
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
+ 2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
```

```
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 - 1) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^
2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (
a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 +
 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*si
nh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] (b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e
^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log
((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) + 2*b^
2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 2*b^2*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^
2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^
4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{3\coth(c+dx)}{2a^2d}+\frac{\coth(c+dx)}{2ad\,(a+b\tanh^2(c+dx))}$$

```
[Out] -3/2*coth(d*x+c)/a^2/d-3/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(5
/2)/d+1/2*coth(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3744, 296, 331, 211}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{3\coth(c+dx)}{2a^2d}+\frac{\coth(c+dx)}{2ad\,(a+b\tanh^2(c+dx))}$$

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*d) - (3*Cot
h[c + d*x])/(2*a^2*d) + Coth[c + d*x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} + \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}\\
&= -\frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2a^2d}\\
&= -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-2\sqrt{a}\coth(c+dx)-\frac{\sqrt{a}b\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{5/2}d}$$

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 2*Sqrt[a]*Coth[c + d*
x] - (Sqrt[a]*b*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*
a^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(68) = 136.

Time = 1.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.07

method	result
risch	$-\frac{2a^2 e^{4dx+4c}+3ab\, e^{4dx+4c}+3\, e^{4dx+4c}b^2+4a^2 e^{2dx+2c}-6\, e^{2dx+2c}b^2+2a^2+5ab+3b^2}{(a+b)d\,a^2\left(a\, e^{4dx+4c}+b\, e^{4dx+4c}+2\, e^{2dx+2c}a-2b\, e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}-1\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+\cdots}{a+b}\cdots\right.}{4a^3d}\cdots$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\cdots\right.}}\cdots\right)}{\cdots}\right)}{a^2\cdots}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\cdots\right.}}\cdots\right)}{\cdots}\right)}{a^2\cdots}}{d}$

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*a^2*exp(4*d*x+4*c)+3*a*b*exp(4*d*x+4*c)+3*exp(4*d*x+4*c)*b^2+4*a^2*exp(
2*d*x+2*c)-6*exp(2*d*x+2*c)*b^2+2*a^2+5*a*b+3*b^2)/(a+b)/d/a^2/(a*exp(4*d*x
+4*c)+b*exp(4*d*x+4*c)+2*exp(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)/(exp(2*d*
x+2*c)-1)+3/4/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+
b))-3/4/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(68) = 136$.

Time = 0.31 (sec) , antiderivative size = 2562, normalized size of antiderivative = 31.24

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + 3*a*b + 3*b^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^
4 + 8*(2*a^2 - 3*b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d
*x + c)^2 + 2*a^2 - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4
*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2
+ 2*a*b + 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))
/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x
 + c))*sinh(d*x + c) + a + b)) + 8*a^2 + 20*a*b + 12*b^2 + 16*((2*a^2 + 3*a
*b + 3*b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))
/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)
*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c
)^6 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b +
```

```
a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^4
 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 - 2
*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh
(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*
d*cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a^4
- 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(2*a^2 + 3*
a*b + 3*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + 2*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(2*a^2 - 3*b^2)
*cosh(d*x + c)^2 + 4*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a^2 - 3
*b^2)*sinh(d*x + c)^2 + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 -
 (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2 + 2*a*b + 3*b^2)*sinh
(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
+ 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a +
 b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a
)/b) + 4*a^2 + 10*a*b + 6*b^2 + 8*((2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^3
+ (2*a^2 - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*
d*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x +
c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 - 2*a^3*b - 3*a^2
*b^2)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
 (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^4 - (a^4 - 2*a^3*b - 3*a^2*b^
2)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (
a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d
*x + c)^2 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b
 + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 -
 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cos
h(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}^2\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(68) = 136$.

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{2a^2+5ab+3b^2+2(2a^2-3b^2)e^{(-2dx-2c)}+(2a^2+3ab+3b^2)e^{(-4dx-4c)}}{(a^4+2a^3b+a^2b^2+(a^4-2a^3b-3a^2b^2)e^{(-2dx-2c)}-(a^4-2a^3b-3a^2b^2)e^{(-4dx-4c)}-(a^4+2a^3b+a} + \frac{3b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}a^2d}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x - 2*c) + (2*a^2 + 3*a
*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b -
3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-4*d*x - 4*c)
- (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 3/2*b*arctan(1/2*((a + b
)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(b\tanh(dx+c)^2+a)^2}\,dx$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{1}{\sinh(c+d\,x)^2\,(b\tanh(c+d\,x)^2+a)^2}\,dx$$

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{(a+4b)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

```
[Out] 1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+b
-b*sech(d*x+c)^2)-b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)-1/2*(3*a+4*b)*a
rctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3745, 482, 541, 536, 213, 214}

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a+4b)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a-b\operatorname{sech}^2(c+dx)+b\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b\operatorname{sech}^2(c+dx)+b\right)}$$

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - (Sqrt[b]*(3*a + 4*b)*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) - (Coth[c + d*
x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)) - (b*Sech[c + d*x])/(
a^2*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+3bx^2}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{2ad} \\
&= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{b\text{sech}(c+dx)}{a^2d\left(a+b-b\text{sech}^2(c+dx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{2(a+b)(a+2b)+4b(a+b)x^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{4a^2(a+b)d} \\
&= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{b\text{sech}(c+dx)}{a^2d\left(a+b-b\text{sech}^2(c+dx)\right)} \\
&\quad - \frac{(a+4b)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{2a^3d} \\
&\quad - \frac{(b(3a+4b))\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{2a^3d} \\
&= \frac{(a+4b)\text{arctanh}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}d} \\
&\quad - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{b\text{sech}(c+dx)}{a^2d\left(a+b-b\text{sech}^2(c+dx)\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.85 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{4i\sqrt{b}(3a+4b)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{4i\sqrt{b}(3a+4b)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{8ab\cosh(c+dx)}{a-b+(a+b)\cosh(2(c+dx))}+$$

[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/8*(((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + ((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + (8*a*b*Cosh[c + d*x])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + a*Csch[(c + d*x)/2]^2 - 4*(a + 4*b)*Log[Cosh[(c + d*x)/2]] + 4*(a + 4*b)*Log[Sinh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^3*d)

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b\left(\frac{\left(-\frac{a}{2}-b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-}\right.}{a^3}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b\left(\frac{\left(-\frac{a}{2}-b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-}\right.}{a^3}}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}+2b\,e^{6dx+6c}+3a\,e^{4dx+4c}-2b\,e^{4dx+4c}+3\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+2b\right)}{d\,a^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)}{2a^2d}+\frac{2\ln\left(e^{d}\right.}{d}$

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^2-1/8/a^2/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-2*a-8*b)*ln(tanh(1/2*d*x+1/2*c))+2*b/a^3*(((-1/2*a-b)*tanh(1/2*d*x+1/2*c)^2-1/2*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1

```
/2*c)^2*b+a)-1/4*(3*a+4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*
c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3368 vs. $2(132) = 264$.

Time = 0.35 (sec) , antiderivative size = 6335, normalized size of antiderivative = 44.93

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] ((a*e^(7*c) + 2*b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) - 2*b*e^(5*c))*e^(5*d*x
) + (3*a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x) + (a*e^c + 2*b*e^c)*e^(d*x))/(4*a
^2*b*d*e^(6*d*x + 6*c) + 4*a^2*b*d*e^(2*d*x + 2*c) - a^3*d - a^2*b*d - (a^3
*d*e^(8*c) + a^2*b*d*e^(8*c))*e^(8*d*x) + 2*(a^3*d*e^(4*c) - 3*a^2*b*d*e^(4
*c))*e^(4*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) - 1/2
*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integrate(1/8*((3*a*b*
e^(3*c) + 4*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 4*b^2*e^c)*e^(d*x))/(a^4
+ a^3*b + (a^4*e^(4*c) + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) - a^3*b*
e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] (a+2*b)*coth(d*x+c)/a^3/d-1/3*coth(d*x+c)^3/a^2/d+1/2*(3*a+5*b)*arctan(b^(1
/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(7/2)/d+1/2*b*(a+b)*tanh(d*x+c)/a^3/d/(a
+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3744, 467, 1275, 211}

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d}$$

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*d)
 + ((a + 2*b)*Coth[c + d*x])/(a^3*d) - Coth[c + d*x]^3/(3*a^2*d) + (b*(a +
b)*Tanh[c + d*x])/(2*a^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\text{Subst}\left(\int \frac{-\frac{2}{ab}+\frac{2(a+b)x^2}{a^2b}-\frac{(a+b)x^4}{a^3}}{x^4(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\text{Subst}\left(\int \left(-\frac{2}{a^2bx^4}+\frac{2(a+2b)}{a^3bx^2}+\frac{-3a-5b}{a^3(a+bx^2)}\right)\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{(b(3a+5b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2a^3d}$$

$$= \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + 2\sqrt{a}\coth(c+dx)\left(2a+6b-a\operatorname{csch}^2(c+dx)\right) + \frac{3\sqrt{a}b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{6a^{7/2}d}$$

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + 2*Sqrt[a]*
Coth[c + d*x]*(2*a + 6*b - a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(a + b)*Sinh[2
*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(6*a^(7/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(99) = 198.

Time = 3.72 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.99

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(3a+5b)\ldots}{\ldots}\right.}{\ldots}$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(3a+5b)\ldots}{\ldots}\right.}{\ldots}$
risch	$-\frac{-9ab\,e^{8dx+8c}-15b^2 e^{8dx+8c}+12a^2 e^{6dx+6c}+6ab\,e^{6dx+6c}+60b^2 e^{6dx+6c}+20a^2 e^{4dx+4c}-4ab\,e^{4dx+4c}-90\,e^{4dx+4c}b^2+4\ldots}{3d\,a^3\left(e^{2dx+2c}-1\right)^3\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}\ldots\right.}$

[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/8/a^3*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-8*b*tanh
(1/2*d*x+1/2*c))-2*b/a^3*(((-1/2*b-1/2*a)*tanh(1/2*d*x+1/2*c)^3+(-1/2*b-1/2
*a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(3*a+5*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/
((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a
+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
c)/((2((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/24/a^2/tanh(1/2*d*x+1/2*c)^3-1
/8/a^3*(-3*a-8*b)/tanh(1/2*d*x+1/2*c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(99) = 198$.

Time = 0.33 (sec) , antiderivative size = 5062, normalized size of antiderivative = 44.80

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^4\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.

Time = 0.38 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.50

$$\begin{aligned}&\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx\\&= \frac{4a^2+19ab+15b^2-2(2a^2+13ab+30b^2)e^{(-2dx-2c)}-2(10a^2-2ab-45b^2)e^{(-4dx-4c)}-6(2a^2+ab\cdots}{3(a^4+a^3b-(a^4+5a^3b)e^{(-2dx-2c)}-2(a^4-5a^3b)e^{(-4dx-4c)}+2(a^4-5a^3b)e^{(-6dx-6c)}+(a^4+5a^3\cdots}\\&\quad-\frac{(3ab+5b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}a^3d}\end{aligned}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*b + 30*b^2)*e^(-2*d*x - 2*c)
 - 2*(10*a^2 - 2*a*b - 45*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + a*b + 10*b^2)*
e^(-6*d*x - 6*c) + 3*(3*a*b + 5*b^2)*e^(-8*d*x - 8*c))/((a^4 + a^3*b - (a^4
 + 5*a^3*b)*e^(-2*d*x - 2*c) - 2*(a^4 - 5*a^3*b)*e^(-4*d*x - 4*c) + 2*(a^4
- 5*a^3*b)*e^(-6*d*x - 6*c) + (a^4 + 5*a^3*b)*e^(-8*d*x - 8*c) - (a^4 + a^3
*b)*e^(-10*d*x - 10*c))*d) - 1/2*(3*a*b + 5*b^2)*arctan(1/2*((a + b)*e^(-2*
d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

3.41 $\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{3(a^2-10ab+5b^2)x}{8(a+b)^5} + \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^5d} - \frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3(a-b)b\tanh(c+dx)}{2(a+b)^4d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 3/8*(a^2-10*a*b+5*b^2)*x/(a+b)^5+3/8*(5*a^2-10*a*b+b^2)*arctan(b^(1/2)*tanh
(d*x+c)/a^(1/2))*b^(1/2)/(a+b)^5/d/a^(1/2)-1/8*(5*a-3*b)*cosh(d*x+c)*sinh(d
*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d
/(a+b*tanh(d*x+c)^2)^2+1/8*(7*a-5*b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+
c)^2)^2+3/2*(a-b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3744, 481, 541, 536, 212, 211}

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a+b)^5} + \frac{3x(a^2-10ab+5b^2)}{8(a+b)^5} + \frac{3b(a-b)\tanh(c+dx)}{2d(a+b)^4\left(a+b\tanh^2(c+dx)\right)} + \frac{b(7a-5b)\tanh(c+dx)}{8d(a+b)^3\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(5a-3b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 10*a*b + 5*b^2)*x)/(8*(a + b)^5) + (3*Sqrt[b]*(5*a^2 - 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^5*d) - ((5*
a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2
)^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)
^2) + ((7*a - 5*b)*b*Tanh[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2)^
2) + (3*(a - b)*b*Tanh[c + d*x])/(2*(a + b)^4*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d} \\
&= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-a(3a-5b)+5(5a-3b)bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^2d}
\end{aligned}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{12a^2(a-3b)-12a(7a-5b)bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{32a(a+b)^3d}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3(a-b)b\tanh(c+dx)}{2(a+b)^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-24a^2(a^2-6ab+b^2)+96a^2(a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{64a^2(a+b)^4d}$$

$$= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3(a-b)b\tanh(c+dx)}{2(a+b)^4d\left(a+b\tanh^2(c+dx)\right)} + \frac{(3b(5a^2-10ab+b^2))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^5d} + \frac{(3(a^2-10ab+5b^2))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^5d}$$

$$= \frac{3(a^2-10ab+5b^2)\,x}{8(a+b)^5} + \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^5d} - \frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3(a-b)b\tanh(c+dx)}{2(a+b)^4d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{12(a^2-10ab+5b^2)(c+dx) + \frac{12\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - 8(a-2b)(a+b)\sinh(2(c+dx)) + \cdots}{32(a+b)^5d}$$

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^2 - 10*a*b + 5*b^2)*(c + d*x) + (12*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*A
rcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - 8*(a - 2*b)*(a + b)*Sinh[
2*(c + d*x)] + (16*a*b^2*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2
*(c + d*x)])^2 + (4*(9*a - 5*b)*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a +
b)*Cosh[2*(c + d*x)]) + (a + b)^2*Sinh[4*(c + d*x)])/(32*(a + b)^5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(220) = 440$.

Time = 193.92 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.54

method	result
derivativedivides	$-\frac{1}{4(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3a-9b}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{11b-a}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(3a^2-3}{}$
default	$-\frac{1}{4(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3a-9b}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{11b-a}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(3a^2-3}{}$
risch	$\frac{3x\,a^2}{8(a^3+3a^2b+3a\,b^2+b^3)(a+b)^2}-\frac{15xab}{4(a^3+3a^2b+3a\,b^2+b^3)(a+b)^2}+\frac{15x\,b^2}{8(a^3+3a^2b+3a\,b^2+b^3)(a+b)^2}+\frac{e^{4dx+4c}}{64(a+b)^3d}-\frac{}{8(}$

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^4+1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2
*c))^3-1/8*(3*a-9*b)/(a+b)^4/(1+tanh(1/2*d*x+1/2*c))-1/8*(11*b-a)/(a+b)^4/(
1+tanh(1/2*d*x+1/2*c))^2+1/8/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(1+tanh(1/2*d*
x+1/2*c))+1/4/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/(a+b)^3/(tanh(1/2*d*x+1
/2*c)-1)^3-1/8*(3*a-9*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)-1/8*(-11*b+a)/(a+b
)^4/(tanh(1/2*d*x+1/2*c)-1)^2+1/8/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(tanh(1/
2*d*x+1/2*c)-1)-2*b/(a+b)^5*((-3/8*a*(3*a^2+2*a*b-b^2)*tanh(1/2*d*x+1/2*c)^
7+(-27/8*a^3-23/4*a^2*b+1/8*a*b^2+5/2*b^3)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^3
```

```
-23/4*a^2*b+1/8*a*b^2+5/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^3-3/4*a^2*b+3/
8*a*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8*(15*a^2-30*a*b+3*b^2)*a*(1/2*(a+((
a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b
)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9248 vs. $2(220) = 440$.

Time = 0.53 (sec) , antiderivative size = 18818, normalized size of antiderivative = 78.41

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3392 vs. $2(220) = 440$.

Time = 0.73 (sec) , antiderivative size = 3392, normalized size of antiderivative = 14.13

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -3/8*(a*b - 3*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) - 3/
4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^4
```

+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/8*(a*b - 3*b^2)*log(2*(a - b)
*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) + 3/4*b*log(2*(a - b)*e^(-2*d*x -
2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d) + 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4 + b^5)*a
rctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a^6*b + 10
*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 3/32*(5*a^3*b -
15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) -
3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4 + b^5)*arctan(1/2*((a +
b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*
a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 -
5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 3/64*(15*a^2
*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a
*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) - 1/64*(9*a^5*b -
65*a^4*b^2 - 134*a^3*b^3 - 34*a^2*b^4 + 29*a*b^5 + 3*b^6 + (9*a^5*b - 183*a
^4*b^2 + 98*a^3*b^3 + 266*a^2*b^4 - 27*a*b^5 - 3*b^6)*e^(6*d*x + 6*c) + (27
*a^5*b - 459*a^4*b^2 + 710*a^3*b^3 - 542*a^2*b^4 + 63*a*b^5 + 9*b^6)*e^(4*d
*x + 4*c) + (27*a^5*b - 341*a^4*b^2 + 86*a^3*b^3 + 398*a^2*b^4 - 65*a*b^5 -
9*b^6)*e^(2*d*x + 2*c))/((a^9 + 7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5
*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*b^7 + (a^9 + 7*a^8*b + 21*a^7*b^2 + 35*
a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*b^7)*e^(8*d*x + 8*c) +
4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a^4*b^5 - 5*a^3*b^
6 - a^2*b^7)*e^(6*d*x + 6*c) + 2*(3*a^9 + 13*a^8*b + 23*a^7*b^2 + 25*a^6*b^
3 + 25*a^5*b^4 + 23*a^4*b^5 + 13*a^3*b^6 + 3*a^2*b^7)*e^(4*d*x + 4*c) + 4*(
a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a^4*b^5 - 5*a^3*b^6 -
a^2*b^7)*e^(2*d*x + 2*c))*d) + 1/64*(9*a^5*b - 65*a^4*b^2 - 134*a^3*b^3 -
34*a^2*b^4 + 29*a*b^5 + 3*b^6 + (27*a^5*b - 341*a^4*b^2 + 86*a^3*b^3 + 398*
a^2*b^4 - 65*a*b^5 - 9*b^6)*e^(-2*d*x - 2*c) + (27*a^5*b - 459*a^4*b^2 + 71
0*a^3*b^3 - 542*a^2*b^4 + 63*a*b^5 + 9*b^6)*e^(-4*d*x - 4*c) + (9*a^5*b - 1
83*a^4*b^2 + 98*a^3*b^3 + 266*a^2*b^4 - 27*a*b^5 - 3*b^6)*e^(-6*d*x - 6*c))
/((a^9 + 7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^
3*b^6 + a^2*b^7 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*
a^4*b^5 - 5*a^3*b^6 - a^2*b^7)*e^(-2*d*x - 2*c) + 2*(3*a^9 + 13*a^8*b + 23*
a^7*b^2 + 25*a^6*b^3 + 25*a^5*b^4 + 23*a^4*b^5 + 13*a^3*b^6 + 3*a^2*b^7)*e^
(-4*d*x - 4*c) + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a
^4*b^5 - 5*a^3*b^6 - a^2*b^7)*e^(-6*d*x - 6*c) + (a^9 + 7*a^8*b + 21*a^7*b^
2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*b^7)*e^(-8*d*x -
8*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(
3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^
4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a
^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 +
6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^
8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e

```
^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2
*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*
a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^
6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) + 1/16*(9*a^4*
b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84
*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x - 2*c) + (27*a^4*b - 156*a^3*b^2 + 1
10*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 -
 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x - 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6
*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^
7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*
d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4
+ 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c))*d) + 3/32*(9*a^3*b + 21*a^2*b^2 +
15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2
*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b
 - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^
4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*
b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*
b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 +
5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*
d) + 3/8*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*((a + b)*e^(4
*d*x + 4*c) + 24*b*e^(2*d*x + 2*c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*d) - 1/64*(24*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c))/((a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d) + 1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d)
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\sinh\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{5(3a-4b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} - \frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{ab\,\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\,\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

```
[Out] -(a-2*b)*cosh(d*x+c)/(a+b)^4/d+1/3*cosh(d*x+c)^3/(a+b)^3/d+1/4*a*b*sech(d*x
+c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)^2+1/8*(7*a-4*b)*b*sech(d*x+c)/(a+b)^4/d
/(a+b-b*sech(d*x+c)^2)+5/8*(3*a-4*b)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2
))*b^(1/2)/(a+b)^(9/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3745, 467, 1273, 1275, 214}

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{5\sqrt{b}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}+\frac{\cosh^3(c+dx)}{3d(a+b)^3}-\frac{(a-2b)\cosh(c+dx)}{d(a+b)^4}+\frac{b(7a-4b)\text{sech}(c+dx)}{8d(a+b)^4\left(a-b\text{sech}^2(c+dx)+b\right)}+\frac{ab\text{sech}(c+dx)}{4d(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}$$

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a
+ b)^(9/2)*d) - ((a - 2*b)*Cosh[c + d*x])/((a + b)^4*d) + Cosh[c + d*x]^3/(
3*(a + b)^3*d) + (a*b*Sech[c + d*x])/(4*(a + b)^3*d*(a + b - b*Sech[c + d*x
]^2)^2) + ((7*a - 4*b)*b*Sech[c + d*x])/(8*(a + b)^4*d*(a + b - b*Sech[c +
d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
```

```
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{b\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}+\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{4d}$$

$$= \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-8b(a+b)+8(a-b)bx^2+\frac{(7a-4b)b^2x^4}{a+b}}{x^4(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{8b(a+b)^3d}$$

$$= \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \left(-\frac{8b}{x^4}+\frac{8(a-2b)b}{(a+b)x^2}+\frac{5(3a-4b)b^2}{(a+b)(a+b-bx^2)}\right)dx, x, \text{sech}(c+dx)\right)}{8b(a+b)^3d}$$

$$= -\frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{(5(3a-4b)b)\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{8(a+b)^4d}$$

$$= \frac{5(3a-4b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} - \frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{ab\operatorname{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\operatorname{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.37

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{15i(3a-4b)\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{9/2}} - \frac{6\cosh(c+dx)\left(3a^3-24a^2b+30ab^2-13b^3+(6a^3-\cdots\right.}{(a+b)\cdots}}{24d}$$

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((15*I)*(3*a - 4*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
d*x)/2]])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]])/S
qrt[b]]))/(a + b)^(9/2) - (6*Cosh[c + d*x]*(3*a^3 - 24*a^2*b + 30*a*b^2 - 1
3*b^3 + (6*a^3 - 27*a^2*b - 11*a*b^2 + 22*b^3)*Cosh[2*(c + d*x)] + 3*(a - 3
*b)*(a + b)^2*Cosh[2*(c + d*x)]^2))/((a + b)^4*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (2*Cosh[3*(c + d*x)])/(a + b)^3)/(24*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(150) = 300$.

Time = 77.41 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.05

method	result
derivativedivides	$\frac{\frac{1}{3(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-5b}{2(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b\left(\frac{-\frac{(9a+20b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}-\frac{(27}{\left(\tanh\right.}\right.}{}}{}$
default	$\frac{\frac{1}{3(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-5b}{2(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b\left(\frac{-\frac{(9a+20b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}-\frac{(27}{\left(\tanh\right.}\right.}{}}{}$
risch	$\frac{e^{3dx+3c}}{24(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{3\,e^{dx+c}a}{8(a^3+3a^2b+3a\,b^2+b^3)(a+b)d}+\frac{9\,e^{dx+c}b}{8(a^3+3a^2b+3a\,b^2+b^3)(a+b)d}-\frac{3\,e^{-dx-c}a}{8(a^4+4a^3b+6a^2b^2+}$

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^3-1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2*
c))^2-1/2*(a-5*b)/(a+b)^4/(1+tanh(1/2*d*x+1/2*c))-2*b/(a+b)^4*((-1/8*(9*a+2
0*b)*a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+66*a^2*b+56*a*b^2-16*b^3)/a*tanh(1
/2*d*x+1/2*c)^4+(-27/8*a^2-11/2*a*b+2*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^2+1/
4*a*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2-5/16*(3*a-4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1
/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^4*(-a+5*b)/(tanh(1/2*d*x+1/2
*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7129 vs. $2(156) = 312$.

Time = 0.45 (sec) , antiderivative size = 13095, normalized size of antiderivative = 78.89

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

3.43 $\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{(a-5b)x}{2(a+b)^4} - \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\,(a+b\tanh^2(c+dx))^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\,(a+b\tanh^2(c+dx))^2} - \frac{(11a-b)b\tanh(c+dx)}{8a(a+b)^3d\,(a+b\tanh^2(c+dx))}$$

```
[Out] -1/2*(a-5*b)*x/(a+b)^4-1/8*(15*a^2-10*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a
^(1/2))*b^(1/2)/a^(3/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*
tanh(d*x+c)^2)^2-3/4*b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2-1/8*(11*
a-b)*b*tanh(d*x+c)/a/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3744, 482, 541, 536, 212, 211}

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^4} - \frac{b(11a-b)\tanh(c+dx)}{8ad(a+b)^3\left(a+b\tanh^2(c+dx)\right)} - \frac{3b\tanh(c+dx)}{4d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{x(a-5b)}{2(a+b)^4}$$

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((a - 5*b)*x)/(a + b)^4 - (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sinh
[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - (3*b*Tanh[c + d*x])/(4
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((11*a - b)*b*Tanh[c + d*x])/(8*a
*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a-5bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-2a(2a-b)+18abx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{8a(a+b)^2d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad - \frac{(11a-b)b\tanh(c+dx)}{8a(a+b)^3d\left(a+b\tanh^2(c+dx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{2a(4a^2-9ab-b^2)-2a(11a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{16a^2(a+b)^3d}
\end{aligned}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(11a-b)b\tanh(c+dx)}{8a(a+b)^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-5b)\mathrm{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{2(a+b)^4d} - \frac{\left(b(15a^2-10ab-b^2)\right)\mathrm{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,\tanh(c+dx)\right)}{8a(a+b)^4d}$$

$$= -\frac{(a-5b)x}{2(a+b)^4} - \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(11a-b)b\tanh(c+dx)}{8a(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.85

$$\int\frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-4(a-5b)(c+dx) + \frac{\sqrt{b}(-15a^2+10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 2(a+b)\sinh(2(c+dx)) - \frac{4b^2(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx))}}{8(a+b)^4d}$$

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-4*(a - 5*b)*(c + d*x) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + 2*(a + b)*Sinh[2*(c + d*x)] - (4*b^2*(a
 + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((9*a - b)
*b*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(
a + b)^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(167) = 334.

Time = 15.26 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^4}+\frac{2b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{}\right.}{}}{}$
default	$\frac{\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^4}+\frac{2b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{}\right.}{}}{}$
risch	$-\frac{xa}{2(a+b)(a^3+3a^2b+3a\,b^2+b^3)}+\frac{5xb}{2(a+b)(a^3+3a^2b+3a\,b^2+b^3)}+\frac{e^{2dx+2c}}{8(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{e^{-2dx-2c}}{8(a^3+3a^2b+3a\,b^2+b^3}$

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)
-1)+1/2*(a-5*b)/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^4*(((-9/8*a^2-5
/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(27*a^3+58*a^2*b+27*a*b^2-4*b^3)/
a*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^3+58*a^2*b+27*a*b^2-4*b^3)/a*tanh(1/2*d*x
+1/2*c)^3+(-9/8*a^2-5/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2
*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8*(15*a^
2-10*a*b-b^2)*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1
/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)
*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a
)^(1/2))))-1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^2+1/2/(a+b)^3/(1+tanh(1/2*d*
x+1/2*c))+1/2/(a+b)^4*(-a+5*b)*ln(1+tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6319 vs. $2(167) = 334$.

Time = 0.45 (sec) , antiderivative size = 12965, normalized size of antiderivative = 70.08

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. $2(167) = 334$.

Time = 0.59 (sec) , antiderivative size = 1806, normalized size of antiderivative = 9.76

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*(
(a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b
^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*
b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 1/16*(15*a^2*b + 10*a*b
^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4*a^3*b^2 -
 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a
```

```
*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*
b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b
^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15
*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a
^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a
^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*
d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 -
 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x
- 2*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*
x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x
- 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5
+ a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)
*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*
b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 1
5*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c)
)*d) - 1/8*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^
2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 +
 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x
 - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 +
4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x -
 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)
*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^
3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) - 1/2*(d*x + c)/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d) + 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) -
1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\sinh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\sinh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d}+\frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

```
[Out] 15/8*cosh(d*x+c)/(a+b)^3/d-1/4*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)^2-
5/8*cosh(d*x+c)/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)-15/8*arctanh(sech(d*x+c)*b^
(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3745, 296, 331, 214}

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}+\frac{15\cosh(c+dx)}{8d(a+b)^3} - \frac{5\cosh(c+dx)}{8d(a+b)^2\left(a-b\,\text{sech}^2(c+dx)+b\right)} - \frac{\cosh(c+dx)}{4d(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}$$

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)
*d) + (15*Cosh[c + d*x])/(8*(a + b)^3*d) - Cosh[c + d*x]/(4*(a + b)*d*(a +
b - b*Sech[c + d*x]^2)^2) - (5*Cosh[c + d*x])/(8*(a + b)^2*d*(a + b - b*Sec
h[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{15\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{8(a+b)^2d}$$

$$= \frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{(15b)\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{8(a+b)^3d}$$

$$= -\frac{15\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} + \frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{-\frac{15i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}} + \frac{2\cosh(c+dx)\left(4-\frac{4b^2}{(a-b+(a+b)\cosh(2(c+dx)))^2}-\frac{9b}{a-b+(a+b)\cosh(2(c+dx))}\right)}{(a+b)^3}}{8d}$$

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((-15*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(
a + b)^(7/2) + (2*Cosh[c + d*x]*(4 - (4*b^2)/(a - b + (a + b)*Cosh[2*(c + d
*x)])^2 - (9*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3)/(8*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

Time = 5.91 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{\left(9a^2+24ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a}-\frac{\left(27a^3+78a^2b+88a\,b^2+16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8a^2}-\frac{(27a^2+56ab+}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{(a+b)^3}}{d}$
default	$\frac{-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{\left(9a^2+24ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a}-\frac{\left(27a^3+78a^2b+88a\,b^2+16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8a^2}-\frac{(27a^2+56ab+}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{(a+b)^3}}{d}$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3a\,b^2+b^3)d}+\frac{e^{-dx-c}}{2(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{e^{dx+c}b(9a\,e^{6dx+6c}+9b\,e^{6dx+6c}+27a\,e^{4dx+4c}-b\,e^{4dx+4c}+27\,e^{2d}}{4(a^3+3a^2b+3a\,b^2+b^3)(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-}$

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^3*((-1/8*(9*a^2+24*a*b+8*
b^2)/a*tanh(1/2*d*x+1/2*c)^6-1/8/a^2*(27*a^3+78*a^2*b+88*a*b^2+16*b^3)*tanh
(1/2*d*x+1/2*c)^4-1/8*(27*a^2+56*a*b+8*b^2)/a*tanh(1/2*d*x+1/2*c)^2-9/8*a-1
/4*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2-15/16/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2
*a+4*b)/(a*b+b^2)^(1/2)))+1/(a+b)^3/(1+tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3792 vs. $2(116) = 232$.

Time = 0.37 (sec) , antiderivative size = 7119, normalized size of antiderivative = 56.50

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(dx+c)}{\left(b\tanh(dx+c)^2+a\right)^3}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/4*(2*a^2 + 4*a*b + 2*b^2 + 2*(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c
))*e^(10*d*x) + 5*(2*a^2*e^(8*c) - a*b*e^(8*c) - 3*b^2*e^(8*c))*e^(8*d*x) +
 5*(4*a^2*e^(6*c) - 7*a*b*e^(6*c) + b^2*e^(6*c))*e^(6*d*x) + 5*(4*a^2*e^(4*
c) - 7*a*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(2*c) - a*b*e^(2*c
) - 3*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(9*c) + 5*a^4*b*d*e^(9*c) + 10*a^3*
b^2*d*e^(9*c) + 10*a^2*b^3*d*e^(9*c) + 5*a*b^4*d*e^(9*c) + b^5*d*e^(9*c))*e
^(9*d*x) + 4*(a^5*d*e^(7*c) + 3*a^4*b*d*e^(7*c) + 2*a^3*b^2*d*e^(7*c) - 2*a
^2*b^3*d*e^(7*c) - 3*a*b^4*d*e^(7*c) - b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*
d*e^(5*c) + 7*a^4*b*d*e^(5*c) + 6*a^3*b^2*d*e^(5*c) + 6*a^2*b^3*d*e^(5*c) +
 7*a*b^4*d*e^(5*c) + 3*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 3*a^4*
b*d*e^(3*c) + 2*a^3*b^2*d*e^(3*c) - 2*a^2*b^3*d*e^(3*c) - 3*a*b^4*d*e^(3*c)
 - b^5*d*e^(3*c))*e^(3*d*x) + (a^5*d*e^c + 5*a^4*b*d*e^c + 10*a^3*b^2*d*e^c
 + 10*a^2*b^3*d*e^c + 5*a*b^4*d*e^c + b^5*d*e^c)*e^(d*x)) + 1/2*integrate(1
5/2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4
*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e
^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.45 $\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^3d} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} + \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

```
[Out] -arctanh(cosh(d*x+c))/a^3/d+1/4*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech(d*x+c)^
2)^2+1/8*b*(7*a+4*b)*sech(d*x+c)/a^2/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)+1/8*(1
5*a^2+20*a*b+8*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3/(a
+b)^(5/2)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{3745, 425, 541, 536, 213, 214\}$

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^3d} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2d(a+b)^2\left(a-b\operatorname{sech}^2(c+dx)+b\right)} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}} + \frac{b\operatorname{sech}(c+dx)}{4ad(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}$$

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^3*d)) + (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcT
anh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) + (b*Sech
[c + d*x])/(4*a*(a + b)*d*(a + b - b*Sech[c + d*x]^2)^2) + (b*(7*a + 4*b)*S
ech[c + d*x])/(8*a^2*(a + b)^2*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{b\text{sech}(c+dx)}{4a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b+3bx^2}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\text{sech}(c+dx)}{4a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\text{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+9ab+4b^2+b(7a+4b)x^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{8a^2(a+b)^2d}$$

$$= \frac{b\text{sech}(c+dx)}{4a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\text{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{a^3d} + \frac{(b(15a^2+20ab+8b^2))\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{8a^3(a+b)^2d}$$

$$= -\frac{\text{arctanh}(\cosh(c+dx))}{a^3d} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} + \frac{b\text{sech}(c+dx)}{4a(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\text{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\frac{i\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}}+\frac{i\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}}+\frac{8a^2b^2 \mathrm{c}}{(a+b)^2(a-b+(a\cdots}}{8a^3d}$$

[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]

```
[Out] ((I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Ta
nh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (I*Sqrt[b]*(15*a^2 + 20*a*b + 8*
b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b
)^(5/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (2*a*b*(9*a + 4*b)*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*
Cosh[2*(c + d*x)])) - 8*Log[Cosh[(c + d*x)/2]] + 8*Log[Sinh[(c + d*x)/2]])/
(8*a^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(142) = 284.

Time = 2.88 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.95

method	result
derivativedivides	$-\frac{\frac{2b\left(\frac{-\frac{(9a^2+28ab+16b^2)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a^2+2ab+b^2)}-\frac{3(9a^3+30a^2b+40a\,b^2+16b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a^2+2ab+b^2)}-\frac{a(27a^2+68ab+32b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8(a^2+2ab+b^2)}-\cdots}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\cdots\right)}{a^3}}{d}$
default	$-\frac{\frac{2b\left(\frac{-\frac{(9a^2+28ab+16b^2)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a^2+2ab+b^2)}-\frac{3(9a^3+30a^2b+40a\,b^2+16b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a^2+2ab+b^2)}-\frac{a(27a^2+68ab+32b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8(a^2+2ab+b^2)}-\cdots}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\cdots\right)}{a^3}}{d}$
risch	$\frac{(9a^2 e^{6dx+6c}+13ab\,e^{6dx+6c}+4b^2e^{6dx+6c}+27a^2e^{4dx+4c}+11ab\,e^{4dx+4c}-4\,e^{4dx+4c}b^2+27a^2e^{2dx+2c}+11ab\,e^{2dx+2c}-4\,e^{2dx\cdots}}{4(a^2+2ab+b^2)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^2 d\,a^2}$

[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

```
[Out] 1/d*(-2*b/a^3*((-1/8*(9*a^2+28*a*b+16*b^2)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^6-3/8*(9*a^3+30*a^2*b+40*a*b^2+16*b^3)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1
```

```
/2*c)^4-1/8*a*(27*a^2+68*a*b+32*b^2)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-
3/8*a^2*(3*a+2*b)/(a^2+2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+
1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/16*(15*a^2+20*a*b+8*b^2)/(a^2+2
*a*b+b^2)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(
a*b+b^2)^(1/2)))+1/a^3*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5783 vs. $2(148) = 296$.

Time = 0.47 (sec) , antiderivative size = 10716, normalized size of antiderivative = 68.69

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(b\tanh(dx+c)^2+a)^3}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/4*((9*a^2*b*e^(7*c) + 13*a*b^2*e^(7*c) + 4*b^3*e^(7*c))*e^(7*d*x) + (27*a
^2*b*e^(5*c) + 11*a*b^2*e^(5*c) - 4*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3
*c) + 11*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 13*a*b^2
*e^c + 4*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d +
 a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a
^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*
b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6
```

```
*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c)
+ 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2
*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*
e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*(
(15*a^2*b*e^(3*c) + 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b
*e^c + 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3 + (a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))
*e^(4*d*x) + 2*(a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c) - a^3*b^3*e^(
2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

3.46 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -15/8*coth(d*x+c)/a^3/d-15/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^
(7/2)/d+1/4*coth(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)^2+5/8*coth(d*x+c)/a^2/d/(a+
b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3744, 296, 331, 211}

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15\coth(c+dx)}{8a^3d} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(7/2)*d) - (15*C
oth[c + d*x])/(8*a^3*d) + Coth[c + d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) +
 (5*Coth[c + d*x])/(8*a^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}\\
&= \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{15\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a^2d}\\
&= -\frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}\\
&\quad + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{(15b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^3d}
\end{aligned}
$$

$$= -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\coth(c+dx) - \frac{\sqrt{a}b(9a-7b+(9a+7b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{7/2}d}$$

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]*Coth[c + d
*x] - (Sqrt[a]*b*(9*a - 7*b + (9*a + 7*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*
x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(7/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(96) = 192.

Time = 5.22 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.73

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\frac{9\tanh(}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\frac{9\tanh(}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{}}{d}$
risch	$-\frac{8\,e^{8dx+8c}a^4+23\,e^{8dx+8c}a^3b+45\,e^{8dx+8c}a^2b^2+45a\,b^3e^{8dx+8c}+15b^4e^{8dx+8c}+32\,e^{6dx+6c}a^4+46b\,a^3e^{6dx+6c}-90a\,b^3e^{6dx}}{4(a^{2}}$

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-1/2/a^3/tanh(1/2*d*x+1/2*c)+2*b/a^3*((-9/
8*a*tanh(1/2*d*x+1/2*c)^7+(-27/8*a-7/2*b)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a-7/
2*b)*tanh(1/2*d*x+1/2*c)^3-9/8*tanh(1/2*d*x+1/2*c)*a)/(tanh(1/2*d*x+1/2*c)^
4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+15/8*a*(1/2*(a
+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((
a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3995 vs. $2(96) = 192$.

Time = 0.38 (sec) , antiderivative size = 8312, normalized size of antiderivative = 74.21

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

[Out] Too large to include

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(96) = 192$.

Time = 0.43 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.27

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{8a^4+41a^3b+73a^2b^2+55ab^3+15b^4+2(16a^4+41a^3b-55ab^3-30b^4)e^{(-2d\ldots}}{4(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+(3a^7+4a^6b-6a^5b^2-12a^4b^3-5a^3b^4)e^{(-2dx-2c)}+2(a^7+2\ldots} + \frac{15b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^3d}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/4*(8*a^4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a^
3*b - 55*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^
2 + 50*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^3 -
 30*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*
b^4)*e^(-8*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 +
(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*c) + 2
*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7 + 2*a^
5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b - 6*a^5*
b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b + 6*a^5*b^2
 + 4*a^4*b^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/8*b*arctan(1/2*((a + b)
*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{\left(b\tanh(dx+c)^2+a\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{1}{\sinh(c+d\,x)^2\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{(a+6b)\operatorname{arctanh}(\cosh(c+dx))}{2a^4d} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

```
[Out] 1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+b
-b*sech(d*x+c)^2)^2-3/4*b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)^2-1/8*b*(
11*a+12*b)*sech(d*x+c)/a^3/(a+b)/d/(a+b-b*sech(d*x+c)^2)-1/8*(15*a^2+40*a*b
+24*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3745, 482, 541, 536, 213, 214}

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(a+6b)\operatorname{arctanh}(\cosh(c+dx))}{2a^4d} - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3d(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}$$

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - (Sqrt[b]*(15*a^2 + 40*a*b +
24*b^2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*
d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)^2) -
(3*b*Sech[c + d*x])/(4*a^2*d*(a + b - b*Sech[c + d*x]^2)^2) - (b*(11*a + 12
*b)*Sech[c + d*x])/(8*a^3*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{(-1+x^2)(a+b-bx^2)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{2ad} \\
&= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} \\
&\quad - \frac{\text{Subst}\left(\int \frac{2(a+b)(2a+3b)+18b(a+b)x^2}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{8a^2(a+b)d}
\end{aligned}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{2(a+b)(4a^2+17ab+12b^2)+2b(a+b)(11a+12b)x^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \operatorname{sech}(c+dx)\right)}{16a^3(a+b)^2d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{(a+6b)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{2a^4d} - \frac{(b(15a^2+40ab+24b^2))\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{8a^4(a+b)d}$$

$$= \frac{(a+6b)\operatorname{arctanh}(\cosh(c+dx))}{2a^4d} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= -\frac{i\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-i\sqrt{a+b}\cosh\left(\frac{1}{2}(c+dx)\right)-\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{b}}\right)}{8a^4(a+b)^{3/2}d}$$
$$-\frac{i\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-i\sqrt{a+b}\cosh\left(\frac{1}{2}(c+dx)\right)+\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{b}}\right)}{8a^4(a+b)^{3/2}d}$$
$$-\frac{b^2\cosh(c+dx)}{a^2(a+b)d(a-b+a\cosh(2(c+dx))+b\cosh(2(c+dx)))^2}$$
$$+\frac{-9ab\cosh(c+dx)-8b^2\cosh(c+dx)}{4a^3(a+b)d(a-b+a\cosh(2(c+dx))+b\cosh(2(c+dx)))}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$
$$+\frac{(a+6b)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}+\frac{(-a-6b)\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-1/8*I)*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sech[(c + d*x)/2]*((-I
)*Sqrt[a + b]*Cosh[(c + d*x)/2] - Sqrt[a]*Sinh[(c + d*x)/2]))/Sqrt[b]])/(a^
4*(a + b)^(3/2)*d) - ((I/8)*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sech
[(c + d*x)/2]*((-I)*Sqrt[a + b]*Cosh[(c + d*x)/2] + Sqrt[a]*Sinh[(c + d*x)/
2]))/Sqrt[b]])/(a^4*(a + b)^(3/2)*d) - (b^2*Cosh[c + d*x])/(a^2*(a + b)*d*(
a - b + a*Cosh[2*(c + d*x)] + b*Cosh[2*(c + d*x)])^2) + (-9*a*b*Cosh[c + d*
x] - 8*b^2*Cosh[c + d*x])/(4*a^3*(a + b)*d*(a - b + a*Cosh[2*(c + d*x)] + b
*Cosh[2*(c + d*x)])) - Csch[(c + d*x)/2]^2/(8*a^3*d) + ((a + 6*b)*Log[Cosh[
(c + d*x)/2]])/(2*a^4*d) + ((-a - 6*b)*Log[Sinh[(c + d*x)/2]])/(2*a^4*d) -
Sech[(c + d*x)/2]^2/(8*a^3*d)
```

Maple [A] (verified)

Time = 7.89 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}+\frac{2b\left(\frac{-\frac{\left(9a^2+32ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a+b)}-\frac{\left(27a^3+102a^2b+152a\,b^2+80b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a+b)}-\frac{a\left(27a^2+80ab+56b^2\right)\mathrm{t}}{8(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{a^4}}{}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}+\frac{2b\left(\frac{-\frac{\left(9a^2+32ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a+b)}-\frac{\left(27a^3+102a^2b+152a\,b^2+80b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a+b)}-\frac{a\left(27a^2+80ab+56b^2\right)\mathrm{t}}{8(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{a^4}}{}$
risch	$-\frac{\left(4a^3e^{10dx+10c}+21a^2b\,e^{10dx+10c}+29a\,b^2e^{10dx+10c}+12b^3e^{10dx+10c}+20a^3e^{8dx+8c}+37a^2b\,e^{8dx+8c}-15a\,b^2e^{8dx+8c}-36b\right.}{}$

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^3+2*b/a^4*((-1/8*(9*a^2+32*a*b+24*b^2)*a/(a+b)*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+102*a^2*b+152*a*b^2+80*b^3)/(a+b)*tanh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2+80*a*b+56*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^2-1/8*a^2*(9*a+10*b)/(a+b))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/16*(15*a^2+40*a*b+24*b^2)/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/8/a^3/tanh(1/2*d*x+1/2*c)^2+1/4/a^4*(-2*a-12*b)*ln(tanh(1/2*d*x+1/2*c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11576 vs. 2(187) = 374.

Time = 0.56 (sec) , antiderivative size = 21301, normalized size of antiderivative = 108.68

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{csch}^3\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/4*((4*a^3*e^(11*c) + 21*a^2*b*e^(11*c) + 29*a*b^2*e^(11*c) + 12*b^3*e^(1
1*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 37*a^2*b*e^(9*c) - 15*a*b^2*e^(9*c) -
36*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 3*a^2*b*e^(7*c) - 7*a*b^2*e
^(7*c) + 12*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e^(5*c) + 3*a^2*b*e^(5*c) -
7*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (20*a^3*e^(3*c) + 37*a^2*b*e^
(3*c) - 15*a*b^2*e^(3*c) - 36*b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 21*a^2*
b*e^c + 29*a*b^2*e^c + 12*b^3*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d + 3*a^4*b^2*
d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*d*e^(12*c)
 + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) - a^5*b*d*e^(10*c) -
5*a^4*b^2*d*e^(10*c) - 3*a^3*b^3*d*e^(10*c))*e^(10*d*x) - (a^6*d*e^(8*c) +
3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*c) - 15*a^3*b^3*d*e^(8*c))*e^(8*d*x)
- 4*(a^6*d*e^(6*c) - a^5*b*d*e^(6*c) + 3*a^4*b^2*d*e^(6*c) + 5*a^3*b^3*d*e^
(6*c))*e^(6*d*x) - (a^6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c
) - 15*a^3*b^3*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c) - a^5*b*d*e^(2*c) -
5*a^4*b^2*d*e^(2*c) - 3*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 6*b)*log((
e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a + 6*b)*log((e^(d*x + c) - 1)*e^(-
c))/(a^4*d) + 8*integrate(1/32*((15*a^2*b*e^(3*c) + 40*a*b^2*e^(3*c) + 24*b
^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 40*a*b^2*e^c + 24*b^3*e^c)*e^(d*x))
/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c
))*e^(4*d*x) + 2*(a^6*e^(2*c) - a^4*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] (a+3*b)*coth(d*x+c)/a^4/d-1/3*coth(d*x+c)^3/a^3/d+5/8*(3*a+7*b)*arctan(b^(1
/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(9/2)/d+1/4*b*(a+b)*tanh(d*x+c)/a^3/d/(a
+b*tanh(d*x+c)^2)^2+1/8*b*(7*a+11*b)*tanh(d*x+c)/a^4/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 467, 1273, 1275, 211}

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} + \frac{(a+3b)\coth(c+dx)}{a^4d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\coth^3(c+dx)}{3a^3d}$$

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*
d) + ((a + 3*b)*Coth[c + d*x])/(a^4*d) - Coth[c + d*x]^3/(3*a^3*d) + (b*(a
+ b)*Tanh[c + d*x])/(4*a^3*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 11*b)*T
anh[c + d*x])/(8*a^4*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{b\text{Subst}\left(\int \frac{-\frac{4}{ab}+\frac{4(a+b)x^2}{a^2b}-\frac{3(a+b)x^4}{a^3}}{x^4(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab+8b(a+2b)x^2-\frac{b^2(7a+11b)x^4}{a}}{x^4(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a^3bd}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(-\frac{8b}{x^4}+\frac{8b(a+3b)}{ax^2}-\frac{5b^2(3a+7b)}{a(a+bx^2)}\right)dx, x, \tanh(c+dx)\right)}{8a^3bd}$$

$$= \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} + \frac{(5b(3a+7b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^4d}$$

$$= \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{15\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\coth(c+dx)\left(-2a-9b+a\operatorname{csch}^2(c+dx)\right) + \frac{3\sqrt{a}b(9a^2+6ab-11b}{}}{24a^{9/2}d}$$

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
[Out] (15*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]
*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(9*a^2 + 6*a
*b - 11*b^2 + (9*a^2 + 20*a*b + 11*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)
])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(24*a^(9/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(135) = 270$.

Time = 11.28 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.63

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{4}}-\ 2b\left(\frac{-\frac{a(9a+13b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8}+\left(-\frac{27}{8}a^{2}-\frac{67}{8}ab-\frac{11}{2}b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\ldots}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}\ldots\right.\right.}\right.\ldots$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{4}}-\ 2b\left(\frac{-\frac{a(9a+13b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8}+\left(-\frac{27}{8}a^{2}-\frac{67}{8}ab-\frac{11}{2}b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\ldots}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}\ldots\right.\right.}\right.\ldots$
risch	$-\frac{900ab^{3}e^{10dx+10c}-64a^{3}b\,e^{2dx+2c}+502a^{2}b^{2}e^{2dx+2c}+1180ab^{3}e^{2dx+2c}+313a^{3}b\,e^{4dx+4c}-1575b^{4}e^{8dx+8c}+630b^{4}e^{10dx+1}\ldots}{\ldots}$

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^4*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-12*b*tan
h(1/2*d*x+1/2*c))-2*b/a^4*((-1/8*a*(9*a+13*b)*tanh(1/2*d*x+1/2*c)^7+(-27/8*
a^2-67/8*a*b-11/2*b^2)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^2-67/8*a*b-11/2*b^2)*
tanh(1/2*d*x+1/2*c)^3+(-9/8*a^2-13/8*a*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*
x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8*(
15*a+35*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*
a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2))))-1/24/a^3/tanh(1/2*d*x+1/2*c)^3-1/8/a^4*(-3*a-12*b)/tanh(1/2*d*x+1
/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7006 vs. 2(135) = 270.

Time = 0.43 (sec) , antiderivative size = 14334, normalized size of antiderivative = 94.93

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}^4\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(135) = 270.

Time = 0.50 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.07

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{16\,a^4+147\,a^3b+351\,a^2b^2+325\,ab^3+105\,b^4+2\,(8\,a^4+32\,a^3b-251\,a^2b^2-590\,ab^3-315\,b^4)e^{(-2\,dx-2\,c)}}{12\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+(a^7-5\,a^6b-13\,a^5b^2-7\,a^4b^3)e^{(-2\,dx-2\,c)}-(3\,a^7+a}$$
$$-\frac{5\,(3\,ab+7\,b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4d}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(16*a^4 + 147*a^3*b + 351*a^2*b^2 + 325*a*b^3 + 105*b^4 + 2*(8*a^4 + 3
2*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*e^(-2*d*x - 2*c) - (96*a^4 + 3
13*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*e^(-4*d*x - 4*c) - 4*(56*a^4
 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*e^(-6*d*x - 6*c) - (176*a^4
 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*e^(-8*d*x - 8*c) - 6*(8*
a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*e^(-10*d*x - 10*c) + 15*(3*a^3*b +
```

```
13*a^2*b^2 + 17*a*b^3 + 7*b^4)*e^(-12*d*x - 12*c))/((a^7 + 3*a^6*b + 3*a^5*
b^2 + a^4*b^3 + (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-2*d*x - 2*c) -
 (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-4*d*x - 4*c) - (3*a^7 - 7*a^
6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-6*d*x - 6*c) + (3*a^7 - 7*a^6*b + 25*a^5
*b^2 + 35*a^4*b^3)*e^(-8*d*x - 8*c) + (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*
b^3)*e^(-10*d*x - 10*c) - (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-12*d
*x - 12*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-14*d*x - 14*c))*d) -
 5/8*(3*a*b + 7*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b
))/(sqrt(a*b)*a^4*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

3.49 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(1+\tanh(c+dx))}{16d} - \frac{3a\tanh(c+dx)}{8d}-\frac{3b\tanh^2(c+dx)}{2d}+\frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d}$$

```
[Out] -3/16*(a+8*b)*ln(1-tanh(d*x+c))/d+3/16*(a-8*b)*ln(1+tanh(d*x+c))/d-3/8*a*ta
nh(d*x+c)/d-3/2*b*tanh(d*x+c)^2/d+1/4*sinh(d*x+c)^4*(b+a*tanh(d*x+c))/d-1/8
*sinh(d*x+c)^2*tanh(d*x+c)*(a+8*b*tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3744, 1818, 815, 647, 31}

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(\tanh(c+dx)+1)}{16d} + \frac{\sinh^4(c+dx)(a\tanh(c+dx)+b)}{4d}-\frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d} - \frac{3a\tanh(c+dx)}{8d}-\frac{3b\tanh^2(c+dx)}{2d}$$

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*(a + 8*b)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a - 8*b)*Log[1 + Tanh[c
+ d*x]])/(16*d) - (3*a*Tanh[c + d*x])/(8*d) - (3*b*Tanh[c + d*x]^2)/(2*d) +
 (Sinh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*d) - (Sinh[c + d*x]^2*Tanh[c +
d*x]*(a + 8*b*Tanh[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^3(-4b-ax-4bx^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d} + \frac{\text{Subst}\left(\int \frac{x^2(3a+24bx)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d} + \frac{\text{Subst}\left(\int \left(-3a-24bx+\frac{3(a+8bx)}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= -\frac{3a\tanh(c+dx)}{8d} - \frac{3b\tanh^2(c+dx)}{2d} + \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d} + \frac{3\text{Subst}\left(\int \frac{a+8bx}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= -\frac{3a\tanh(c+dx)}{8d} - \frac{3b\tanh^2(c+dx)}{2d} + \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d} - \frac{(3(a-8b))\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{16d} + \frac{(3(a+8b))\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{16d}$$

$$= -\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-8b)\log(1+\tanh(c+dx))}{16d} - \frac{3a\tanh(c+dx)}{8d} - \frac{3b\tanh^2(c+dx)}{2d} + \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{3a(c+dx)}{8d}+\frac{b\left(12\log(\cosh(c+dx))+2\operatorname{sech}^2(c+dx)-4\sinh^2(c+dx)+\sinh^4(c+dx)\right)}{4d}-\frac{a\sinh(2(c+dx))}{4d}+\frac{a\sinh(4(c+dx))}{32d}$$

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*(12*Log[Cosh[c + d*x]] + 2*Sech[c + d*x]^2 - 4*S
inh[c + d*x]^2 + Sinh[c + d*x]^4))/(4*d) - (a*Sinh[2*(c + d*x)])/(4*d) + (a
*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh(dx+c}{2}\right.}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh(dx+c}{2}\right.}{d}$
risch	$\frac{3ax}{8}-3bx+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}a}{8d}-\frac{5\,e^{2dx+2c}b}{16d}+\frac{e^{-2dx-2c}a}{8d}-\frac{5\,e^{-2dx-2c}b}{16d}-\frac{e^{-4dx-4c}a}{64d}+$

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln(cosh(d*
x+c))-3/2*tanh(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. 2(120) = 240.

Time = 0.28 (sec) , antiderivative size = 1530, normalized size of antiderivative = 11.59

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/64*((a + b)*cosh(d*x + c)^12 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^11
+ (a + b)*sinh(d*x + c)^12 - 6*(a + 3*b)*cosh(d*x + c)^10 + 6*(11*(a + b)*c
osh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^10 + 20*(11*(a + b)*cosh(d*x + c)^3
 - 3*(a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(8*(a - 8*b)*d*x - 5*a -
13*b)*cosh(d*x + c)^8 + 3*(165*(a + b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d*x -
90*(a + 3*b)*cosh(d*x + c)^2 - 5*a - 13*b)*sinh(d*x + c)^8 + 24*(33*(a + b)
*cosh(d*x + c)^5 - 30*(a + 3*b)*cosh(d*x + c)^3 + (8*(a - 8*b)*d*x - 5*a -
13*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x
+ c)^6 + 4*(231*(a + b)*cosh(d*x + c)^6 - 315*(a + 3*b)*cosh(d*x + c)^4 + 1
2*(a - 8*b)*d*x + 21*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^2 + 22*b)
*sinh(d*x + c)^6 + 24*(33*(a + b)*cosh(d*x + c)^7 - 63*(a + 3*b)*cosh(d*x +
 c)^5 + 7*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^3 + 2*(6*(a - 8*b)*d
*x + 11*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b
)*cosh(d*x + c)^4 + 3*(165*(a + b)*cosh(d*x + c)^8 - 420*(a + 3*b)*cosh(d*x
 + c)^6 + 70*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d
*x + 40*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^2 + 5*a - 13*b)*sinh(d*x + c
)^4 + 4*(55*(a + b)*cosh(d*x + c)^9 - 180*(a + 3*b)*cosh(d*x + c)^7 + 42*(8
*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^5 + 40*(6*(a - 8*b)*d*x + 11*b)*
cosh(d*x + c)^3 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*(a - 3*b)*cosh(d*x + c)^2 + 6*(11*(a + b)*cosh(d*x + c)^10 - 45*
(a + 3*b)*cosh(d*x + c)^8 + 14*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)
^6 + 20*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^4 + 3*(8*(a - 8*b)*d*x + 5*a
 - 13*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c)^2 + 192*(b*cosh(d*x + c)^
8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 + 2*b*cosh(d*x +
c)^6 + 2*(14*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^6 + 4*(14*b*cosh(d*x + c)
^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + b*cosh(d*x + c)^4 + (70*b*cosh(d*
x + c)^4 + 30*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(14*b*cosh(d*x + c
)^5 + 10*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*b*cos
h(d*x + c)^6 + 15*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*(2*b*cosh(d*x + c)^7 + 3*b*cosh(d*x + c)^5 + b*cosh(d*x + c)^3)*sinh(d*
x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*((a + b)*
cosh(d*x + c)^11 - 5*(a + 3*b)*cosh(d*x + c)^9 + 2*(8*(a - 8*b)*d*x - 5*a -
 13*b)*cosh(d*x + c)^7 + 4*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^5 + (8*(a
 - 8*b)*d*x + 5*a - 13*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d
*x + c) - a + b)/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d
```

```
*sinh(d*x + c)^8 + 2*d*cosh(d*x + c)^6 + 2*(14*d*cosh(d*x + c)^2 + d)*sinh(
d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 +
 d*cosh(d*x + c)^4 + (70*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh
(d*x + c)^4 + 4*(14*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(14*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 3*d
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 3*d*cosh(d*x +
 c)^5 + d*cosh(d*x + c)^3)*sinh(d*x + c))
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)\sinh^4{(c+dx)}\,dx$$

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.47

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{1}{64}b\left(\frac{192(dx+c)}{d}-\frac{20e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}+\frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{18e^{(-2dx-2c)}+39e^{(-4dx-}}{d(e^{(-4dx-4c)}+2e^{(-6}}\right.$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*
c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) +
 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.54

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{24\,(dx+c)(a-8\,b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-20\,be^{(2\,dx+2\,c)}+192\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-}{64\,d}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*(a - 8*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*
e^(2*d*x + 2*c) - 20*b*e^(2*d*x + 2*c) + 192*b*log(e^(2*d*x + 2*c) + 1) - (
9*a*e^(8*d*x + 8*c) + 72*b*e^(8*d*x + 8*c) + 10*a*e^(6*d*x + 6*c) + 36*b*e^
(6*d*x + 6*c) - 6*a*e^(4*d*x + 4*c) + 111*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x
+ 2*c) + 18*b*e^(2*d*x + 2*c) + a - b)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c))^
2)/d
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=x\left(\frac{3\,a}{8}-3\,b\right)+\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,\left(a+b\right)}{64\,d}-\frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,\left(a-b\right)}{64\,d}+\frac{3\,b\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(2\,a-5\,b\right)}{16\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a+5\,b\right)}{16\,d}$$

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] x*((3*a)/8 - 3*b) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) + (exp(4*c + 4*d*x)*(a
 + b))/(64*d) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (ex
p(- 4*c - 4*d*x)*(a - b))/(64*d) + (3*b*log(exp(2*c)*exp(2*d*x) + 1))/d + (
exp(- 2*c - 2*d*x)*(2*a - 5*b))/(16*d) - (exp(2*c + 2*d*x)*(2*a + 5*b))/(16
*d)
```

3.50 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{5b\arctan(\sinh(c+dx))}{2d}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5b\sinh(c+dx)}{2d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}$$

```
[Out] 5/2*b*arctan(sinh(d*x+c))/d-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d-5/2*b*sin
h(d*x+c)/d+5/6*b*sinh(d*x+c)^3/d-1/2*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3747, 2713, 2672, 294, 308, 209}

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{5b\arctan(\sinh(c+dx))}{2d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{5b\sinh(c+dx)}{2d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}$$

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^
3)/(3*d) - (5*b*Sinh[c + d*x])/(2*d) + (5*b*Sinh[c + d*x]^3)/(6*d) - (b*Sin
h[c + d*x]^3*Tanh[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i\int\left(-ia\sinh^3(c+dx)-ib\sinh^3(c+dx)\tanh^3(c+dx)\right)\,dx\\
&= a\int\sinh^3(c+dx)\,dx+b\int\sinh^3(c+dx)\tanh^3(c+dx)\,dx\\
&= -\frac{a\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(c+dx)\right)}{d}+\frac{b\text{Subst}\left(\int\frac{x^6}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}\\
&\quad+\frac{(5b)\text{Subst}\left(\int\frac{x^4}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{2d}\\
&= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}\\
&\quad+\frac{(5b)\text{Subst}\left(\int\left(-1+x^2+\frac{1}{1+x^2}\right)\,dx,x,\sinh(c+dx)\right)}{2d}\\
&= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5b\sinh(c+dx)}{2d}+\frac{5b\sinh^3(c+dx)}{6d}\\
&\quad-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{(5b)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{2d}\\
&= \frac{5b\arctan(\sinh(c+dx))}{2d}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}\\
&\quad-\frac{5b\sinh(c+dx)}{2d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int\sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx &= \frac{5b\arctan(\sinh(c+dx))}{2d}\\
&\quad-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}\\
&\quad-\frac{5b\text{sech}(c+dx)\tanh(c+dx)}{2d}\\
&\quad-\frac{5b\sinh(c+dx)\tanh^2(c+dx)}{3d}\\
&\quad+\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{3d}
\end{aligned}
$$

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (3*a*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(
c + d*x)])/(12*d) - (5*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) - (5*b*Sinh[c +
 d*x]*Tanh[c + d*x]^2)/(3*d) + (b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(3*d)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\mathrm{e}^{d}\right.}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\mathrm{e}^{d}\right.}{d}$
risch	$\frac{\mathrm{e}^{3dx+3c}a}{24d}+\frac{\mathrm{e}^{3dx+3c}b}{24d}-\frac{3\,\mathrm{e}^{dx+c}a}{8d}-\frac{9\,\mathrm{e}^{dx+c}b}{8d}-\frac{3\,\mathrm{e}^{-dx-c}a}{8d}+\frac{9\,\mathrm{e}^{-dx-c}b}{8d}+\frac{\mathrm{e}^{-3dx-3c}a}{24d}-\frac{\mathrm{e}^{-3dx-3c}b}{24d}-\frac{b\,\mathrm{e}^{dx}}{d}$

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^5/cosh(d*x+c
)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(d*
x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 1070, normalized size of antiderivative = 10.92

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*((a + b)*cosh(d*x + c)^10 + 10*(a + b)*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a + b)*sinh(d*x + c)^10 - (7*a + 25*b)*cosh(d*x + c)^8 + (45*(a + b)*cosh
(d*x + c)^2 - 7*a - 25*b)*sinh(d*x + c)^8 + 8*(15*(a + b)*cosh(d*x + c)^3 -
 (7*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a + 25*b)*cosh(d*x + c
)^6 + 2*(105*(a + b)*cosh(d*x + c)^4 - 14*(7*a + 25*b)*cosh(d*x + c)^2 - 13
*a - 25*b)*sinh(d*x + c)^6 + 4*(63*(a + b)*cosh(d*x + c)^5 - 14*(7*a + 25*b
)*cosh(d*x + c)^3 - 3*(13*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*
a - 25*b)*cosh(d*x + c)^4 + 2*(105*(a + b)*cosh(d*x + c)^6 - 35*(7*a + 25*b
)*cosh(d*x + c)^4 - 15*(13*a + 25*b)*cosh(d*x + c)^2 - 13*a + 25*b)*sinh(d*
x + c)^4 + 8*(15*(a + b)*cosh(d*x + c)^7 - 7*(7*a + 25*b)*cosh(d*x + c)^5 -
 5*(13*a + 25*b)*cosh(d*x + c)^3 - (13*a - 25*b)*cosh(d*x + c))*sinh(d*x +
c)^3 - (7*a - 25*b)*cosh(d*x + c)^2 + (45*(a + b)*cosh(d*x + c)^8 - 28*(7*a
 + 25*b)*cosh(d*x + c)^6 - 30*(13*a + 25*b)*cosh(d*x + c)^4 - 12*(13*a - 25
```

```
*b)*cosh(d*x + c)^2 - 7*a + 25*b)*sinh(d*x + c)^2 + 120*(b*cosh(d*x + c)^7
+ 7*b*cosh(d*x + c)*sinh(d*x + c)^6 + b*sinh(d*x + c)^7 + 2*b*cosh(d*x + c)
^5 + (21*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^5 + 5*(7*b*cosh(d*x + c)^3
+ 2*b*cosh(d*x + c))*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (35*b*cosh(d*x +
 c)^4 + 20*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + (21*b*cosh(d*x + c)^5 +
 20*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + (7*b*cosh(d*x
+ c)^6 + 10*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a + b)*cosh(d*x + c)^9 - 4*(7*a + 25
*b)*cosh(d*x + c)^7 - 6*(13*a + 25*b)*cosh(d*x + c)^5 - 4*(13*a - 25*b)*cos
h(d*x + c)^3 - (7*a - 25*b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d
*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cos
h(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d
*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*
cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*
x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d
*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c
))
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int\left(a+b\tanh^3\left(c+dx\right)\right)\sinh^3\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{1}{24}\,b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)$$
$$+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] 1/24*b*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d
 - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*
(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a*(e^(3
*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{120\,b\arctan\left(e^{(dx+c)}\right)+ae^{(3\,dx+3\,c)}+be^{(3\,dx+3\,c)}-9\,ae^{(dx+c)}-27\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}-27\,be^{(2\,dx+2\,c)}-\right.}{24\,d}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/24*(120*b*arctan(e^(d*x + c)) + a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) - 9
*a*e^(d*x + c) - 27*b*e^(d*x + c) - (9*a*e^(2*d*x + 2*c) - 27*b*e^(2*d*x +
2*c) - a + b)*e^(-3*d*x - 3*c) - 24*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^
(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{e^{3\,c+3\,d\,x}\,(a+b)}{24\,d}+\frac{5\,\mathrm{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}+\frac{e^{-3\,c-3\,d\,x}\,(a-b)}{24\,d}-\frac{e^{c+d\,x}\,(3\,a+9\,b)}{8\,d}-\frac{e^{-c-d\,x}\,(3\,a-9\,b)}{8\,d}-\frac{b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{2\,b\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b))/(24*d) + (5*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))
/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(- 3*c - 3*d*x)*(a - b))/(
24*d) - (exp(c + d*x)*(3*a + 9*b))/(8*d) - (exp(- c - d*x)*(3*a - 9*b))/(8*
d) - (b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2
*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.51 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{(a+4b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-4b)\log(1+\tanh(c+dx))}{4d} + \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d}$$

```
[Out] 1/4*(a+4*b)*ln(1-tanh(d*x+c))/d-1/4*(a-4*b)*ln(1+tanh(d*x+c))/d+1/2*a*tanh(
d*x+c)/d+1/2*b*tanh(d*x+c)^2/d+1/2*sinh(d*x+c)^2*(b+a*tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3744, 1818, 1816, 647, 31}

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{(a+4b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-4b)\log(\tanh(c+dx)+1)}{4d} + \frac{\sinh^2(c+dx)(a\tanh(c+dx)+b)}{2d} + \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d}$$

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] ((a + 4*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 4*b)*Log[1 + Tanh[c + d*x]
])/(4*d) + (a*Tanh[c + d*x])/(2*d) + (b*Tanh[c + d*x]^2)/(2*d) + (Sinh[c +
d*x]^2*(b + a*Tanh[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{x(-2b-ax-2bx^2)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{\text{Subst}\left(\int \left(a+2bx-\frac{a+4bx}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} - \frac{\text{Subst}\left(\int \frac{a+4bx}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{(a-4b)\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{4d} - \frac{(a+4b)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= \frac{(a+4b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-4b)\log(1+\tanh(c+dx))}{4d} + \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{a(-c-dx)}{2d} - \frac{b\left(4\log(\cosh(c+dx))+\text{sech}^2(c+dx)-\sinh^2(c+dx)\right)}{2d} + \frac{a\sinh(2(c+dx))}{4d}$$

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*(-c - d*x))/(2*d) - (b*(4*Log[Cosh[c + d*x]] + Sech[c + d*x]^2 - Sinh[c
+ d*x]^2))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)}{d}$
risch	$-\frac{ax}{2}+2bx+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{8d}+\frac{4bc}{d}-\frac{2b\,e^{2dx+2c}}{d\left(e^{2dx+2c}+1\right)^2}-\frac{2b\ln(e^{2dx+2c}}{d}$

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^4/cos
h(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(90) = 180.

Time = 0.32 (sec) , antiderivative size = 924, normalized size of antiderivative = 9.24

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/8*((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
 + b)*sinh(d*x + c)^8 - 2*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^6 - 2*(2*
(a - 4*b)*d*x - 14*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 4*(14
*(a + b)*cosh(d*x + c)^3 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c))*sinh(
d*x + c)^5 - 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh
(d*x + c)^4 - 4*(a - 4*b)*d*x - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^
2 - 7*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 5*(2*(a - 4*b)*d*
x - a - b)*cosh(d*x + c)^3 - (4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c))*sinh(d*
x + c)^3 - 2*(2*(a - 4*b)*d*x + a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*cosh
(d*x + c)^6 - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^4 - 2*(a - 4*b)*d*
x - 6*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - 16
*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6
 + 2*b*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^
2 + (15*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c))
*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*cosh(d
x + c)^7 - 3(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^5 - 2*(4*(a - 4*b)*d*

```
x + 7*b)*cosh(d*x + c)^3 - (2*(a - 4*b)*d*x + a - b)*cosh(d*x + c))*sinh(d*
x + c) - a + b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*
sinh(d*x + c)^6 + 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 2*d)*sinh(d
*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^3 + d
*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))
*sinh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int\left(a+b\tanh^3(c+dx)\right)\sinh^2(c+dx)\,dx$$

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{8}\,b\left(\frac{16\,(dx+c)}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}+\frac{16\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+1}{d\left(e^{(-2\,dx-2\,c)}+2\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}\right)$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(16*(d*x + c)
/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x - 2
*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c))))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{4\,(dx+c)(a-4\,b)-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-2\,dx-2\,c)}+16\,b\log}{8\,d}$$

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] -1/8*(4*(d*x + c)*(a - 4*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (2*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c) - a + b)*e^(-2*d*x - 2*c) + 16*b*log(e^(2*d*x + 2*c) + 1) - 8*(3*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + 3*b)/(e^(2*d*x + 2*c) + 1)^2)/d

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{2\,c+2\,d\,x}\,(a+b)}{8\,d}-\frac{2\,b}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-x\,\left(\frac{a}{2}-2\,b\right)+\frac{2\,b}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{e^{-2\,c-2\,d\,x}\,(a-b)}{8\,d}-\frac{2\,b\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+1\right)}{d}$$

[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3),x)

[Out] (exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a/2 - 2*b) + (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a - b))/(8*d) - (2*b*log(exp(2*c)*exp(2*d*x) + 1))/d

3.52 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{3b\arctan(\sinh(c+dx))}{2d}+\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

```
[Out] -3/2*b*arctan(sinh(d*x+c))/d+a*cosh(d*x+c)/d+3/2*b*sinh(d*x+c)/d-1/2*b*sinh
(d*x+c)*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3747, 2718, 2672, 294, 327, 209}

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{a\cosh(c+dx)}{d}-\frac{3b\arctan(\sinh(c+dx))}{2d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c + d*x])/d + (3*b*Sinh[c + d*
x])/(2*d) - (b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \left(ia\sinh(c+dx)+ib\sinh(c+dx)\tanh^3(c+dx)\right)\,dx\right)\\
&= a\int \sinh(c+dx)\,dx+b\int \sinh(c+dx)\tanh^3(c+dx)\,dx\\
&= \frac{a\cosh(c+dx)}{d}+\frac{b\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}
\end{aligned}$$

$$= \frac{a\cosh(c+dx)}{d} - \frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d} + \frac{(3b)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d}$$

$$= \frac{a\cosh(c+dx)}{d} + \frac{3b\sinh(c+dx)}{2d} - \frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d} - \frac{(3b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d}$$

$$= -\frac{3b\arctan(\sinh(c+dx))}{2d} + \frac{a\cosh(c+dx)}{d} + \frac{3b\sinh(c+dx)}{2d} - \frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3b\arctan(\sinh(c+dx))}{2d} + \frac{a\cosh(c)\cosh(dx)}{d} + \frac{a\sinh(c)\sinh(dx)}{d} + \frac{3b\text{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{b\sinh(c+dx)\tanh^2(c+dx)}{d}$$

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c]*Cosh[d*x])/d + (a*Sinh[c]*S
inh[d*x])/d + (3*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sinh[c + d*x]*Ta
nh[c + d*x]^2)/d
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\text{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(e^{dx+c}\right)\right)}{d}$	73
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\text{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(e^{dx+c}\right)\right)}{d}$	73
risch	$\frac{e^{dx+c}a}{2d}+\frac{e^{dx+c}b}{2d}+\frac{e^{-dx-c}a}{2d}-\frac{e^{-dx-c}b}{2d}+\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^2}+\frac{3ib\ln\left(e^{dx+c}-i\right)}{2d}-\frac{3ib\ln\left(e^{dx+c}+i\right)}{2d}$	125

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(57) = 114.

Time = 0.33 (sec) , antiderivative size = 528, normalized size of antiderivative = 8.38

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{(a+b)\cosh\left(dx+c\right)^6+6\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+\left(a+b\right)\sinh\left(dx+c\right)^6+3\left(a+b\right)\cosh\left(dx\right.}{}$$

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/2*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 + 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 + 6*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 6*(b*cosh(d*x + c)^5 + 5*b*cosh(d*x + c)*sinh(d*x + c)^4 + b*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + 2*(5*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + 2*(5*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + b*cosh(d*x + c) + (5*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3),x)

[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right) + \frac{a\cosh\left(dx+c\right)}{d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^
(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c
)))) + a*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{6\,b\arctan\left(e^{(dx+c)}\right)-ae^{(dx+c)}-be^{(dx+c)}-(a-b)e^{(-dx-c)}-\frac{2\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/2*(6*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) - (a - b)*e^(
-d*x - c) - 2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/
d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{\mathrm{e}^{-c-d\,x}\,(a-b)}{2\,d}-\frac{3\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}+\frac{\mathrm{e}^{c+d\,x}\,(a+b)}{2\,d}+\frac{b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] (exp(- c - d*x)*(a - b))/(2*d) - (3*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d
*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a + b))/(2*d) + (b
*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.53 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{b\arctan(\sinh(c+dx))}{2d}-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b*arctan(sinh(d*x+c))/d-a*arctanh(cosh(d*x+c))/d-1/2*b*sech(d*x+c)*tanh
(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3747, 3855, 2691}

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b\arctan(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*ArcTanh[Cosh[c + d*x]])/d - (b*Sech[c
+ d*x]*Tanh[c + d*x])/(2*d)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \left(-ia\operatorname{csch}(c+dx) - ib\operatorname{sech}(c+dx)\tanh^2(c+dx)\right)\,dx \\
&= a\int \operatorname{csch}(c+dx)\,dx + b\int \operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx \\
&= -\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{1}{2}b\int \operatorname{sech}(c+dx)\,dx \\
&= \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = {} & \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} \\
& + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} \\
& - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}
\end{aligned}$$

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[S
inh[c/2 + (d*x)/2]])/d - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	56
default	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	56
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{2d}-\frac{ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{2d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	101

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c)
*tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 522, normalized size of antiderivative = 10.65

$$\int \mathrm{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3-\left(b\cosh(dx+c)^4+4\,b\cosh(dx+\right.}{}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*
(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*arctan(cosh(d*x +
c) + sinh(d*x + c)) - b*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh
(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*
sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x
+ c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) -
1) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh
(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x
+ c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)\operatorname{csch}{(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3),x)
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan\left(e^{(dx+c)}\right)-a\log\left(e^{(dx+c)}+1\right)+a\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{be^{(3\,dx+3\,c)}-be^{(dx+c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] (b*arctan(e^(d*x + c)) - a*log(e^(d*x + c) + 1) + a*log(abs(e^(d*x + c) - 1
)) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}-\frac{a\,\ln\left(-8\,a\,b^2-32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-8\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d}$$
$$+\frac{a\,\ln\left(8\,a\,b^2+32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-8\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d}$$
$$-\frac{b\left(\ln\left(4\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^2\,b\,16\mathrm{i}-b^3\,4\mathrm{i}\right)\,1\mathrm{i}-\ln\left(4\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+a^2\,b\,16\mathrm{i}+b^3\,4\mathrm{i}\right)\,1\mathrm{i}\right)}{2\,d}$$
$$-\frac{b\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}}$$

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x),x)
```

```
[Out] (2*b*exp(c + d*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*log
(- 8*a*b^2 - 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*exp(c)))/d
+ (a*log(8*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*exp(c
)))/d - (b*(log(4*b^3*exp(d*x)*exp(c) - b^3*4i - a^2*b*16i + 16*a^2*b*exp(d
*x)*exp(c))*1i - log(a^2*b*16i + b^3*4i + 4*b^3*exp(d*x)*exp(c) + 16*a^2*b*
exp(d*x)*exp(c))*1i))/(2*d) - (b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x))
```

3.54 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] -a*coth(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3744, 14}

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{b\tanh^2(c+dx)}{2d}-\frac{a\coth(c+dx)}{d}$$

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
```

```
+ 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+bx^3}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+bx\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}-\frac{b\operatorname{sech}^2(c+dx)}{2d}$$

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
[Out] -((a*Coth[c + d*x])/d) - (b*Sech[c + d*x]^2)/(2*d)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{a}{\tanh(dx+c)}+\frac{b\tanh(dx+c)^2}{2}}{d}$	28
default	$\frac{-\frac{a}{\tanh(dx+c)}+\frac{b\tanh(dx+c)^2}{2}}{d}$	28
risch	$-\frac{2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-b\,e^{2dx+2c}+a\right)}{d\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)}$	80

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a/tanh(d*x+c)+1/2*b*tanh(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(27) = 54.

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.86

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{2\left((2a+b)\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+(2a+b)\right.}{d\cosh(dx+c)^4+6d\cosh(dx+c)^2\sinh(dx+c)^2+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -2*((2*a + b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (2*a + b)
*sinh(d*x + c)^2 + 2*a - b)/(d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{2a}{d\left(e^{(-2dx-2c)}-1\right)} - \frac{2b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) - 2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{2\left(\frac{a}{e^{(2\,dx+2\,c)}-1}+\frac{be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}\right)}{d}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] -2*(a/(e^(2*d*x + 2*c) - 1) + b*e^(2*d*x + 2*c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= -\frac{2\left(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+a\,\mathrm{e}^{4\,c+4\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}+b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^2}$$

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^2,x)
[Out] -(2*(a + 2*a*exp(2*c + 2*d*x) + a*exp(4*c + 4*d*x) - b*exp(2*c + 2*d*x) + b
*exp(4*c + 4*d*x)))/(d*(exp(2*c + 2*d*x) - 1)*(exp(2*c + 2*d*x) + 1)^2)
```

3.55 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{b\arctan(\sinh(c+dx))}{2d}+\frac{a\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b*arctan(sinh(d*x+c))/d+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*
csch(d*x+c)/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3747, 3853, 3855}

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{a\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\arctan(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Cot
h[c + d*x]*Csch[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i \int \left(ia\text{csch}^3(c+dx) + ib\text{sech}^3(c+dx)\right)\,dx\right)\\
&= a\int \text{csch}^3(c+dx)\,dx + b\int \text{sech}^3(c+dx)\,dx\\
&= -\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} + \frac{b\text{sech}(c+dx)\tanh(c+dx)}{2d}\\
&\quad -\frac{1}{2}a\int \text{csch}(c+dx)\,dx + \frac{1}{2}b\int \text{sech}(c+dx)\,dx\\
&= \frac{b\arctan(\sinh(c+dx))}{2d} + \frac{a\text{arctanh}(\cosh(c+dx))}{2d}\\
&\quad -\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} + \frac{b\text{sech}(c+dx)\tanh(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[Co
sh[(c + d*x)/2]])/(2*d) - (a*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d
*x)/2]^2)/(8*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}-b\,e^{6dx+6c}+3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+3\,e^{2dx+2c}a-3b\,e^{2dx+2c}+a+b\right)}{d\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)^2}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{ib\ln(e^{d}}{2}$

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/2*sech(d*x+c
)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. 2(63) = 126.

Time = 0.27 (sec) , antiderivative size = 1188, normalized size of antiderivative = 16.73

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - b)*cosh(d*x + c)^7 + 14*(a - b)*cosh(d*x + c)*sinh(d*x + c)^6
+ 2*(a - b)*sinh(d*x + c)^7 + 6*(a + b)*cosh(d*x + c)^5 + 6*(7*(a - b)*cosh
(d*x + c)^2 + a + b)*sinh(d*x + c)^5 + 10*(7*(a - b)*cosh(d*x + c)^3 + 3*(a
 + b)*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^3 + 2*(35*(a
 - b)*cosh(d*x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a - 3*b)*sinh(d*x +
c)^3 + 6*(7*(a - b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a - b
)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^8 + 56*b*cosh(d*x + c
)^3*sinh(d*x + c)^5 + 28*b*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b*cosh(d*x +
 c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 2*b*cosh(d*x + c)^4 + 2*(35*b*cos
h(d*x + c)^4 - b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - b*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*b*cosh(d*x + c)^6 - 3*b*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(b*cosh(d*x + c)^7 - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)*arct
an(cosh(d*x + c) + sinh(d*x + c)) + 2*(a + b)*cosh(d*x + c) - (a*cosh(d*x +
 c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*
x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh
(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*
x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3
)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a*cosh(d*x +
 c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*
x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh
(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*
x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3
)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a - b)*
cosh(d*x + c)^6 + 15*(a + b)*cosh(d*x + c)^4 + 9*(a - b)*cosh(d*x + c)^2 +
a + b)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x +
c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 -
d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*
cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \int \left(a+b\tanh^3{(c+dx)}\right)\operatorname{csch}^3{(c+dx)}\, dx$$

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.20

$$\begin{aligned} &\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx \\ &= -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) \\ &\quad + \frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right) \end{aligned}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d - log
(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

$$\begin{aligned} &\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx \\ &= \frac{2\,b\arctan\left(e^{(dx+c)}\right) + a\log\left(e^{(dx+c)}+1\right) - a\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(ae^{(7\,dx+7\,c)} - be^{(7\,dx+7\,c)} + 3\,ae^{(5\,dx+5\,c)} + 3\,be^{(5\,dx+5\,\cdots}}{(e^{(4\,dx+4\,\cdots}}}{2\,d} \end{aligned}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] 1/2*(2*b*arctan(e^(d*x + c)) + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x +
c) - 1)) - 2*(a*e^(7*d*x + 7*c) - b*e^(7*d*x + 7*c) + 3*a*e^(5*d*x + 5*c) +
 3*b*e^(5*d*x + 5*c) + 3*a*e^(3*d*x + 3*c) - 3*b*e^(3*d*x + 3*c) + a*e^(d*x
 + c) + b*e^(d*x + c))/(e^(4*d*x + 4*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{a\,\ln\left(e^{c+d\,x}+1\right)}{2\,d}-\frac{\frac{4\,e^{3\,c+3\,d\,x}\,(a-b)}{d}+\frac{4\,e^{c+d\,x}\,(a+b)}{d}}{e^{8\,c+8\,d\,x}-2\,e^{4\,c+4\,d\,x}+1}-\frac{a\,\ln\left(e^{c+d\,x}-1\right)}{2\,d}-\frac{\frac{e^{3\,c+3\,d\,x}\,(a-b)}{d}+\frac{3\,e^{c+d\,x}\,(a+b)}{d}}{e^{4\,c+4\,d\,x}-1}-\frac{b\,\ln\left(e^{c+d\,x}-\mathrm{i}\right)\,1\mathrm{i}}{2\,d}+\frac{b\,\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

```
[Out] (a*log(exp(c + d*x) + 1))/(2*d) - ((4*exp(3*c + 3*d*x)*(a - b))/d + (4*exp(
c + d*x)*(a + b))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (a*log(e
xp(c + d*x) - 1))/(2*d) - ((exp(3*c + 3*d*x)*(a - b))/d + (3*exp(c + d*x)*(
a + b))/d)/(exp(4*c + 4*d*x) - 1) - (b*log(exp(c + d*x) - 1i)*1i)/(2*d) + (
b*log(exp(c + d*x) + 1i)*1i)/(2*d)
```

3.56 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+\frac{b\log(\tanh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d+b*ln(tanh(d*x+c))/d-1/2*b*tanh(d*x+c)
^2/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3744, 1816}

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^2(c+dx)}{2d}+\frac{b\log(\tanh(c+dx))}{d}$$

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) + (b*Log[Tanh[c + d*x]])/d
- (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^3)}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{a}{x^2}+\frac{b}{x}-bx\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+\frac{b\log(\tanh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$
\begin{aligned}
&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx\\
&= \frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}\\
&\quad -\frac{b\left(2\log(\cosh(c+dx))-2\log(\sinh(c+dx))-\operatorname{sech}^2(c+dx)\right)}{2d}
\end{aligned}
$$

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*(2
*Log[Cosh[c + d*x]] - 2*Log[Sinh[c + d*x]] - Sech[c + d*x]^2))/(2*d)
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+b\left(\frac{1}{2\cosh(dx+c)^{2}}+\ln(\tanh(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+b\left(\frac{1}{2\cosh(dx+c)^{2}}+\ln(\tanh(dx+c))\right)}{d}$
risch	$-\frac{2\left(-3b\,e^{8dx+8c}+6a\,e^{6dx+6c}+9b\,e^{6dx+6c}+10a\,e^{4dx+4c}-9b\,e^{4dx+4c}+2\,e^{2dx+2c}a+3b\,e^{2dx+2c}-2a\right)}{3d\left(e^{2dx+2c}-1\right)^{3}\left(e^{2dx+2c}+1\right)^{2}}-\frac{b\ln(e^{2dx+2c}+1)}{d}$

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(1/2/cosh(d*x+c)^2+ln(tanh(d*x
+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1739 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 1739, normalized size of antiderivative = 31.05

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
[Out] 1/3*(6*b*cosh(d*x + c)^8 + 48*b*cosh(d*x + c)*sinh(d*x + c)^7 + 6*b*sinh(d*
x + c)^8 - 6*(2*a + 3*b)*cosh(d*x + c)^6 + 6*(28*b*cosh(d*x + c)^2 - 2*a -
3*b)*sinh(d*x + c)^6 + 12*(28*b*cosh(d*x + c)^3 - 3*(2*a + 3*b)*cosh(d*x +
c))*sinh(d*x + c)^5 - 2*(10*a - 9*b)*cosh(d*x + c)^4 + 2*(210*b*cosh(d*x +
c)^4 - 45*(2*a + 3*b)*cosh(d*x + c)^2 - 10*a + 9*b)*sinh(d*x + c)^4 + 8*(42
*b*cosh(d*x + c)^5 - 15*(2*a + 3*b)*cosh(d*x + c)^3 - (10*a - 9*b)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(84*b*cosh(d*x +
 c)^6 - 45*(2*a + 3*b)*cosh(d*x + c)^4 - 6*(10*a - 9*b)*cosh(d*x + c)^2 - 2
*a - 3*b)*sinh(d*x + c)^2 - 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh
(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^8 + (45*b*cosh(d*x + c)^
2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*
x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*b*cosh(d*x +
 c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3
 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^4 + 2*(105*b*cosh
(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c
)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*cosh(d*x + c)^3 +
 b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (45*b*cosh(d*x + c)
^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b
```

)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*cosh(d*x + c)^7 - 6*b*cosh (d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log (2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^ 8 + (45*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^ 5 - 14*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d* x + c)^4 + 2*(105*b*cosh(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c) ^2 + (45*b*cosh(d*x + c)^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*co sh(d*x + c)^7 - 6*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(12*b*cosh(d*x + c)^7 - 9*(2*a + 3*b)*cosh(d*x + c)^5 - 2*(10*a - 9*b)*c osh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*a)/(d*cosh(d* x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 - d*cos h(d*x + c)^8 + (45*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d* x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105* d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cos h(d*x + c)^5 - 14*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^4 + 2*(105*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 - 15* d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 - 7*d*cosh (d*x + c)^5 - 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + d*co sh(d*x + c)^2 + (45*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 - 30*d*cosh(d* x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c) ^9 - 4*d*cosh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*co sh(d*x + c))*sinh(d*x + c) - d)

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)\operatorname{csch}^4\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(52) = 104$.

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.29

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)
)) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(52) = 104$.

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.66

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx =$$
$$-\frac{6\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)-\frac{3\left(3\,be^{(4\,dx+4\,c)}+10\,be^{(2\,dx+2\,c)}+3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4\,c)}+24\,ae^{(2\,dx+2\,c)}+33\,be^{(2\,dx+2\,c)}-8\,a-11\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*log(e^(2*d*x + 2*c) + 1) - 6*b*log(abs(e^(2*d*x + 2*c) - 1)) - 3*
(3*b*e^(4*d*x + 4*c) + 10*b*e^(2*d*x + 2*c) + 3*b)/(e^(2*d*x + 2*c) + 1)^2
+ (11*b*e^(6*d*x + 6*c) - 33*b*e^(4*d*x + 4*c) + 24*a*e^(2*d*x + 2*c) + 33*
b*e^(2*d*x + 2*c) - 8*a - 11*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{4\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{8\,a}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

```
[Out] (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1)) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*
a)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) -
 (2*atan((b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))
/(-d^2)^(1/2)
```

3.57 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\begin{aligned}\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{3}{8}\left(a^2+21b^2\right)x+\frac{6ab\log(\cosh(c+dx))}{d}-\frac{6b^2\tanh(c+dx)}{d}\\&\quad-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{d}-\frac{b^2\tanh^5(c+dx)}{5d}\\&\quad+\frac{\cosh^3(c+dx)\sinh(c+dx)\left(a^2+b^2+2ab\tanh(c+dx)\right)}{4d}\\&\quad-\frac{\cosh(c+dx)\sinh(c+dx)\left(5a^2+17b^2+20ab\tanh(c+dx)\right)}{8d}\end{aligned}$$

```
[Out] 3/8*(a^2+21*b^2)*x+6*a*b*ln(cosh(d*x+c))/d-6*b^2*tanh(d*x+c)/d-a*b*tanh(d*x
+c)^2/d-b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d+1/4*cosh(d*x+c)^3*sinh(
d*x+c)*(a^2+b^2+2*a*b*tanh(d*x+c))/d-1/8*cosh(d*x+c)*sinh(d*x+c)*(5*a^2+17*
b^2+20*a*b*tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3744, 1818, 1816, 647, 31\}$

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= -\frac{3(a^2+21b^2)\tanh(c+dx)}{8d} - \frac{3(a^2+16ab+21b^2)\log(1-\tanh(c+dx))}{16d}$$
$$+ \frac{3(a^2-16ab+21b^2)\log(\tanh(c+dx)+1)}{16d}$$
$$+ \frac{\sinh^4(c+dx)\left((a^2+b^2)\tanh(c+dx)+2ab\right)}{4d}$$
$$- \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+16ab\tanh(c+dx)+13b^2\right)}{8d}$$
$$- \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^3(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-3*(a^2 + 16*a*b + 21*b^2)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a^2 - 16*a
*b + 21*b^2)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*(a^2 + 21*b^2)*Tanh[c + d*
x])/(8*d) - (3*a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c + d*x]^3)/d - (b^2*Tanh
[c + d*x]^5)/(5*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a^2 + 13*b^2 + 16*a*b*
Tanh[c + d*x]))/(8*d) + (Sinh[c + d*x]^4*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]
))/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)^2}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\sinh^4(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{4d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x^3(-8ab-(a^2+5b^2)x-8abx^2-4b^2x^3-4b^2x^5)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}\\
&= -\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d}\\
&\quad+\frac{\sinh^4(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{4d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x^2(3(a^2+13b^2)+48abx+16b^2x^2+8b^2x^4)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}\\
&= -\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d}\\
&\quad+\frac{\sinh^4(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{4d}\\
&\quad+\frac{\text{Subst}\left(\int \left(-3(a^2+21b^2)-48abx-24b^2x^2-8b^2x^4+\frac{3(a^2+21b^2+16abx)}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{8d}\\
&= -\frac{3(a^2+21b^2)\tanh(c+dx)}{8d}-\frac{3ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{d}\\
&\quad-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d}\\
&\quad+\frac{\sinh^4(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{4d}\\
&\quad+\frac{3\text{Subst}\left(\int \frac{a^2+21b^2+16abx}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}
\end{aligned}$$

$$= -\frac{3(a^2+21b^2)\tanh(c+dx)}{8d} - \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{d} - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d} + \frac{\sinh^4(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{4d} - \frac{(3(a^2-16ab+21b^2))\operatorname{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{16d} + \frac{(3(a^2+16ab+21b^2))\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{16d}$$

$$= -\frac{3(a^2+16ab+21b^2)\log(1-\tanh(c+dx))}{16d} + \frac{3(a^2-16ab+21b^2)\log(1+\tanh(c+dx))}{16d} - \frac{3(a^2+21b^2)\tanh(c+dx)}{8d} - \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{d} - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)}{8d} + \frac{\sinh^4(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{4d}$$

Mathematica [A] (verified)

Time = 7.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{60(a^2+21b^2)(c+dx) - 200ab\cosh(2(c+dx)) + 10ab\cosh(4(c+dx)) + 960ab\log(\cosh(c+dx)) + 160ab}{}$$

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (60*(a^2 + 21*b^2)*(c + d*x) - 200*a*b*Cosh[2*(c + d*x)] + 10*a*b*Cosh[4*(c
 + d*x)] + 960*a*b*Log[Cosh[c + d*x]] + 160*a*b*Sech[c + d*x]^2 - 40*(a^2 +
 4*b^2)*Sinh[2*(c + d*x)] + 5*(a^2 + b^2)*Sinh[4*(c + d*x)] - 1152*b^2*Tanh
[c + d*x] + 224*b^2*Sech[c + d*x]^2*Tanh[c + d*x] - 32*b^2*Sech[c + d*x]^4*
Tanh[c + d*x])/(160*d)
```

Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh}{}\right.}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh}{}\right.}{d}$
risch	$\frac{3a^2x}{8}-6abx+\frac{63b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}a^2}{8d}-\frac{5\,e^{2dx+2c}ab}{8d}-\frac{e^{2dx+2c}b^2}{2d}$

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln(c
osh(d*x+c))-3/2*tanh(d*x+c)^2)+b^2*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*sin
h(d*x+c)^7/cosh(d*x+c)^5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x+c)^
3-63/40*tanh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5034 vs. 2(162) = 324.

Time = 0.31 (sec) , antiderivative size = 5034, normalized size of antiderivative = 29.61

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2\sinh^4(c+dx)\,dx$$

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(162) = 324.

Time = 0.28 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.23

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{1}{320}b^2\left(\frac{2520(dx+c)}{d}+\frac{5\left(32e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{135e^{(-2dx-2c)}+5358e^{(-4dx-4c)}+18190e^{(-6c}}{d(e^{(-4dx-4c)}+5e^{(-6dx-6c)}+10e^{(}}\right.$$
$$+\frac{1}{32}ab\left(\frac{192(dx+c)}{d}-\frac{20e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}+\frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{18e^{(-2dx-2c)}+39e^{(-4dx}}{d(e^{(-4dx-4c)}+2e^{(-}}\right.$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c
) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10
*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c)))) + 1/32*a*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^
(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c)
+ 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(162) = 324.

Time = 0.47 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.20

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{5a^2e^{(4dx+4c)}+10abe^{(4dx+4c)}+5b^2e^{(4dx+4c)}-40a^2e^{(2dx+2c)}-200abe^{(2dx+2c)}-160b^2e^{(2dx+2c)}+1920ab}{}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/320*(5*a^2*e^(4*d*x + 4*c) + 10*a*b*e^(4*d*x + 4*c) + 5*b^2*e^(4*d*x + 4*
c) - 40*a^2*e^(2*d*x + 2*c) - 200*a*b*e^(2*d*x + 2*c) - 160*b^2*e^(2*d*x +
2*c) + 1920*a*b*log(e^(2*d*x + 2*c) + 1) + 120*(a^2 - 16*a*b + 21*b^2)*(d*x
```

```
 + c) - 5*(18*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 378*b^2*e^(4*
d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 40*a*b*e^(2*d*x + 2*c) - 32*b^2*e^(2*d
*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c) - 32*(137*a*b*e^(10*d*x + 1
0*c) + 645*a*b*e^(8*d*x + 8*c) - 200*b^2*e^(8*d*x + 8*c) + 1250*a*b*e^(6*d*
x + 6*c) - 600*b^2*e^(6*d*x + 6*c) + 1250*a*b*e^(4*d*x + 4*c) - 840*b^2*e^(
4*d*x + 4*c) + 645*a*b*e^(2*d*x + 2*c) - 520*b^2*e^(2*d*x + 2*c) + 137*a*b
- 144*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.11

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = x\left(\frac{3a^2}{8}-6ab+\frac{63b^2}{8}\right)+\frac{4(5b^2+ab)}{d\,(e^{2c+2dx}+1)}+\frac{e^{-2c-2dx}(a^2-5ab+4b^2)}{8d}-\frac{e^{2c+2dx}(a^2+5ab+4b^2)}{8d}+\frac{e^{4c+4dx}(a+b)^2}{64d}-\frac{4(5b^2+ab)}{d\,(2e^{2c+2dx}+e^{4c+4dx}+1)}+\frac{24b^2}{d\,(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}-\frac{e^{-4c-4dx}(a-b)^2}{64d}-\frac{16b^2}{d\,(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1)}+\frac{32b^2}{5d\,(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1)}+\frac{6ab\ln\left(e^{2c}e^{2dx}+1\right)}{d}$$

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] x*((3*a^2)/8 - 6*a*b + (63*b^2)/8) + (4*(a*b + 5*b^2))/(d*(exp(2*c + 2*d*x)
 + 1)) + (exp(- 2*c - 2*d*x)*(a^2 - 5*a*b + 4*b^2))/(8*d) - (exp(2*c + 2*d*
x)*(5*a*b + a^2 + 4*b^2))/(8*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d) - (4*
(a*b + 5*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (24*b^2)/(
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(
- 4*c - 4*d*x)*(a - b)^2)/(64*d) - (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (32*b^2)/(5*d*
(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c
 + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*a*b*log(exp(2*c)*exp(2*d*x) + 1))
/d
```

3.58 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{5ab\arctan(\sinh(c+dx))}{d}-\frac{a^2\cosh(c+dx)}{d}\\ &\quad-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}\\ &\quad+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{6b^2\operatorname{sech}(c+dx)}{d}\\ &\quad+\frac{4b^2\operatorname{sech}^3(c+dx)}{3d}-\frac{b^2\operatorname{sech}^5(c+dx)}{5d}\\ &\quad-\frac{5ab\sinh(c+dx)}{d}+\frac{5ab\sinh^3(c+dx)}{3d}\\ &\quad-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}\end{aligned}$$

```
[Out] 5*a*b*arctan(sinh(d*x+c))/d-a^2*cosh(d*x+c)/d-4*b^2*cosh(d*x+c)/d+1/3*a^2*c
osh(d*x+c)^3/d+1/3*b^2*cosh(d*x+c)^3/d-6*b^2*sech(d*x+c)/d+4/3*b^2*sech(d*x
+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-5*a*b*sinh(d*x+c)/d+5/3*a*b*sinh(d*x+c)^3/d
-a*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3747, 2713, 2672, 294, 308, 209, 2670, 276}

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{5ab\arctan(\sinh(c+dx))}{d}+\frac{5ab\sinh^3(c+dx)}{3d}-\frac{5ab\sinh(c+dx)}{d}-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{4b^2\cosh(c+dx)}{d}-\frac{b^2\text{sech}^5(c+dx)}{5d}+\frac{4b^2\text{sech}^3(c+dx)}{3d}-\frac{6b^2\text{sech}(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*Cosh[c + d*x])/d - (4*b^2*Cosh[c + d
*x])/d + (a^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^3)/(3*d) - (6*b^2
*Sech[c + d*x])/d + (4*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(
5*d) - (5*a*b*Sinh[c + d*x])/d + (5*a*b*Sinh[c + d*x]^3)/(3*d) - (a*b*Sinh[
c + d*x]^3*Tanh[c + d*x]^2)/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = i\int \left(-ia^2\sinh^3(c+dx)-2iab\sinh^3(c+dx)\tanh^3(c+dx)-ib^2\sinh^3(c+dx)\tanh^6(c+dx)\right)\,dx$$

$$= a^2\int \sinh^3(c+dx)\,dx+(2ab)\int \sinh^3(c+dx)\tanh^3(c+dx)\,dx+b^2\int \sinh^3(c+dx)\tanh^6(c+dx)\,dx$$

$$= -\frac{a^2\text{Subst}\left(\int (1-x^2)\,dx, x, \cosh(c+dx)\right)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{(1-x^2)^4}{x^6}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} - \frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d} + \frac{(5ab)\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{b^2\text{Subst}\left(\int \left(-4+\frac{1}{x^6}-\frac{4}{x^4}+\frac{6}{x^2}+x^2\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{d} - \frac{4b^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^3(c+dx)}{3d} - \frac{6b^2\text{sech}(c+dx)}{d} + \frac{4b^2\text{sech}^3(c+dx)}{3d} - \frac{b^2\text{sech}^5(c+dx)}{5d} - \frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d} + \frac{(5ab)\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{d} - \frac{4b^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^3(c+dx)}{3d} - \frac{6b^2\text{sech}(c+dx)}{d} + \frac{4b^2\text{sech}^3(c+dx)}{3d} - \frac{b^2\text{sech}^5(c+dx)}{5d} - \frac{5ab\sinh(c+dx)}{d} + \frac{5ab\sinh^3(c+dx)}{3d} - \frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d} + \frac{(5ab)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{5ab\arctan(\sinh(c+dx))}{d} - \frac{a^2\cosh(c+dx)}{d} - \frac{4b^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^3(c+dx)}{3d} - \frac{6b^2\text{sech}(c+dx)}{d} + \frac{4b^2\text{sech}^3(c+dx)}{3d} - \frac{b^2\text{sech}^5(c+dx)}{5d} - \frac{5ab\sinh(c+dx)}{d} + \frac{5ab\sinh^3(c+dx)}{3d} - \frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.66

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{-45(a^2+5b^2)\cosh(c+dx)+5(a^2+b^2)\cosh(3(c+dx))-2b\left(-40b\operatorname{sech}^3(c+dx)+6b\operatorname{sech}^5(c+dx)-5a(\right.}{6}$$

[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] (-45*(a^2 + 5*b^2)*Cosh[c + d*x] + 5*(a^2 + b^2)*Cosh[3*(c + d*x)] - 2*b*(-40*b*Sech[c + d*x]^3 + 6*b*Sech[c + d*x]^5 - 5*a*(60*ArcTan[Tanh[(c + d*x)/2]] - 27*Sinh[c + d*x] + Sinh[3*(c + d*x)]) + 30*Sech[c + d*x]*(6*b + a*Tanh[c + d*x])))/(60*d)

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\mathrm{e}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\mathrm{e}\right.}{d}$
risch	$\frac{\mathrm{e}^{3dx+3c}a^2}{24d}+\frac{\mathrm{e}^{3dx+3c}ab}{12d}+\frac{\mathrm{e}^{3dx+3c}b^2}{24d}-\frac{3\,\mathrm{e}^{dx+c}a^2}{8d}-\frac{9\,\mathrm{e}^{dx+c}ab}{4d}-\frac{15b^2\mathrm{e}^{dx+c}}{8d}-\frac{3\,\mathrm{e}^{-dx-c}a^2}{8d}+\frac{9\,\mathrm{e}^{-dx-c}ab}{4d}-$

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c)))+b^2*(1/3*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-64/3*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. $2(172) = 344$.

Time = 0.29 (sec) , antiderivative size = 3341, normalized size of antiderivative = 18.36

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
[Out] 1/120*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^16 + 80*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^15 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^16 - 20*(a
^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^14 + 20*(30*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^2 - a^2 - 11*a*b - 10*b^2)*sinh(d*x + c)^14 + 280*(10*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^3 - (a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^13 - 20*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^12 + 20*(455*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^4 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 - 11
*a^2 - 61*a*b - 137*b^2)*sinh(d*x + c)^12 + 80*(273*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^5 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 + 61*
a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 20*(31*a^2 + 87*a*b + 390*
b^2)*cosh(d*x + c)^10 + 20*(2002*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 1001
*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 - 66*(11*a^2 + 61*a*b + 137*b^2)*c
osh(d*x + c)^2 - 31*a^2 - 87*a*b - 390*b^2)*sinh(d*x + c)^10 + 40*(1430*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x +
c)^5 - 110*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^3 - 5*(31*a^2 + 87*a*b
 + 390*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(425*a^2 + 5649*b^2)*cosh(d*
x + c)^8 + 2*(32175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 30030*(a^2 + 11*a
*b + 10*b^2)*cosh(d*x + c)^6 - 4950*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x +
c)^4 - 450*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^2 - 425*a^2 - 5649*b^2
)*sinh(d*x + c)^8 + 16*(3575*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 4290*(a^
2 + 11*a*b + 10*b^2)*cosh(d*x + c)^7 - 990*(11*a^2 + 61*a*b + 137*b^2)*cosh
(d*x + c)^5 - 150*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^3 - (425*a^2 +
5649*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(31*a^2 - 87*a*b + 390*b^2)*c
osh(d*x + c)^6 + 4*(10010*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 15015*(a^2
 + 11*a*b + 10*b^2)*cosh(d*x + c)^8 - 4620*(11*a^2 + 61*a*b + 137*b^2)*cosh
(d*x + c)^6 - 1050*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^4 - 14*(425*a^
2 + 5649*b^2)*cosh(d*x + c)^2 - 155*a^2 + 435*a*b - 1950*b^2)*sinh(d*x + c)
^6 + 8*(2730*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 - 5005*(a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^9 - 1980*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^7 -
630*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^5 - 14*(425*a^2 + 5649*b^2)*c
osh(d*x + c)^3 - 15*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 - 20*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^4 + 20*(455*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^12 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^10 -
495*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^8 - 210*(31*a^2 + 87*a*b + 39
0*b^2)*cosh(d*x + c)^6 - 7*(425*a^2 + 5649*b^2)*cosh(d*x + c)^4 - 15*(31*a^
```

2 - 87*a*b + 390*b^2)*cosh(d*x + c)^2 - 11*a^2 + 61*a*b - 137*b^2)*sinh(d*x + c)^4 + 16*(175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 455*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^11 - 275*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^9 - 150*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^7 - 7*(425*a^2 + 5649*b^2)*cosh(d*x + c)^5 - 25*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c)^3 - 5*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 20*(a^2 - 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 4*(150*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 - 455*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^12 - 330*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^10 - 225*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^8 - 14*(425*a^2 + 5649*b^2)*cosh(d*x + c)^6 - 75*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c)^4 - 30*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^2 - 5*a^2 + 55*a*b - 50*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 + 1200*(a*b*cosh(d*x + c)^13 + 13*a*b*cosh(d*x + c)*sinh(d*x + c)^12 + a*b*sinh(d*x + c)^13 + 5*a*b*cosh(d*x + c)^11 + (78*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^11 + 10*a*b*cosh(d*x + c)^9 + 11*(26*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^10 + 5*(143*a*b*cosh(d*x + c)^4 + 55*a*b*cosh(d*x + c)^2 + 2*a*b)*sinh(d*x + c)^9 + 10*a*b*cosh(d*x + c)^7 + 3*(429*a*b*cosh(d*x + c)^5 + 275*a*b*cosh(d*x + c)^3 + 30*a*b*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(858*a*b*cosh(d*x + c)^6 + 825*a*b*cosh(d*x + c)^4 + 180*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^7 + 5*a*b*cosh(d*x + c)^5 + 2*(858*a*b*cosh(d*x + c)^7 + 1155*a*b*cosh(d*x + c)^5 + 420*a*b*cosh(d*x + c)^3 + 35*a*b*cosh(d*x + c))*sinh(d*x + c)^6 + (1287*a*b*cosh(d*x + c)^8 + 2310*a*b*cosh(d*x + c)^6 + 1260*a*b*cosh(d*x + c)^4 + 210*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^5 + a*b*cosh(d*x + c)^3 + 5*(143*a*b*cosh(d*x + c)^9 + 330*a*b*cosh(d*x + c)^7 + 252*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^4 + (286*a*b*cosh(d*x + c)^10 + 825*a*b*cosh(d*x + c)^8 + 840*a*b*cosh(d*x + c)^6 + 350*a*b*cosh(d*x + c)^4 + 50*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^3 + (78*a*b*cosh(d*x + c)^11 + 275*a*b*cosh(d*x + c)^9 + 360*a*b*cosh(d*x + c)^7 + 210*a*b*cosh(d*x + c)^5 + 50*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + (13*a*b*cosh(d*x + c)^12 + 55*a*b*cosh(d*x + c)^10 + 90*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x + c)^6 + 25*a*b*cosh(d*x + c)^4 + 3*a*b*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 8*(10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^15 - 35*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^13 - 30*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^11 - 25*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^9 - 2*(425*a^2 + 5649*b^2)*cosh(d*x + c)^7 - 15*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c)^5 - 10*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^3 - 5*(a^2 - 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^13 + 13*d*cosh(d*x + c)*sinh(d*x + c)^12 + d*sinh(d*x + c)^13 + 5*d*cosh(d*x + c)^11 + (78*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^11 + 11*(26*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^10 + 10*d*cosh(d*x + c)^9 + 5*(143*d*cosh(d*x + c)^4 + 55*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^9 + 3*(429*d*cosh(d*x + c)^5 + 275*d*cosh(d*x + c)^3 + 30*d*cosh(d*x + c))*sinh(d*x + c)^8 + 10*d*cosh(d*x + c)^7 + 2*(858*d*cosh(d*x + c)^6 + 825*d*cosh(d*x + c)^4 + 180*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^7 + 2*(858*d*cosh(d*x + c)^7 + 1155*d*cosh(d*x +

```
c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^6 + 5*d*co
sh(d*x + c)^5 + (1287*d*cosh(d*x + c)^8 + 2310*d*cosh(d*x + c)^6 + 1260*d*c
osh(d*x + c)^4 + 210*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(143*d*co
sh(d*x + c)^9 + 330*d*cosh(d*x + c)^7 + 252*d*cosh(d*x + c)^5 + 70*d*cosh(d
*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (286*d
*cosh(d*x + c)^10 + 825*d*cosh(d*x + c)^8 + 840*d*cosh(d*x + c)^6 + 350*d*c
osh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (78*d*cosh(d*x
 + c)^11 + 275*d*cosh(d*x + c)^9 + 360*d*cosh(d*x + c)^7 + 210*d*cosh(d*x +
 c)^5 + 50*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (13*d*c
osh(d*x + c)^12 + 55*d*cosh(d*x + c)^10 + 90*d*cosh(d*x + c)^8 + 70*d*cosh(
d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)^2\sinh^3\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(172) = 344$.

Time = 0.29 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.91

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{1}{120}b^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx\cdots}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9\cdots}}\right.$$
$$+\frac{1}{12}ab\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\cdots\right.$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
[Out] -1/120*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c
) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) +
 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c
) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-
11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/12*a*b*((27*e^(-d*x - c) - e^(-3
```

```
*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(
-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x
- 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/
d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.59

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{1200\,ab\arctan\left(e^{(dx+c)}\right)+5\,a^2e^{(3\,dx+3\,c)}+10\,abe^{(3\,dx+3\,c)}+5\,b^2e^{(3\,dx+3\,c)}-45\,a^2e^{(dx+c)}-270\,abe^{(dx+c)}-225}{}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/120*(1200*a*b*arctan(e^(d*x + c)) + 5*a^2*e^(3*d*x + 3*c) + 10*a*b*e^(3*d
*x + 3*c) + 5*b^2*e^(3*d*x + 3*c) - 45*a^2*e^(d*x + c) - 270*a*b*e^(d*x + c
) - 225*b^2*e^(d*x + c) - 5*(9*a^2*e^(2*d*x + 2*c) - 54*a*b*e^(2*d*x + 2*c)
 + 45*b^2*e^(2*d*x + 2*c) - a^2 + 2*a*b - b^2)*e^(-3*d*x - 3*c) - 16*(15*a*
b*e^(9*d*x + 9*c) + 90*b^2*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) + 280*b
^2*e^(7*d*x + 7*c) + 428*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) + 280
*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 90*b^2*e^(d*x + c))/(e^(2*d*x +
 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.18

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{\mathrm{e}^{3\,c+3\,d\,x}\,{(a+b)}^2}{24\,d}-\frac{\mathrm{e}^{c+d\,x}\,\left(3\,a^2+18\,a\,b+15\,b^2\right)}{8\,d}\\ &\quad-\frac{\mathrm{e}^{-c-d\,x}\,\left(3\,a^2-18\,a\,b+15\,b^2\right)}{8\,d}+\frac{\mathrm{e}^{-3\,c-3\,d\,x}\,{(a-b)}^2}{24\,d}\\ &\quad+\frac{10\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}-\frac{256\,b^2\,\mathrm{e}^{c+d\,x}}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}\\ &\quad+\frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}\\ &\quad-\frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}\\ &\quad-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(6\,b^2+a\,b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{4\,\mathrm{e}^{c+d\,x}\,\left(8\,b^2+3\,a\,b\right)}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}\end{aligned}$$

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(18*a*b + 3*a^2 + 15*b^
2))/(8*d) - (exp(- c - d*x)*(3*a^2 - 18*a*b + 15*b^2))/(8*d) + (exp(- 3*c -
 3*d*x)*(a - b)^2)/(24*d) + (10*atan((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(
a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (256*b^2*exp(c + d*x))/(15*
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (64*b
^2*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2*exp(c + d*x))/(5*d*(5*exp(2*c +
 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) + 1)) - (2*exp(c + d*x)*(a*b + 6*b^2))/(d*(exp(2*c + 2*d
*x) + 1)) + (4*exp(c + d*x)*(3*a*b + 8*b^2))/(3*d*(2*exp(2*c + 2*d*x) + exp
(4*c + 4*d*x) + 1))
```

3.59 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= -\frac{1}{2}\left(a^2+7b^2\right)x-\frac{4ab\log(\cosh(c+dx))}{d}+\frac{3b^2\tanh(c+dx)}{d}\\&\quad+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}\\&\quad+\frac{\cosh(c+dx)\sinh(c+dx)\left(a^2+b^2+2ab\tanh(c+dx)\right)}{2d}\end{aligned}$$

```
[Out] -1/2*(a^2+7*b^2)*x-4*a*b*ln(cosh(d*x+c))/d+3*b^2*tanh(d*x+c)/d+a*b*tanh(d*x
+c)^2/d+2/3*b^2*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d+1/2*cosh(d*x+c)*sin
h(d*x+c)*(a^2+b^2+2*a*b*tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 1818, 1816, 647, 31}

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{(a^2+7b^2)\tanh(c+dx)}{2d} + \frac{\sinh^2(c+dx)\left((a^2+b^2)\tanh(c+dx)+2ab\right)}{2d} + \frac{ab\tanh^2(c+dx)}{d} + \frac{(a+b)(a+7b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-7b)(a-b)\log(\tanh(c+dx)+1)}{4d} + \frac{b^2\tanh^5(c+dx)}{5d} + \frac{2b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] ((a + b)*(a + 7*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 7*b)*(a - b)*Log[1
 + Tanh[c + d*x]])/(4*d) + ((a^2 + 7*b^2)*Tanh[c + d*x])/(2*d) + (a*b*Tanh[
c + d*x]^2)/d + (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^5)/(5*d)
 + (Sinh[c + d*x]^2*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\sinh^2(c+dx)\,(2ab+(a^2+b^2)\tanh(c+dx))}{2d} \\
&\quad+ \frac{\text{Subst}\left(\int \frac{x(-4ab-(a^2+3b^2)x-4abx^2-2b^2x^3-2b^2x^5)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{\sinh^2(c+dx)\,(2ab+(a^2+b^2)\tanh(c+dx))}{2d} \\
&\quad+ \frac{\text{Subst}\left(\int \left(a^2+7b^2+4abx+4b^2x^2+2b^2x^4-\frac{a^2+7b^2+8abx}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{(a^2+7b^2)\tanh(c+dx)}{2d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d} \\
&\quad+ \frac{\sinh^2(c+dx)\,(2ab+(a^2+b^2)\tanh(c+dx))}{2d}-\frac{\text{Subst}\left(\int \frac{a^2+7b^2+8abx}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{(a^2+7b^2)\tanh(c+dx)}{2d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx)}{3d} \\
&\quad+ \frac{b^2\tanh^5(c+dx)}{5d}+\frac{\sinh^2(c+dx)\,(2ab+(a^2+b^2)\tanh(c+dx))}{2d} \\
&\quad+ \frac{((a-7b)(a-b))\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{4d} \\
&\quad- \frac{((a+b)(a+7b))\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{4d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+b)(a+7b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-7b)(a-b)\log(1+\tanh(c+dx))}{4d} \\
&+ \frac{(a^2+7b^2)\tanh(c+dx)}{2d} + \frac{ab\tanh^2(c+dx)}{d} + \frac{2b^2\tanh^3(c+dx)}{3d} \\
&+ \frac{b^2\tanh^5(c+dx)}{5d} + \frac{\sinh^2(c+dx)\,(2ab+(a^2+b^2)\tanh(c+dx))}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.06

$$
\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx
$$

$$
= \frac{-30a^2c - 210b^2c - 30a^2dx - 210b^2dx + 30ab\cosh(2(c+dx)) - 240ab\log(\cosh(c+dx)) + 15a^2\sinh(2(c
$$

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-30*a^2*c - 210*b^2*c - 30*a^2*d*x - 210*b^2*d*x + 30*a*b*Cosh[2*(c + d*x)
] - 240*a*b*Log[Cosh[c + d*x]] + 15*a^2*Sinh[2*(c + d*x)] + 15*b^2*Sinh[2*(
c + d*x)] + 232*b^2*Tanh[c + d*x] + 12*b^2*Sech[c + d*x]^4*Tanh[c + d*x] -
4*b*Sech[c + d*x]^2*(15*a + 16*b*Tanh[c + d*x]))/(60*d)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+b^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}-\frac{7c}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+b^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}-\frac{7c}{2}\right.}{d}$
risch	$-\frac{a^2x}{2}+4abx-\frac{7b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}b}{8d}$

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c)
^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+b^2*(1/2*sinh(d*x+c)^7/co
sh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*tanh(d*x+c
)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3649 vs. 2(121) = 242.

Time = 0.28 (sec) , antiderivative size = 3649, normalized size of antiderivative = 28.29

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
[Out] 1/120*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 + 210*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)*sinh(d*x + c)^13 + 15*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^14 - 15
*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^12 -
15*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 91*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 -
5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^12 + 60*(91*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^3 - 3*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(
d*x + c))*sinh(d*x + c)^11 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*
a*b + 87*b^2)*cosh(d*x + c)^10 + 15*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)
^4 - 20*(a^2 - 8*a*b + 7*b^2)*d*x - 66*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2
 - 10*a*b - 5*b^2)*cosh(d*x + c)^2 + 9*a^2 - 10*a*b - 87*b^2)*sinh(d*x + c)
^10 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 110*(4*(a^2 - 8*a*b +
7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^3 - 5*(20*(a^2 - 8*a*b +
 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 15*
(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^8 +
 15*(3003*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 495*(4*(a^2 - 8*a*b + 7*b^2
)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^4 - 40*(a^2 - 8*a*b + 7*b^2)*
d*x - 45*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x
+ c)^2 + 5*a^2 - 66*a*b - 251*b^2)*sinh(d*x + c)^8 + 120*(429*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^7 - 99*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b -
 5*b^2)*cosh(d*x + c)^5 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b
 + 87*b^2)*cosh(d*x + c)^3 - (40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b
 + 251*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d
*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^6 + 5*(9009*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^8 - 2772*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b -
 5*b^2)*cosh(d*x + c)^6 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*
b + 87*b^2)*cosh(d*x + c)^4 - 120*(a^2 - 8*a*b + 7*b^2)*d*x - 84*(40*(a^2 -
 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^2 - 15*a^2 -
198*a*b - 1103*b^2)*sinh(d*x + c)^6 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^9 - 396*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d
*x + c)^7 - 126*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*co
sh(d*x + c)^5 - 28*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2
)*cosh(d*x + c)^3 - (120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110
3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 5*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 2
7*a^2 + 30*a*b + 667*b^2)*cosh(d*x + c)^4 + 5*(3003*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^10 - 1485*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)
```

*cosh(d*x + c)^8 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*
b^2)*cosh(d*x + c)^6 - 210*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b +
251*b^2)*cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 15*(120*(a^2 - 8
*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^2 - 27*a^2 -
30*a*b - 667*b^2)*sinh(d*x + c)^4 + 20*(273*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^11 - 165*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*
x + c)^9 - 90*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh
(d*x + c)^7 - 42*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*
cosh(d*x + c)^5 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110
3*b^2)*cosh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b +
667*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 7
5*a^2 - 150*a*b + 1003*b^2)*cosh(d*x + c)^2 + (1365*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^12 - 990*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*
cosh(d*x + c)^10 - 675*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*
b^2)*cosh(d*x + c)^8 - 420*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b +
251*b^2)*cosh(d*x + c)^6 - 75*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 19
8*a*b + 1103*b^2)*cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 30*(60*(
a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x + c)^2 - 75*
a^2 + 150*a*b - 1003*b^2)*sinh(d*x + c)^2 - 15*a^2 + 30*a*b - 15*b^2 - 480*
(a*b*cosh(d*x + c)^12 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^11 + a*b*sinh(d*
x + c)^12 + 5*a*b*cosh(d*x + c)^10 + (66*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(
d*x + c)^10 + 10*a*b*cosh(d*x + c)^8 + 10*(22*a*b*cosh(d*x + c)^3 + 5*a*b*c
osh(d*x + c))*sinh(d*x + c)^9 + 5*(99*a*b*cosh(d*x + c)^4 + 45*a*b*cosh(d*x
+ c)^2 + 2*a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 8*(99*a*b*cosh(
d*x + c)^5 + 75*a*b*cosh(d*x + c)^3 + 10*a*b*cosh(d*x + c))*sinh(d*x + c)^7
+ 2*(462*a*b*cosh(d*x + c)^6 + 525*a*b*cosh(d*x + c)^4 + 140*a*b*cosh(d*x
+ c)^2 + 5*a*b)*sinh(d*x + c)^6 + 5*a*b*cosh(d*x + c)^4 + 4*(198*a*b*cosh(d
*x + c)^7 + 315*a*b*cosh(d*x + c)^5 + 140*a*b*cosh(d*x + c)^3 + 15*a*b*cosh
(d*x + c))*sinh(d*x + c)^5 + 5*(99*a*b*cosh(d*x + c)^8 + 210*a*b*cosh(d*x +
c)^6 + 140*a*b*cosh(d*x + c)^4 + 30*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c)^4 + a*b*cosh(d*x + c)^2 + 20*(11*a*b*cosh(d*x + c)^9 + 30*a*b*cosh(d*x +
c)^7 + 28*a*b*cosh(d*x + c)^5 + 10*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c)
)*sinh(d*x + c)^3 + (66*a*b*cosh(d*x + c)^10 + 225*a*b*cosh(d*x + c)^8 + 28
0*a*b*cosh(d*x + c)^6 + 150*a*b*cosh(d*x + c)^4 + 30*a*b*cosh(d*x + c)^2 +
a*b)*sinh(d*x + c)^2 + 2*(6*a*b*cosh(d*x + c)^11 + 25*a*b*cosh(d*x + c)^9 +
40*a*b*cosh(d*x + c)^7 + 30*a*b*cosh(d*x + c)^5 + 10*a*b*cosh(d*x + c)^3 +
a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 90*(4*(a^2 - 8
*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^11 - 75*(20*(a^2
- 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^9 - 60*(40*(a
^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^7 - 15*(1
20*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^5
- 10*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x +
c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 75*a^2 - 150*a*b + 1003*b^2)*cosh(d
*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x +

```
c)^11 + d*sinh(d*x + c)^12 + 5*d*cosh(d*x + c)^10 + (66*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c)^10 + 10*(22*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sin
h(d*x + c)^9 + 10*d*cosh(d*x + c)^8 + 5*(99*d*cosh(d*x + c)^4 + 45*d*cosh(d
*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(99*d*cosh(d*x + c)^5 + 75*d*cosh(d*x
+ c)^3 + 10*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 2*(46
2*d*cosh(d*x + c)^6 + 525*d*cosh(d*x + c)^4 + 140*d*cosh(d*x + c)^2 + 5*d)*
sinh(d*x + c)^6 + 4*(198*d*cosh(d*x + c)^7 + 315*d*cosh(d*x + c)^5 + 140*d*
cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 5*d*cosh(d*x + c)^4
 + 5*(99*d*cosh(d*x + c)^8 + 210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4
+ 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 30
*d*cosh(d*x + c)^7 + 28*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (66*d*cosh(d*x + c)^10 + 225
*d*cosh(d*x + c)^8 + 280*d*cosh(d*x + c)^6 + 150*d*cosh(d*x + c)^4 + 30*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(6*d*cosh(d*x + c)^11 + 25*d*cosh(d
*x + c)^9 + 40*d*cosh(d*x + c)^7 + 30*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int\left(a+b\tanh^3\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(121) = 242$.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.33

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{120}b^2\left(\frac{420(dx+c)}{d}+\frac{15e^{(-2dx-2c)}}{d}-\frac{1003e^{(-2dx-2c)}+3350e^{(-4dx-4c)}+5590e^{(-6dx-6c)}+3915e^{(-8c}}{d(e^{(-2dx-2c)}+5e^{(-4dx-4c)}+10e^{(-6dx-6c)}+10e^{(-8dx-8c)}+5}\right.-\frac{1}{4}ab\left(\frac{16(dx+c)}{d}-\frac{e^{(-2dx-2c)}}{d}+\frac{16\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2e^{(-2dx-2c)}-15e^{(-4dx-4c)}+1}{d(e^{(-2dx-2c)}+2e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^2*(420*(d
*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*x
```

```
 - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x -
 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1/
4*a*b*(16*(d*x + c)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d
 - (2*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*
e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(121) = 242$.

Time = 0.42 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.29

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{15\,a^2e^{(2\,dx+2\,c)}+30\,abe^{(2\,dx+2\,c)}+15\,b^2e^{(2\,dx+2\,c)}-480\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-60\,(a^2-8\,ab+7\,b^2)(dx+c}{}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^2*e^(2*d*x + 2*c) + 30*a*b*e^(2*d*x + 2*c) + 15*b^2*e^(2*d*x +
2*c) - 480*a*b*log(e^(2*d*x + 2*c) + 1) - 60*(a^2 - 8*a*b + 7*b^2)*(d*x + c
) + 15*(2*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x +
2*c) - a^2 + 2*a*b - b^2)*e^(-2*d*x - 2*c) + 8*(137*a*b*e^(10*d*x + 10*c) +
 625*a*b*e^(8*d*x + 8*c) - 180*b^2*e^(8*d*x + 8*c) + 1190*a*b*e^(6*d*x + 6*
c) - 480*b^2*e^(6*d*x + 6*c) + 1190*a*b*e^(4*d*x + 4*c) - 680*b^2*e^(4*d*x
+ 4*c) + 625*a*b*e^(2*d*x + 2*c) - 400*b^2*e^(2*d*x + 2*c) + 137*a*b - 116*
b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.37

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{e^{2\,c+2\,d\,x}\,(a+b)^2}{8\,d}-\frac{4\,(3\,b^2+a\,b)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-x\left(\frac{a^2}{2}-4\,a\,b+\frac{7\,b^2}{2}\right)\\&\quad+\frac{4\,(4\,b^2+a\,b)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{64\,b^2}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}\\&\quad-\frac{e^{-2\,c-2\,d\,x}\,(a-b)^2}{8\,d}+\frac{16\,b^2}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}\\&\quad-\frac{32\,b^2}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}\\&\quad-\frac{4\,a\,b\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+1\right)}{d}\end{aligned}$$

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (4*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x)
 + 1)) - x*(a^2/2 - 4*a*b + (7*b^2)/2) + (4*(a*b + 4*b^2))/(d*(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a - b)^2)/(8*
d) + (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4
*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
 + 1)) - (4*a*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.60 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{3ab\arctan(\sinh(c+dx))}{d} + \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{3b^2\operatorname{sech}(c+dx)}{d} - \frac{b^2\operatorname{sech}^3(c+dx)}{d} + \frac{b^2\operatorname{sech}^5(c+dx)}{5d} + \frac{3ab\sinh(c+dx)}{d} - \frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}$$

```
[Out] -3*a*b*arctan(sinh(d*x+c))/d+a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+3*b^2*sech
(d*x+c)/d-b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+3*a*b*sinh(d*x+c)/d-a
*b*sinh(d*x+c)*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

= {3747, 2718, 2672, 294, 327, 209, 2670, 276}

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{a^2\cosh(c+dx)}{d} - \frac{3ab\arctan(\sinh(c+dx))}{d} + \frac{3ab\sinh(c+dx)}{d} - \frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\text{sech}^5(c+dx)}{5d} - \frac{b^2\text{sech}^3(c+dx)}{d} + \frac{3b^2\text{sech}(c+dx)}{d}$$

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-3*a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*
x])/d + (3*b^2*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/d + (b^2*Sech[c + d
*x]^5)/(5*d) + (3*a*b*Sinh[c + d*x])/d - (a*b*Sinh[c + d*x]*Tanh[c + d*x]^2
)/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \left(ia^2\sinh(c+dx)+2iab\sinh(c+dx)\tanh^3(c+dx)\right.\right.\\
&\qquad\left.\left.+ib^2\sinh(c+dx)\tanh^6(c+dx)\right)\,dx\right)\\
&= a^2\int \sinh(c+dx)\,dx+(2ab)\int \sinh(c+dx)\tanh^3(c+dx)\,dx\\
&\quad+b^2\int \sinh(c+dx)\tanh^6(c+dx)\,dx\\
&= \frac{a^2\cosh(c+dx)}{d}+\frac{(2ab)\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&\quad-\frac{b^2\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx,x,\cosh(c+dx)\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^2\cosh(c+dx)}{d} - \frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d} \\
&\quad + \frac{(3ab)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&\quad - \frac{b^2\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{3b^2\text{sech}(c+dx)}{d} \\
&\quad - \frac{b^2\text{sech}^3(c+dx)}{d} + \frac{b^2\text{sech}^5(c+dx)}{5d} + \frac{3ab\sinh(c+dx)}{d} \\
&\quad - \frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d} - \frac{(3ab)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{3ab\arctan(\sinh(c+dx))}{d} + \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{3b^2\text{sech}(c+dx)}{d} \\
&\quad - \frac{b^2\text{sech}^3(c+dx)}{d} + \frac{b^2\text{sech}^5(c+dx)}{5d} + \frac{3ab\sinh(c+dx)}{d} - \frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$
\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx
= \frac{5(a^2+b^2)\cosh(c+dx) + b\left(-5b\text{sech}^3(c+dx) + b\text{sech}^5(c+dx) + 10a\left(-3\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sin\right.\right.}{5d}
$$

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*(a^2 + b^2)*Cosh[c + d*x] + b*(-5*b*Sech[c + d*x]^3 + b*Sech[c + d*x]^5
+ 10*a*(-3*ArcTan[Tanh[(c + d*x)/2]] + Sinh[c + d*x]) + 5*Sech[c + d*x]*(3*
b + a*Tanh[c + d*x])))/(5*d)
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(e^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sinh(dx}{\cosh(dx}\right.}{d}$
default	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(e^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sinh(dx}{\cosh(dx}\right.}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{b^2e^{dx+c}}{2d}+\frac{e^{-dx-c}a^2}{2d}-\frac{e^{-dx-c}ab}{d}+\frac{e^{-dx-c}b^2}{2d}+\frac{2b\,e^{dx+c}\left(5a\,e^{8dx+8c}+15b\,e^{8dx+8c}+1\right.}{}$

[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(cosh(d*x+c)*a^2+2*a*b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c)))+b^2*(sinh(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sinh(d*x+c)^2/cosh(d*x+c)^5+16/5/cosh(d*x+c)^5))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2230 vs. $2(121) = 242$.

Time = 0.28 (sec) , antiderivative size = 2230, normalized size of antiderivative = 18.13

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] 1/10*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 60*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 30*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^10 + 30*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^10 + 100*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 5*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^8 + 5*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 270*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2 + 18*a*b + 47*b^2)*sinh(d*x + c)^8 + 40*(99*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 90*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^6 + 4*(1155*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 1575*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^2 + 25*a^2 + 91*b^2)*sinh(d*x + c)^6 + 8*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 945*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^3 + 3*(25*a^2 + 91*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(15*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^4 + 5*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 1260*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^6 + 70*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^4 + 12*(25*a^2 + 91*b^2)*cosh(d*x + c)^2 + 15*a^2 - 18*a*b + 47*b^2)*sinh(d*x + c)^4 + 20*(55*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 180*(a

```
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*
x + c)^5 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^3 + (15*a^2 - 18*a*b + 47*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ 10*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 135*(a^2 + 2*a*b + 3*b^2)*c
osh(d*x + c)^8 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^6 + 6*(25*a^2
+ 91*b^2)*cosh(d*x + c)^4 + 3*(15*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^2 +
3*a^2 - 6*a*b + 9*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 - 60*(a*b*c
osh(d*x + c)^11 + 11*a*b*cosh(d*x + c)*sinh(d*x + c)^10 + a*b*sinh(d*x + c)
^11 + 5*a*b*cosh(d*x + c)^9 + 5*(11*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c
)^9 + 10*a*b*cosh(d*x + c)^7 + 15*(11*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x
+ c))*sinh(d*x + c)^8 + 10*(33*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2
 + a*b)*sinh(d*x + c)^7 + 10*a*b*cosh(d*x + c)^5 + 14*(33*a*b*cosh(d*x + c)
^5 + 30*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(231
*a*b*cosh(d*x + c)^6 + 315*a*b*cosh(d*x + c)^4 + 105*a*b*cosh(d*x + c)^2 +
5*a*b)*sinh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + 10*(33*a*b*cosh(d*x + c)^7
 + 63*a*b*cosh(d*x + c)^5 + 35*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*s
inh(d*x + c)^4 + 5*(33*a*b*cosh(d*x + c)^8 + 84*a*b*cosh(d*x + c)^6 + 70*a*
b*cosh(d*x + c)^4 + 20*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^3 + a*b*cos
h(d*x + c) + 5*(11*a*b*cosh(d*x + c)^9 + 36*a*b*cosh(d*x + c)^7 + 42*a*b*co
sh(d*x + c)^5 + 20*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + (11*a*b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x +
c)^6 + 50*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)
)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 4*(15*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^11 + 75*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^9 + 10*(15*a^2 + 18*a*b
+ 47*b^2)*cosh(d*x + c)^7 + 6*(25*a^2 + 91*b^2)*cosh(d*x + c)^5 + 5*(15*a^2
 - 18*a*b + 47*b^2)*cosh(d*x + c)^3 + 15*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c
))*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10
 + d*sinh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 5*(11*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^9 + 15*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^8 + 10*d*cosh(d*x + c)^7 + 10*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^7 + 14*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 +
 5*d*cosh(d*x + c))*sinh(d*x + c)^6 + 10*d*cosh(d*x + c)^5 + 2*(231*d*cosh(
d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x
+ c)^5 + 10*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x +
c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(33*d*c
osh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^3 + 5*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c
)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + d*cosh(d*x + c) + (11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)^2\sinh{(c+dx)}\,dx$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(121) = 242$.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=ab\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d\left(e^{(-dx-c)}+2e^{(-3dx-3c)}+e^{(-5dx-5c)}\right)}\right)$$
$$+\frac{1}{10}b^2\left(\frac{5e^{(-dx-c)}}{d}+\frac{85e^{(-2dx-2c)}+210e^{(-4dx-4c)}+314e^{(-6dx-6c)}+185e^{(-8dx-8c)}+65e^{(-10dx-10c)}+}{d\left(e^{(-dx-c)}+5e^{(-3dx-3c)}+10e^{(-5dx-5c)}+10e^{(-7dx-7c)}+5e^{(-9dx-9c)}+e^{(-11dx-11}\right.}\right.$$
$$+\frac{a^2\cosh{(dx+c)}}{d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c))
)) + 1/10*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*
c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) +
5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d
*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + a^2*cosh(d*x + c)/
d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.65

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=$$
$$-\frac{60\,ab\arctan\left(e^{(dx+c)}\right)-5\,a^2e^{(dx+c)}-10\,abe^{(dx+c)}-5\,b^2e^{(dx+c)}-5\,(a^2-2\,ab+b^2)e^{(-dx-c)}-\frac{4\,(5\,abe^{(9\,dx+}}{}}{10\,d}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/10*(60*a*b*arctan(e^(d*x + c)) - 5*a^2*e^(d*x + c) - 10*a*b*e^(d*x + c)
- 5*b^2*e^(d*x + c) - 5*(a^2 - 2*a*b + b^2)*e^(-d*x - c) - 4*(5*a*b*e^(9*d*
x + 9*c) + 15*b^2*e^(9*d*x + 9*c) + 10*a*b*e^(7*d*x + 7*c) + 40*b^2*e^(7*d*
x + 7*c) + 66*b^2*e^(5*d*x + 5*c) - 10*a*b*e^(3*d*x + 3*c) + 40*b^2*e^(3*d*
x + 3*c) - 5*a*b*e^(d*x + c) + 15*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)
/d
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.75

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{e^{c+dx}(a+b)^2}{2d}+\frac{e^{-c-dx}(a-b)^2}{2d}-\frac{6\,\mathrm{atan}\left(\frac{a\,b\,e^{dx}\,e^{c}\sqrt{d^2}}{d\sqrt{a^2b^2}}\right)\sqrt{a^2b^2}}{\sqrt{d^2}}$$
$$+\frac{72\,b^2\,e^{c+dx}}{5\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)}$$
$$-\frac{64\,b^2\,e^{c+dx}}{5\,d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)}$$
$$+\frac{32\,b^2\,e^{c+dx}}{5\,d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)}$$
$$+\frac{2\,e^{c+dx}\,(3\,b^2+a\,b)}{d\,(e^{2c+2dx}+1)}-\frac{4\,e^{c+dx}\,(2\,b^2+a\,b)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}$$

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] (exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a - b)^2)/(2*d) - (6*atan
((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d
^2)^(1/2) + (72*b^2*exp(c + d*x))/(5*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1)) - (64*b^2*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d
*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3
2*b^2*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (2*exp(c +
d*x)*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (4*exp(c + d*x)*(a*b + 2*b
^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.61 $\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 \, dx = \frac{ab\arctan(\sinh(c+dx))}{d} - \frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b^2\operatorname{sech}(c+dx)}{d} + \frac{2b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{b^2\operatorname{sech}^5(c+dx)}{5d} - \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

```
[Out] a*b*arctan(sinh(d*x+c))/d-a^2*arctanh(cosh(d*x+c))/d-b^2*sech(d*x+c)/d+2/3*
b^2*sech(d*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-a*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3747, 3855, 2691, 2686, 200}

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 \, dx = -\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{ab\arctan(\sinh(c+dx))}{d} - \frac{ab\tanh(c+dx)\operatorname{sech}(c+dx)}{d} - \frac{b^2\operatorname{sech}^5(c+dx)}{5d} + \frac{2b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{b^2\operatorname{sech}(c+dx)}{d}$$

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*ArcTanh[Cosh[c + d*x]])/d - (b^2*Sech[
c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) -
 (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i\int \left(-ia^2\operatorname{csch}(c+dx) - 2iab\operatorname{sech}(c+dx)\tanh^2(c+dx) \right. \\
&\qquad \left. - ib^2\operatorname{sech}(c+dx)\tanh^5(c+dx)\right)\,dx \\
&= a^2\int \operatorname{csch}(c+dx)\,dx + (2ab)\int \operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx \\
&\quad + b^2\int \operatorname{sech}(c+dx)\tanh^5(c+dx)\,dx
\end{aligned}
$$

$$= -\frac{a^2\text{arctanh}(\cosh(c+dx))}{d} - \frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d} + (ab)\int \text{sech}(c+dx)\,dx - \frac{b^2\text{Subst}\left(\int (-1+x^2)^2\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\arctan(\sinh(c+dx))}{d} - \frac{a^2\text{arctanh}(\cosh(c+dx))}{d} - \frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d} - \frac{b^2\text{Subst}\left(\int (1-2x^2+x^4)\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\arctan(\sinh(c+dx))}{d} - \frac{a^2\text{arctanh}(\cosh(c+dx))}{d} - \frac{b^2\text{sech}(c+dx)}{d} + \frac{2b^2\text{sech}^3(c+dx)}{3d} - \frac{b^2\text{sech}^5(c+dx)}{5d} - \frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int \text{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{2ab\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b^2\text{sech}(c+dx)}{d} + \frac{2b^2\text{sech}^3(c+dx)}{3d} - \frac{b^2\text{sech}^5(c+dx)}{5d} - \frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}$$

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Log[Cosh[(c + d*x)/2]])/d + (a^2
*Log[Sinh[(c + d*x)/2]])/d - (b^2*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]^3
)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}(\mathrm{e}^{dx+c})+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(\mathrm{e}^{dx+c})\right)+b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{1}{}\right.}{d}$
default	$\frac{-2a^2 \operatorname{arctanh}(\mathrm{e}^{dx+c})+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(\mathrm{e}^{dx+c})\right)+b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{1}{}\right.}{d}$
risch	$-\frac{2b\,\mathrm{e}^{dx+c}(15a\,\mathrm{e}^{8dx+8c}+15b\,\mathrm{e}^{8dx+8c}+30a\,\mathrm{e}^{6dx+6c}+20b\,\mathrm{e}^{6dx+6c}+58b\,\mathrm{e}^{4dx+4c}-30\,\mathrm{e}^{2dx+2c}a+20b\,\mathrm{e}^{2dx+2c}-15a+15b)}{15d(\mathrm{e}^{2dx+2c}+1)^5}+$

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(
d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/
3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(94) = 188$.

Time = 0.29 (sec) , antiderivative size = 2498, normalized size of antiderivative = 25.49

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*(a*b + b^2)*cosh(d*x + c)^9 + 270*(a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^8 + 30*(a*b + b^2)*sinh(d*x + c)^9 + 20*(3*a*b + 2*b^2)*cosh(d*x +
 c)^7 + 20*(54*(a*b + b^2)*cosh(d*x + c)^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^7
 + 116*b^2*cosh(d*x + c)^5 + 140*(18*(a*b + b^2)*cosh(d*x + c)^3 + (3*a*b +
 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(945*(a*b + b^2)*cosh(d*x + c)^4
 + 105*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 29*b^2)*sinh(d*x + c)^5 + 20*(189*
(a*b + b^2)*cosh(d*x + c)^5 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^3 + 29*b^2*c
osh(d*x + c))*sinh(d*x + c)^4 - 20*(3*a*b - 2*b^2)*cosh(d*x + c)^3 + 20*(12
6*(a*b + b^2)*cosh(d*x + c)^6 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^4 + 58*b^2
*cosh(d*x + c)^2 - 3*a*b + 2*b^2)*sinh(d*x + c)^3 + 20*(54*(a*b + b^2)*cosh
(d*x + c)^7 + 21*(3*a*b + 2*b^2)*cosh(d*x + c)^5 + 58*b^2*cosh(d*x + c)^3 -
 3*(3*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 30*(a*b*cosh(d*x + c)^1
0 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 + 5*a*b*cos
h(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^8 + 10*a*b*cos
h(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)
^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x
```

+ c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x + c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b + 10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 30*(a*b - b^2)*cosh(d*x + c) + 15*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8 + 5*(9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6 + 40*(3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a^2*cosh(d*x + c)^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2*cosh(d*x + c)^4 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2 + 40*(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8 + 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x + c)^7 + 6*a^2*cosh(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 15*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8 + 5*(9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6 + 40*(3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a^2*cosh(d*x + c)^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2*cosh(d*x + c)^4 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2 + 40*(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8 + 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x + c)^7 + 6*a^2*cosh(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(27*(a*b + b^2)*cosh(d*x + c)^8 + 14*(3*a*b + 2*b^2)*cosh(d*x + c)^6 + 58*b^2*cosh(d*x + c)^4 - 6*(3*a*b - 2*b^2)*cosh(d*x + c)^2 - 3*a*b + 3*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +

```
c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6
+ 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*
cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int\left(a+b\tanh^3\left(c+dx\right)\right)^2\operatorname{csch}\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.

Time $= 0.29$ (sec) , antiderivative size $= 447$, normalized size of antiderivative $= 4.56$

$$\begin{aligned}
&\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= -2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) \\
&\quad -\frac{2}{15}\,b^2\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right. \\
&\quad +\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}
\end{aligned}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 2/15*b^2*(15*e^(-d*x - c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-
4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*
e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*log(tanh(1/
2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$= \frac{30\,ab\arctan\left(e^{(dx+c)}\right) - 15\,a^2\log\left(e^{(dx+c)}+1\right) + 15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(15\,abe^{(9\,dx+9\,c)}+15\,b^2e^{(9\,dx+9\,c)}+30\,ab\right.}{}}{15\,d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(30*a*b*arctan(e^(d*x + c)) - 15*a^2*log(e^(d*x + c) + 1) + 15*a^2*log
(abs(e^(d*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c)
 + 30*a*b*e^(7*d*x + 7*c) + 20*b^2*e^(7*d*x + 7*c) + 58*b^2*e^(5*d*x + 5*c)
 - 30*a*b*e^(3*d*x + 3*c) + 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 1
5*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.33

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$= \frac{a^2\,\ln\left(32\,a^6+32\,a^4\,b^2-32\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-32\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d}$$
$$- \frac{176\,b^2\,\mathrm{e}^{c+d\,x}}{15\,\left(d+3\,d\,\mathrm{e}^{2\,c+2\,d\,x}+3\,d\,\mathrm{e}^{4\,c+4\,d\,x}+d\,\mathrm{e}^{6\,c+6\,d\,x}\right)}$$
$$- \frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+10\,d\,\mathrm{e}^{4\,c+4\,d\,x}+10\,d\,\mathrm{e}^{6\,c+6\,d\,x}+5\,d\,\mathrm{e}^{8\,c+8\,d\,x}+d\,\mathrm{e}^{10\,c+10\,d\,x}\right)}$$
$$- \frac{a^2\,\ln\left(-32\,a^6-32\,a^4\,b^2-32\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-32\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d}$$
$$- \frac{2\,b^2\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}} + \frac{16\,b^2\,\mathrm{e}^{c+d\,x}}{3\,\left(d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}\right)}$$
$$+ \frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+4\,d\,\mathrm{e}^{2\,c+2\,d\,x}+6\,d\,\mathrm{e}^{4\,c+4\,d\,x}+4\,d\,\mathrm{e}^{6\,c+6\,d\,x}+d\,\mathrm{e}^{8\,c+8\,d\,x}\right)}$$
$$- \frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}} + \frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}$$
$$- \frac{a\,b\,\left(\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+32\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^5\,b\,32\mathrm{i}-a^3\,b^3\,32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+32\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+a^5\,b\right.\right.}{d}$$

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x),x)
```

```
[Out] (a^2*log(32*a^6 + 32*a^4*b^2 - 32*a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x)
*exp(c)))/d - (176*b^2*exp(c + d*x))/(15*(d + 3*d*exp(2*c + 2*d*x) + 3*d*ex
p(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) - (32*b^2*exp(c + d*x))/(5*(d + 5*d*e
xp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 10*d*exp(6*c + 6*d*x) + 5*d*exp(8
*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2*log(- 32*a^6 - 32*a^4*b^2 - 32*
a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x)*exp(c)))/d - (2*b^2*exp(c + d*x))
/(d + d*exp(2*c + 2*d*x)) + (16*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c + 2*d
*x) + d*exp(4*c + 4*d*x))) + (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c + 2*
d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x))) -
 (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) + (4*a*b*exp(c + d*x))/(d +
2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*exp(d*x)*
exp(c) - a^3*b^3*32i - a^5*b*32i + 32*a^5*b*exp(d*x)*exp(c))*1i - log(a^5*b
*32i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 32*a^5*b*exp(d*x)*exp(c))
*1i))/d
```

3.62 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] -a^2*coth(d*x+c)/d+a*b*tanh(d*x+c)^2/d+1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 276}

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) + (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^5)/
(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^3)^2}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^2}+2abx+b^2x^4\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{a^2\coth(c+dx)}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{5d}-\frac{2b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}+\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) - (a*b*Sech[c + d*x]^2)/d + (b^2*Tanh[c + d*x])/(5
*d) - (2*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) + (b^2*Sech[c + d*x]^4*Ta
nh[c + d*x])/(5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 4.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{-a^{2} \coth(dx+c)-\frac{ab}{\cosh(dx+c)^{2}}+b^{2}\left(-\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)^{5}}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^{5}}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^{4}}{5}+\frac{4\operatorname{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{-a^{2} \coth(dx+c)-\frac{ab}{\cosh(dx+c)^{2}}+b^{2}\left(-\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)^{5}}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^{5}}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^{4}}{5}+\frac{4\operatorname{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
risch	$-\frac{2(5a^{2}e^{10dx+10c}+10ab\,e^{10dx+10c}+5b^{2}e^{10dx+10c}+25a^{2}e^{8dx+8c}+20ab\,e^{8dx+8c}-5b^{2}e^{8dx+8c}+50a^{2}e^{6dx+6c}+10b^{2}e^{6dx+}}{5d\left(e^{2dx+2c}+1\right)^{5}\left(e^{2dx+2}\right.}$

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)-a*b/cosh(d*x+c)^2+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)
^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+
c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(45) = 90$.

Time = 0.25 (sec) , antiderivative size = 518, normalized size of antiderivative = 11.02

$$\int \operatorname{csch}^{2}(c+dx)\left(a+b\tanh^{3}(c+dx)\right)^{2}\,dx =$$

$$-\frac{4\left(\left(5a^{2}+5ab+2b^{2}\right)\cosh\left(dx+c\right)^{5}+5\left(5a^{2}+5ab+2b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+\right.}{5\left(d\cosh\left(dx+c\right)^{7}+7d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+3d\cosh\left(dx+c\right)^{5}+\left(21d\cos\right.\right.}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 5*a*b + 2*b^2)*c
osh(d*x + c)*sinh(d*x + c)^4 + (5*a*b + 3*b^2)*sinh(d*x + c)^5 + (25*a^2 +
5*a*b - 2*b^2)*cosh(d*x + c)^3 + (10*(5*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a
*b - 3*b^2)*sinh(d*x + c)^3 + (10*(5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 +
 3*(25*a^2 + 5*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 - a*
b)*cosh(d*x + c) + (5*(5*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(5*a*b - b^2)*cos
h(d*x + c)^2 + 10*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + (21*
d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 3*d*cos
h(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 5
0*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c)^5 + 10*d*
```

```
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d*x + c) + (7
*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*sin
h(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int\left(a+b\tanh^3(c+dx)\right)^2\operatorname{csch}^2(c+dx)\,dx$$

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(45) = 90$.

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{2}{5}b^2\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}+1}\right.$$
$$+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^
(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) - 4*a*b/(d*(e^(
d*x + c) + e^(-d*x - c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.60

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{2\left(\frac{5a^2}{e^{(2dx+2c)}-1}+\frac{10abe^{(8dx+8c)}+5b^2e^{(8dx+8c)}+30abe^{(6dx+6c)}+30abe^{(4dx+4c)}+10b^2e^{(4dx+4c)}+10abe^{(2dx+2c)}+b^2}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{5d}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^2/(e^(2*d*x + 2*c) - 1) + (10*a*b*e^(8*d*x + 8*c) + 5*b^2*e^(8*d*
x + 8*c) + 30*a*b*e^(6*d*x + 6*c) + 30*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*
x + 4*c) + 10*a*b*e^(2*d*x + 2*c) + b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 483, normalized size of antiderivative = 10.28

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2\,e^{8c+8dx}\,(b^2+2ab)}{5d}-\frac{8\,e^{2c+2dx}\,(b^2+ab)}{5d}-\frac{2\,(2ab-b^2)}{5d}+\frac{8\,e^{6c+6dx}\,(ab-b^2)}{5d}+\frac{12\,b^2\,e^{4c+4dx}}{5d}}{5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{\frac{2b^2}{5d}+\frac{2\,e^{4c+4dx}\,(b^2+2ab)}{5d}+\frac{4\,e^{2c+2dx}\,(ab-b^2)}{5d}}{3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{2\,(ab-b^2)}{5d}+\frac{2\,e^{2c+2dx}\,(b^2+2ab)}{5d}}{2\,e^{2c+2dx}+e^{4c+4dx}+1}$$
$$-\frac{\frac{2\,e^{6c+6dx}\,(b^2+2ab)}{5d}-\frac{2\,(b^2+ab)}{5d}+\frac{6\,e^{4c+4dx}\,(ab-b^2)}{5d}+\frac{6\,b^2\,e^{2c+2dx}}{5d}}{4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{2a^2}{d\,\left(e^{2c+2dx}-1\right)}-\frac{2\,(b^2+2ab)}{5d\,\left(e^{2c+2dx}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

```
[Out] - ((2*exp(8*c + 8*d*x)*(2*a*b + b^2))/(5*d) - (8*exp(2*c + 2*d*x)*(a*b + b^
2))/(5*d) - (2*(2*a*b - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a*b - b^2))/(5*d
) + (12*b^2*exp(4*c + 4*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d
*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) -
((2*b^2)/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) + (4*exp(2*c + 2*
d*x)*(a*b - b^2))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1) - ((2*(a*b - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2)
)/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*exp(6*c + 6*d*x)
*(2*a*b + b^2))/(5*d) - (2*(a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(a*b -
b^2))/(5*d) + (6*b^2*exp(2*c + 2*d*x))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4
*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a^2)/(d*(exp(
2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.63 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{ab\arctan(\sinh(c+dx))}{d}+\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{b^2\operatorname{sech}^5(c+dx)}{5d}+\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

```
[Out] a*b*arctan(sinh(d*x+c))/d+1/2*a^2*arctanh(cosh(d*x+c))/d-1/2*a^2*coth(d*x+c
)*csch(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+a*b*sech(d*
x+c)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {3747, 3853, 3855, 2686, 14}

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{ab\arctan(\sinh(c+dx))}{d}+\frac{ab\tanh(c+dx)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^5(c+dx)}{5d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech
[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\left(i\int\left(ia^2\text{csch}^3(c+dx)+2iab\text{sech}^3(c+dx)+ib^2\text{sech}^3(c+dx)\tanh^3(c+dx)\right)\,dx\right)$$
$$= a^2\int\text{csch}^3(c+dx)\,dx+(2ab)\int\text{sech}^3(c+dx)\,dx+b^2\int\text{sech}^3(c+dx)\tanh^3(c+dx)\,dx$$
$$= -\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}-\frac{1}{2}a^2\int\text{csch}(c+dx)\,dx+(ab)\int\text{sech}(c+dx)\,dx+\frac{b^2\text{Subst}\left(\int x^2(-1+x^2)\,dx,x,\text{sech}(c+dx)\right)}{d}$$
$$= \frac{ab\arctan(\sinh(c+dx))}{d}+\frac{a^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}+\frac{b^2\text{Subst}\left(\int(-x^2+x^4)\,dx,x,\text{sech}(c+dx)\right)}{d}$$
$$= \frac{ab\arctan(\sinh(c+dx))}{d}+\frac{a^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}-\frac{b^2\text{sech}^3(c+dx)}{3d}+\frac{b^2\text{sech}^5(c+dx)}{5d}+\frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

$$\int\text{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{2ab\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b^2\text{sech}^3(c+dx)}{3d}+\frac{b^2\text{sech}^5(c+dx)}{5d}+\frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}$$

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) + (a^
2*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^2*Log[Sinh[(c + d*x)/2]])/(2*d) - (a^2
*Sech[(c + d*x)/2]^2)/(8*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech[c + d
*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 8.68 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15\,\mathrm{co}}\right.}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15\,\mathrm{co}}\right.}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}\left(15a^2\mathrm{e}^{12dx+12c}-30ab\,\mathrm{e}^{12dx+12c}+90a^2\mathrm{e}^{10dx+10c}+40b^2\mathrm{e}^{10dx+10c}+225a^2\mathrm{e}^{8dx+8c}+90ab\,\mathrm{e}^{8dx+8c}-96b^2\mathrm{e}^{8dx+8c}\right.}{15d\left(\mathrm{e}^{2dx+2c}-\right.}$

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/2*sech
(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)
^5-2/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4642 vs. 2(99) = 198.

Time = 0.32 (sec) , antiderivative size = 4642, normalized size of antiderivative = 43.38

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^2 - 2*a*b)*cosh(d*x + c)^13 + 390*(a^2 - 2*a*b)*cosh(d*x + c)*
sinh(d*x + c)^12 + 30*(a^2 - 2*a*b)*sinh(d*x + c)^13 + 20*(9*a^2 + 4*b^2)*c
osh(d*x + c)^11 + 20*(117*(a^2 - 2*a*b)*cosh(d*x + c)^2 + 9*a^2 + 4*b^2)*si
nh(d*x + c)^11 + 220*(39*(a^2 - 2*a*b)*cosh(d*x + c)^3 + (9*a^2 + 4*b^2)*co
sh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^
9 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^4 + 550*(9*a^2 + 4*b^2)*cosh(d*x +
 c)^2 + 225*a^2 + 90*a*b - 96*b^2)*sinh(d*x + c)^9 + 6*(6435*(a^2 - 2*a*b)*
cosh(d*x + c)^5 + 550*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 9*(75*a^2 + 30*a*b
- 32*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^2 + 28*b^2)*cosh(d*x + c
)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^6 + 825*(9*a^2 + 4*b^2)*cosh(d*x
+ c)^4 + 27*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 75*a^2 + 28*b^2)*s
inh(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^7 + 1155*(9*a^2 + 4*b^
2)*cosh(d*x + c)^5 + 63*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 7*(75*
a^2 + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75*a^2 - 30*a*b - 32*b^2)
*cosh(d*x + c)^5 + 6*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^8 + 1540*(9*a^2 + 4*
b^2)*cosh(d*x + c)^6 + 126*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^4 + 28*
(75*a^2 + 28*b^2)*cosh(d*x + c)^2 + 75*a^2 - 30*a*b - 32*b^2)*sinh(d*x + c)
```

^5 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^9 + 3300*(9*a^2 + 4*b^2)*cosh(d*x
 + c)^7 + 378*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^5 + 140*(75*a^2 + 28
*b^2)*cosh(d*x + c)^3 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c))*sinh(d
x + c)^4 + 20(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 4*(2145*(a^2 - 2*a*b)*cosh
(d*x + c)^10 + 825*(9*a^2 + 4*b^2)*cosh(d*x + c)^8 + 126*(75*a^2 + 30*a*b -
 32*b^2)*cosh(d*x + c)^6 + 70*(75*a^2 + 28*b^2)*cosh(d*x + c)^4 + 15*(75*a^
2 - 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 45*a^2 + 20*b^2)*sinh(d*x + c)^3 + 4
(585(a^2 - 2*a*b)*cosh(d*x + c)^11 + 275*(9*a^2 + 4*b^2)*cosh(d*x + c)^9
+ 54*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^7 + 42*(75*a^2 + 28*b^2)*cosh
(d*x + c)^5 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 15*(9*a^2 + 4
*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 60*(a*b*cosh(d*x + c)^14 + 14*a*b*co
sh(d*x + c)*sinh(d*x + c)^13 + a*b*sinh(d*x + c)^14 + 3*a*b*cosh(d*x + c)^1
2 + (91*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^12 + a*b*cosh(d*x + c)^1
0 + 4*(91*a*b*cosh(d*x + c)^3 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^11 + (10
01*a*b*cosh(d*x + c)^4 + 198*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^10 -
5*a*b*cosh(d*x + c)^8 + 2*(1001*a*b*cosh(d*x + c)^5 + 330*a*b*cosh(d*x + c)
^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a*b*cosh(d*x + c)^6 + 148
5*a*b*cosh(d*x + c)^4 + 45*a*b*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^8 - 5
*a*b*cosh(d*x + c)^6 + 8*(429*a*b*cosh(d*x + c)^7 + 297*a*b*cosh(d*x + c)^5
 + 15*a*b*cosh(d*x + c)^3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a*
b*cosh(d*x + c)^8 + 2772*a*b*cosh(d*x + c)^6 + 210*a*b*cosh(d*x + c)^4 - 14
0*a*b*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^6 + a*b*cosh(d*x + c)^4 + 2*(1
001*a*b*cosh(d*x + c)^9 + 1188*a*b*cosh(d*x + c)^7 + 126*a*b*cosh(d*x + c)^
5 - 140*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + (1001
*a*b*cosh(d*x + c)^10 + 1485*a*b*cosh(d*x + c)^8 + 210*a*b*cosh(d*x + c)^6
- 350*a*b*cosh(d*x + c)^4 - 75*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 +
 3*a*b*cosh(d*x + c)^2 + 4*(91*a*b*cosh(d*x + c)^11 + 165*a*b*cosh(d*x + c)
^9 + 30*a*b*cosh(d*x + c)^7 - 70*a*b*cosh(d*x + c)^5 - 25*a*b*cosh(d*x + c)
^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (91*a*b*cosh(d*x + c)^12 + 198*a*
b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 - 140*a*b*cosh(d*x + c)^6 - 75*
a*b*cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^2 + a*b
+ 2*(7*a*b*cosh(d*x + c)^13 + 18*a*b*cosh(d*x + c)^11 + 5*a*b*cosh(d*x + c)
^9 - 20*a*b*cosh(d*x + c)^7 - 15*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^
3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c
)) + 30*(a^2 + 2*a*b)*cosh(d*x + c) - 15*(a^2*cosh(d*x + c)^14 + 14*a^2*cos
h(d*x + c)*sinh(d*x + c)^13 + a^2*sinh(d*x + c)^14 + 3*a^2*cosh(d*x + c)^12
 + (91*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^12 + a^2*cosh(d*x + c)^10
 + 4*(91*a^2*cosh(d*x + c)^3 + 9*a^2*cosh(d*x + c))*sinh(d*x + c)^11 + (100
1*a^2*cosh(d*x + c)^4 + 198*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 - 5
*a^2*cosh(d*x + c)^8 + 2*(1001*a^2*cosh(d*x + c)^5 + 330*a^2*cosh(d*x + c)^
3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a^2*cosh(d*x + c)^6 + 1485
*a^2*cosh(d*x + c)^4 + 45*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^8 - 5*
a^2*cosh(d*x + c)^6 + 8*(429*a^2*cosh(d*x + c)^7 + 297*a^2*cosh(d*x + c)^5
+ 15*a^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a^2
*cosh(d*x + c)^8 + 2772*a^2*cosh(d*x + c)^6 + 210*a^2*cosh(d*x + c)^4 - 140

*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^6 + a^2*cosh(d*x + c)^4 + 2*(10
01*a^2*cosh(d*x + c)^9 + 1188*a^2*cosh(d*x + c)^7 + 126*a^2*cosh(d*x + c)^5
 - 140*a^2*cosh(d*x + c)^3 - 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + (1001*
a^2*cosh(d*x + c)^10 + 1485*a^2*cosh(d*x + c)^8 + 210*a^2*cosh(d*x + c)^6 -
 350*a^2*cosh(d*x + c)^4 - 75*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 +
3*a^2*cosh(d*x + c)^2 + 4*(91*a^2*cosh(d*x + c)^11 + 165*a^2*cosh(d*x + c)^
9 + 30*a^2*cosh(d*x + c)^7 - 70*a^2*cosh(d*x + c)^5 - 25*a^2*cosh(d*x + c)^
3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + (91*a^2*cosh(d*x + c)^12 + 198*a^2
*cosh(d*x + c)^10 + 45*a^2*cosh(d*x + c)^8 - 140*a^2*cosh(d*x + c)^6 - 75*a
^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^2 + a^2 +
 2*(7*a^2*cosh(d*x + c)^13 + 18*a^2*cosh(d*x + c)^11 + 5*a^2*cosh(d*x + c)^
9 - 20*a^2*cosh(d*x + c)^7 - 15*a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3
 + 3*a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + 15*(a^2*cosh(d*x + c)^14 + 14*a^2*cosh(d*x + c)*sinh(d*x + c)^13 + a^2
*sinh(d*x + c)^14 + 3*a^2*cosh(d*x + c)^12 + (91*a^2*cosh(d*x + c)^2 + 3*a^
2)*sinh(d*x + c)^12 + a^2*cosh(d*x + c)^10 + 4*(91*a^2*cosh(d*x + c)^3 + 9*
a^2*cosh(d*x + c))*sinh(d*x + c)^11 + (1001*a^2*cosh(d*x + c)^4 + 198*a^2*c
osh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 - 5*a^2*cosh(d*x + c)^8 + 2*(1001*a^
2*cosh(d*x + c)^5 + 330*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x
 + c)^9 + (3003*a^2*cosh(d*x + c)^6 + 1485*a^2*cosh(d*x + c)^4 + 45*a^2*cos
h(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^8 - 5*a^2*cosh(d*x + c)^6 + 8*(429*a^2*
cosh(d*x + c)^7 + 297*a^2*cosh(d*x + c)^5 + 15*a^2*cosh(d*x + c)^3 - 5*a^2*
cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a^2*cosh(d*x + c)^8 + 2772*a^2*cosh(
d*x + c)^6 + 210*a^2*cosh(d*x + c)^4 - 140*a^2*cosh(d*x + c)^2 - 5*a^2)*sin
h(d*x + c)^6 + a^2*cosh(d*x + c)^4 + 2*(1001*a^2*cosh(d*x + c)^9 + 1188*a^2
*cosh(d*x + c)^7 + 126*a^2*cosh(d*x + c)^5 - 140*a^2*cosh(d*x + c)^3 - 15*a
^2*cosh(d*x + c))*sinh(d*x + c)^5 + (1001*a^2*cosh(d*x + c)^10 + 1485*a^2*c
osh(d*x + c)^8 + 210*a^2*cosh(d*x + c)^6 - 350*a^2*cosh(d*x + c)^4 - 75*a^2
*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(91*a^2
*cosh(d*x + c)^11 + 165*a^2*cosh(d*x + c)^9 + 30*a^2*cosh(d*x + c)^7 - 70*a
^2*cosh(d*x + c)^5 - 25*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x +
 c)^3 + (91*a^2*cosh(d*x + c)^12 + 198*a^2*cosh(d*x + c)^10 + 45*a^2*cosh(d
*x + c)^8 - 140*a^2*cosh(d*x + c)^6 - 75*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d
*x + c)^2 + 3*a^2)*sinh(d*x + c)^2 + a^2 + 2*(7*a^2*cosh(d*x + c)^13 + 18*a
^2*cosh(d*x + c)^11 + 5*a^2*cosh(d*x + c)^9 - 20*a^2*cosh(d*x + c)^7 - 15*a
^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x
+ c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(195*(a^2 - 2*a*b)*cosh(d*
x + c)^12 + 110*(9*a^2 + 4*b^2)*cosh(d*x + c)^10 + 27*(75*a^2 + 30*a*b - 32
*b^2)*cosh(d*x + c)^8 + 28*(75*a^2 + 28*b^2)*cosh(d*x + c)^6 + 15*(75*a^2 -
 30*a*b - 32*b^2)*cosh(d*x + c)^4 + 30*(9*a^2 + 4*b^2)*cosh(d*x + c)^2 + 15
*a^2 + 30*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh
(d*x + c)^13 + d*sinh(d*x + c)^14 + 3*d*cosh(d*x + c)^12 + (91*d*cosh(d*x +
 c)^2 + 3*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c)
)*sinh(d*x + c)^11 + d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 + 198*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 + 330*d*co

```
sh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^9 - 5*d*cosh(d*x + c)^8 +
(3003*d*cosh(d*x + c)^6 + 1485*d*cosh(d*x + c)^4 + 45*d*cosh(d*x + c)^2 - 5
*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 + 297*d*cosh(d*x + c)^5 + 15
*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 5*d*cosh(d*x + c)
^6 + (3003*d*cosh(d*x + c)^8 + 2772*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)
^4 - 140*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)
^9 + 1188*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 - 140*d*cosh(d*x + c)^3
 - 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + d*cosh(d*x + c)^4 + (1001*d*cosh(d
*x + c)^10 + 1485*d*cosh(d*x + c)^8 + 210*d*cosh(d*x + c)^6 - 350*d*cosh(d*
x + c)^4 - 75*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c
)^11 + 165*d*cosh(d*x + c)^9 + 30*d*cosh(d*x + c)^7 - 70*d*cosh(d*x + c)^5
- 25*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
c)^2 + (91*d*cosh(d*x + c)^12 + 198*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 - 140*d*cosh(d*x + c)^6 - 75*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + 3
*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 + 18*d*cosh(d*x + c)^11 + 5*d
*cosh(d*x + c)^9 - 20*d*cosh(d*x + c)^7 - 15*d*cosh(d*x + c)^5 + 2*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)^2\operatorname{csch}^3{(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-\frac{8}{15}\,b^2\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right.$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/15*b^2*(5*e^(-3*d*x - 3*c)/(d*(5
*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4
*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{60\,ab\arctan\left(e^{(dx+c)}\right)+15\,a^2\log\left(e^{(dx+c)}+1\right)-15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{30\,\left(a^2e^{(3\,dx+3\,c)}+a^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}+\frac{4\,(15\,ab}{}}{30\,d}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*arctan(e^(d*x + c)) + 15*a^2*log(e^(d*x + c) + 1) - 15*a^2*log
(abs(e^(d*x + c) - 1)) - 30*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(e^(2*d
*x + 2*c) - 1)^2 + 4*(15*a*b*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) - 20*
b^2*e^(7*d*x + 7*c) + 8*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) - 20*b
^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.24

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{a^2\,\mathrm{e}^{c+d\,x}}{d-d\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{136\,b^2\,\mathrm{e}^{c+d\,x}}{15\,\left(d+3\,d\,\mathrm{e}^{2\,c+2\,d\,x}+3\,d\,\mathrm{e}^{4\,c+4\,d\,x}+d\,\mathrm{e}^{6\,c+6\,d\,x}\right)}$$
$$+\frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+10\,d\,\mathrm{e}^{4\,c+4\,d\,x}+10\,d\,\mathrm{e}^{6\,c+6\,d\,x}+5\,d\,\mathrm{e}^{8\,c+8\,d\,x}+d\,\mathrm{e}^{10\,c+10\,d\,x}\right)}$$
$$-\frac{a^2\,\ln\left(4\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-16\,a^4\,b^2-4\,a^6+16\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{2\,d}$$
$$+\frac{a^2\,\ln\left(4\,a^6+16\,a^4\,b^2+4\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{2\,d}$$
$$-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{d-2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}-\frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,\left(d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}\right)}$$
$$-\frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+4\,d\,\mathrm{e}^{2\,c+2\,d\,x}+6\,d\,\mathrm{e}^{4\,c+4\,d\,x}+4\,d\,\mathrm{e}^{6\,c+6\,d\,x}+d\,\mathrm{e}^{8\,c+8\,d\,x}\right)}$$
$$+\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}}-\frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}$$
$$-\frac{a\,b\,\left(\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+8\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^5\,b\,8\mathrm{i}-a^3\,b^3\,32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+8\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+a^5\,b\,8\mathrm{i}+\right.\right.}{d}$$

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

```
[Out] (a^2*exp(c + d*x))/(d - d*exp(2*c + 2*d*x)) + (136*b^2*exp(c + d*x))/(15*(d
 + 3*d*exp(2*c + 2*d*x) + 3*d*exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) + (32
*b^2*exp(c + d*x))/(5*(d + 5*d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 1
0*d*exp(6*c + 6*d*x) + 5*d*exp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2
*log(4*a^6*exp(d*x)*exp(c) - 16*a^4*b^2 - 4*a^6 + 16*a^4*b^2*exp(d*x)*exp(c
)))/(2*d) + (a^2*log(4*a^6 + 16*a^4*b^2 + 4*a^6*exp(d*x)*exp(c) + 16*a^4*b^
2*exp(d*x)*exp(c)))/(2*d) - (2*a^2*exp(c + d*x))/(d - 2*d*exp(2*c + 2*d*x)
+ d*exp(4*c + 4*d*x)) - (8*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c + 2*d*x) +
 d*exp(4*c + 4*d*x))) - (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c + 2*d*x)
+ 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x))) + (2*a
*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) - (4*a*b*exp(c + d*x))/(d + 2*d*e
xp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*exp(d*x)*exp(c
) - a^3*b^3*32i - a^5*b*8i + 8*a^5*b*exp(d*x)*exp(c))*1i - log(a^5*b*8i + a
^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 8*a^5*b*exp(d*x)*exp(c))*1i))/d
```

3.64 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d+2*a*b*ln(tanh(d*x+c))/d-a*b*tanh(
d*x+c)^2/d+1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 1816}

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) + (2*a*b*Log[Tanh[c + d
*x]])/d - (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh
[c + d*x]^5)/(5*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^3)^2}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}-\frac{a^2}{x^2}+\frac{2ab}{x}-2abx+b^2x^2-b^2x^4\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{2ab\log(\tanh(c+dx))}{d}\\
&\quad -\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\begin{aligned}
&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx\\
&= \frac{2a^2\coth(c+dx)}{3d}-\frac{a^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{2ab\log(\cosh(c+dx))}{d}\\
&\quad+\frac{2ab\log(\sinh(c+dx))}{d}+\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{2b^2\tanh(c+dx)}{15d}\\
&\quad+\frac{b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d}-\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}
\end{aligned}$$

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a^2*Coth[c + d*x])/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (
2*a*b*Log[Cosh[c + d*x]])/d + (2*a*b*Log[Sinh[c + d*x]])/d + (a*b*Sech[c +
d*x]^2)/d + (2*b^2*Tanh[c + d*x])/(15*d) + (b^2*Sech[c + d*x]^2*Tanh[c + d*
x])/(15*d) - (b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Maple [A] (verified)

Time = 20.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sec}}{}\right.}{4}\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sec}}{}\right.}{4}\right.}{d}$
risch	$-\frac{4\left(-15ab\,e^{14dx+14c}+15a^2e^{12dx+12c}+15b^2e^{12dx+12c}+70a^2e^{10dx+10c}+45ab\,e^{10dx+10c}-50b^2e^{10dx+10c}+125a^2e^{8dx+8}\right.}{15c}$

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(1/2/cosh(d*x+c)^2+ln(ta
nh(d*x+c)))+b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4
+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4125 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 4125, normalized size of antiderivative = 42.53

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 2/15*(30*a*b*cosh(d*x + c)^14 + 420*a*b*cosh(d*x + c)*sinh(d*x + c)^13 + 30
*a*b*sinh(d*x + c)^14 - 30*(a^2 + b^2)*cosh(d*x + c)^12 + 30*(91*a*b*cosh(d
*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^12 + 120*(91*a*b*cosh(d*x + c)^3 - 3*(
a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 10*(14*a^2 + 9*a*b - 10*b^2)*c
osh(d*x + c)^10 + 10*(3003*a*b*cosh(d*x + c)^4 - 198*(a^2 + b^2)*cosh(d*x +
 c)^2 - 14*a^2 - 9*a*b + 10*b^2)*sinh(d*x + c)^10 + 20*(3003*a*b*cosh(d*x +
 c)^5 - 330*(a^2 + b^2)*cosh(d*x + c)^3 - 5*(14*a^2 + 9*a*b - 10*b^2)*cosh(
d*x + c))*sinh(d*x + c)^9 - 10*(25*a^2 + 13*b^2)*cosh(d*x + c)^8 + 10*(9009
*a*b*cosh(d*x + c)^6 - 1485*(a^2 + b^2)*cosh(d*x + c)^4 - 45*(14*a^2 + 9*a*
b - 10*b^2)*cosh(d*x + c)^2 - 25*a^2 - 13*b^2)*sinh(d*x + c)^8 + 80*(1287*a
*b*cosh(d*x + c)^7 - 297*(a^2 + b^2)*cosh(d*x + c)^5 - 15*(14*a^2 + 9*a*b -
 10*b^2)*cosh(d*x + c)^3 - (25*a^2 + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7
 - 2*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^6 + 2*(45045*a*b*cosh(d*x +
c)^8 - 13860*(a^2 + b^2)*cosh(d*x + c)^6 - 1050*(14*a^2 + 9*a*b - 10*b^2)*c
osh(d*x + c)^4 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^2 - 100*a^2 + 45*a*b +
```

44*b^2)*sinh(d*x + c)^6 + 4*(15015*a*b*cosh(d*x + c)^9 - 5940*(a^2 + b^2)*
cosh(d*x + c)^7 - 630*(14*a^2 + 9*a*b - 10*b^2)*cosh(d*x + c)^5 - 140*(25*a
^2 + 13*b^2)*cosh(d*x + c)^3 - 3*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c))
*sinh(d*x + c)^5 - 2*(25*a^2 + 17*b^2)*cosh(d*x + c)^4 + 2*(15015*a*b*cosh(
d*x + c)^10 - 7425*(a^2 + b^2)*cosh(d*x + c)^8 - 1050*(14*a^2 + 9*a*b - 10*
b^2)*cosh(d*x + c)^6 - 350*(25*a^2 + 13*b^2)*cosh(d*x + c)^4 - 15*(100*a^2
- 45*a*b - 44*b^2)*cosh(d*x + c)^2 - 25*a^2 - 17*b^2)*sinh(d*x + c)^4 + 8*(
1365*a*b*cosh(d*x + c)^11 - 825*(a^2 + b^2)*cosh(d*x + c)^9 - 150*(14*a^2 +
9*a*b - 10*b^2)*cosh(d*x + c)^7 - 70*(25*a^2 + 13*b^2)*cosh(d*x + c)^5 - 5
*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^3 - (25*a^2 + 17*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 2*(10*a^2 - 15*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(136
5*a*b*cosh(d*x + c)^12 - 990*(a^2 + b^2)*cosh(d*x + c)^10 - 225*(14*a^2 + 9
*a*b - 10*b^2)*cosh(d*x + c)^8 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^6 - 15
*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^4 - 6*(25*a^2 + 17*b^2)*cosh(d*x
+ c)^2 + 10*a^2 - 15*a*b + 2*b^2)*sinh(d*x + c)^2 + 10*a^2 + 2*b^2 - 15*(a
*b*cosh(d*x + c)^16 + 16*a*b*cosh(d*x + c)*sinh(d*x + c)^15 + a*b*sinh(d*x
+ c)^16 + 2*a*b*cosh(d*x + c)^14 + 2*(60*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*
x + c)^14 - 2*a*b*cosh(d*x + c)^12 + 28*(20*a*b*cosh(d*x + c)^3 + a*b*cosh(
d*x + c))*sinh(d*x + c)^13 + 2*(910*a*b*cosh(d*x + c)^4 + 91*a*b*cosh(d*x +
c)^2 - a*b)*sinh(d*x + c)^12 - 6*a*b*cosh(d*x + c)^10 + 8*(546*a*b*cosh(d*
x + c)^5 + 91*a*b*cosh(d*x + c)^3 - 3*a*b*cosh(d*x + c))*sinh(d*x + c)^11 +
2*(4004*a*b*cosh(d*x + c)^6 + 1001*a*b*cosh(d*x + c)^4 - 66*a*b*cosh(d*x +
c)^2 - 3*a*b)*sinh(d*x + c)^10 + 4*(2860*a*b*cosh(d*x + c)^7 + 1001*a*b*co
sh(d*x + c)^5 - 110*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x +
c)^9 + 6*(2145*a*b*cosh(d*x + c)^8 + 1001*a*b*cosh(d*x + c)^6 - 165*a*b*cos
h(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 6*a*b*cosh(d*x + c
)^6 + 16*(715*a*b*cosh(d*x + c)^9 + 429*a*b*cosh(d*x + c)^7 - 99*a*b*cosh(d
*x + c)^5 - 45*a*b*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 2*(4004*a*b*cosh(d*x
+ c)^10 + 3003*a*b*cosh(d*x + c)^8 - 924*a*b*cosh(d*x + c)^6 - 630*a*b*cosh
(d*x + c)^4 + 3*a*b)*sinh(d*x + c)^6 + 2*a*b*cosh(d*x + c)^4 + 4*(1092*a*b*
cosh(d*x + c)^11 + 1001*a*b*cosh(d*x + c)^9 - 396*a*b*cosh(d*x + c)^7 - 378
*a*b*cosh(d*x + c)^5 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(910*a*b*co
sh(d*x + c)^12 + 1001*a*b*cosh(d*x + c)^10 - 495*a*b*cosh(d*x + c)^8 - 630*
a*b*cosh(d*x + c)^6 + 45*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 - 2*a*b
*cosh(d*x + c)^2 + 8*(70*a*b*cosh(d*x + c)^13 + 91*a*b*cosh(d*x + c)^11 - 5
5*a*b*cosh(d*x + c)^9 - 90*a*b*cosh(d*x + c)^7 + 15*a*b*cosh(d*x + c)^3 + a
*b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(60*a*b*cosh(d*x + c)^14 + 91*a*b*cos
h(d*x + c)^12 - 66*a*b*cosh(d*x + c)^10 - 135*a*b*cosh(d*x + c)^8 + 45*a*b*
cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 - a*b + 4*(4
*a*b*cosh(d*x + c)^15 + 7*a*b*cosh(d*x + c)^13 - 6*a*b*cosh(d*x + c)^11 - 1
5*a*b*cosh(d*x + c)^9 + 9*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^3 - a*b
*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*
x + c))) + 15*(a*b*cosh(d*x + c)^16 + 16*a*b*cosh(d*x + c)*sinh(d*x + c)^15
+ a*b*sinh(d*x + c)^16 + 2*a*b*cosh(d*x + c)^14 + 2*(60*a*b*cosh(d*x + c)^
2 + a*b)*sinh(d*x + c)^14 - 2*a*b*cosh(d*x + c)^12 + 28*(20*a*b*cosh(d*x +

c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^13 + 2*(910*a*b*cosh(d*x + c)^4 + 9
1*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^12 - 6*a*b*cosh(d*x + c)^10 + 8*
(546*a*b*cosh(d*x + c)^5 + 91*a*b*cosh(d*x + c)^3 - 3*a*b*cosh(d*x + c))*si
nh(d*x + c)^11 + 2*(4004*a*b*cosh(d*x + c)^6 + 1001*a*b*cosh(d*x + c)^4 - 6
6*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^10 + 4*(2860*a*b*cosh(d*x + c)
^7 + 1001*a*b*cosh(d*x + c)^5 - 110*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x +
c))*sinh(d*x + c)^9 + 6*(2145*a*b*cosh(d*x + c)^8 + 1001*a*b*cosh(d*x + c)
^6 - 165*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 6*
a*b*cosh(d*x + c)^6 + 16*(715*a*b*cosh(d*x + c)^9 + 429*a*b*cosh(d*x + c)^7
- 99*a*b*cosh(d*x + c)^5 - 45*a*b*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 2*(40
04*a*b*cosh(d*x + c)^10 + 3003*a*b*cosh(d*x + c)^8 - 924*a*b*cosh(d*x + c)^
6 - 630*a*b*cosh(d*x + c)^4 + 3*a*b)*sinh(d*x + c)^6 + 2*a*b*cosh(d*x + c)^
4 + 4*(1092*a*b*cosh(d*x + c)^11 + 1001*a*b*cosh(d*x + c)^9 - 396*a*b*cosh(
d*x + c)^7 - 378*a*b*cosh(d*x + c)^5 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(910*a*b*cosh(d*x + c)^12 + 1001*a*b*cosh(d*x + c)^10 - 495*a*b*cosh(d
*x + c)^8 - 630*a*b*cosh(d*x + c)^6 + 45*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*
x + c)^4 - 2*a*b*cosh(d*x + c)^2 + 8*(70*a*b*cosh(d*x + c)^13 + 91*a*b*cosh
(d*x + c)^11 - 55*a*b*cosh(d*x + c)^9 - 90*a*b*cosh(d*x + c)^7 + 15*a*b*cos
h(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(60*a*b*cosh(d*x + c)
^14 + 91*a*b*cosh(d*x + c)^12 - 66*a*b*cosh(d*x + c)^10 - 135*a*b*cosh(d*x
+ c)^8 + 45*a*b*cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c
)^2 - a*b + 4*(4*a*b*cosh(d*x + c)^15 + 7*a*b*cosh(d*x + c)^13 - 6*a*b*cosh
(d*x + c)^11 - 15*a*b*cosh(d*x + c)^9 + 9*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(
d*x + c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*
x + c) - sinh(d*x + c))) + 4*(105*a*b*cosh(d*x + c)^13 - 90*(a^2 + b^2)*cos
h(d*x + c)^11 - 25*(14*a^2 + 9*a*b - 10*b^2)*cosh(d*x + c)^9 - 20*(25*a^2 +
13*b^2)*cosh(d*x + c)^7 - 3*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^5 -
2*(25*a^2 + 17*b^2)*cosh(d*x + c)^3 + (10*a^2 - 15*a*b + 2*b^2)*cosh(d*x +
c))*sinh(d*x + c))/(d*cosh(d*x + c)^16 + 16*d*cosh(d*x + c)*sinh(d*x + c)^1
5 + d*sinh(d*x + c)^16 + 2*d*cosh(d*x + c)^14 + 2*(60*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c)^14 + 28*(20*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
c)^13 - 2*d*cosh(d*x + c)^12 + 2*(910*d*cosh(d*x + c)^4 + 91*d*cosh(d*x +
c)^2 - d)*sinh(d*x + c)^12 + 8*(546*d*cosh(d*x + c)^5 + 91*d*cosh(d*x + c)^
3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^11 - 6*d*cosh(d*x + c)^10 + 2*(4004*d*
cosh(d*x + c)^6 + 1001*d*cosh(d*x + c)^4 - 66*d*cosh(d*x + c)^2 - 3*d)*sinh
(d*x + c)^10 + 4*(2860*d*cosh(d*x + c)^7 + 1001*d*cosh(d*x + c)^5 - 110*d*c
osh(d*x + c)^3 - 15*d*cosh(d*x + c))*sinh(d*x + c)^9 + 6*(2145*d*cosh(d*x +
c)^8 + 1001*d*cosh(d*x + c)^6 - 165*d*cosh(d*x + c)^4 - 45*d*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 429*d*cosh(d*x + c)^7 - 9
9*d*cosh(d*x + c)^5 - 45*d*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 6*d*cosh(d*x
+ c)^6 + 2*(4004*d*cosh(d*x + c)^10 + 3003*d*cosh(d*x + c)^8 - 924*d*cosh(d
*x + c)^6 - 630*d*cosh(d*x + c)^4 + 3*d)*sinh(d*x + c)^6 + 4*(1092*d*cosh(d
*x + c)^11 + 1001*d*cosh(d*x + c)^9 - 396*d*cosh(d*x + c)^7 - 378*d*cosh(d*
x + c)^5 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^4 + 2*(91
0*d*cosh(d*x + c)^12 + 1001*d*cosh(d*x + c)^10 - 495*d*cosh(d*x + c)^8 - 63

```
0*d*cosh(d*x + c)^6 + 45*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 8*(70*d*c
osh(d*x + c)^13 + 91*d*cosh(d*x + c)^11 - 55*d*cosh(d*x + c)^9 - 90*d*cosh(
d*x + c)^7 + 15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*
cosh(d*x + c)^2 + 2*(60*d*cosh(d*x + c)^14 + 91*d*cosh(d*x + c)^12 - 66*d*c
osh(d*x + c)^10 - 135*d*cosh(d*x + c)^8 + 45*d*cosh(d*x + c)^4 + 6*d*cosh(d
*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(4*d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c
)^13 - 6*d*cosh(d*x + c)^11 - 15*d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)^5 +
2*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)^2\operatorname{csch}^4\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(91) = 182$.

Time = 0.30 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{4}{15}\,b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right.$$
$$+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x -
2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
+ 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*
d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*
```

```
c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-1
0*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.57

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{60\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-60\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{10\,\left(11\,abe^{(6\,dx+6\,c)}-33\,abe^{(4\,dx+4\,c)}+12\,a^2e^{(2\,dx+2\,c)}+33\,abe^{(2\,d}}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*a*b*log(e^(2*d*x + 2*c) + 1) - 60*a*b*log(abs(e^(2*d*x + 2*c) - 1
)) + 10*(11*a*b*e^(6*d*x + 6*c) - 33*a*b*e^(4*d*x + 4*c) + 12*a^2*e^(2*d*x
+ 2*c) + 33*a*b*e^(2*d*x + 2*c) - 4*a^2 - 11*a*b)/(e^(2*d*x + 2*c) - 1)^3 -
 (137*a*b*e^(10*d*x + 10*c) + 805*a*b*e^(8*d*x + 8*c) + 1730*a*b*e^(6*d*x +
 6*c) - 120*b^2*e^(6*d*x + 6*c) + 1730*a*b*e^(4*d*x + 4*c) + 40*b^2*e^(4*d*
x + 4*c) + 805*a*b*e^(2*d*x + 2*c) - 40*b^2*e^(2*d*x + 2*c) + 137*a*b - 8*b
^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.55

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{40\,b^2}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{4\,a^2}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}\\ &\quad-\frac{8\,a^2}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(b^2+a\,b\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}\\ &\quad-\frac{16\,b^2}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}\\ &\quad+\frac{32\,b^2}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}\\ &\quad-\frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}+\frac{4\,a\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}\end{aligned}$$

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

```
[Out] (40*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
 1)) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(3
*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(
a*b + b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (16*b^2)/(d*(
4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*
d*x) + 1)) + (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (4*atan((
a*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))
/(-d^2)^(1/2) + (4*a*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.65 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 275

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx\\
&=\frac{3}{8}a\left(a^2+63b^2\right)x+\frac{3b(3a^2+5b^2)\log(\cosh(c+dx))}{d}-\frac{18ab^2\tanh(c+dx)}{d}\\
&\quad-\frac{b(3a^2+10b^2)\tanh^2(c+dx)}{2d}-\frac{3ab^2\tanh^3(c+dx)}{d}-\frac{3b^3\tanh^4(c+dx)}{2d}\\
&\quad-\frac{3ab^2\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^6(c+dx)}{2d}-\frac{b^3\tanh^8(c+dx)}{8d}\\
&\quad+\frac{\cosh^3(c+dx)\sinh(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\tanh(c+dx)\right)}{4d}\\
&\quad-\frac{\cosh(c+dx)\sinh(c+dx)\left(a(5a^2+51b^2)+2b(15a^2+11b^2)\tanh(c+dx)\right)}{8d}
\end{aligned}$$

```
[Out] 3/8*a*(a^2+63*b^2)*x+3*b*(3*a^2+5*b^2)*ln(cosh(d*x+c))/d-18*a*b^2*tanh(d*x+
c)/d-1/2*b*(3*a^2+10*b^2)*tanh(d*x+c)^2/d-3*a*b^2*tanh(d*x+c)^3/d-3/2*b^3*t
anh(d*x+c)^4/d-3/5*a*b^2*tanh(d*x+c)^5/d-1/2*b^3*tanh(d*x+c)^6/d-1/8*b^3*ta
nh(d*x+c)^8/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*ta
nh(d*x+c))/d-1/8*cosh(d*x+c)*sinh(d*x+c)*(a*(5*a^2+51*b^2)+2*b*(15*a^2+11*b
^2)*tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3744, 1818, 1816, 647, 31}

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= -\frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d}-\frac{3a(a^2+63b^2)\tanh(c+dx)}{8d}$$
$$-\frac{3(a+b)\left(a^2+23ab+40b^2\right)\log(1-\tanh(c+dx))}{16d}$$
$$+\frac{3(a-b)\left(a^2-23ab+40b^2\right)\log(\tanh(c+dx)+1)}{16d}$$
$$+\frac{\sinh^4(c+dx)\left(a(a^2+3b^2)\tanh(c+dx)+b(3a^2+b^2)\right)}{4d}$$
$$-\frac{\sinh^2(c+dx)\tanh(c+dx)\left(4b(6a^2+5b^2)\tanh(c+dx)+a(a^2+39b^2)\right)}{8d}$$
$$-\frac{3ab^2\tanh^5(c+dx)}{5d}-\frac{3ab^2\tanh^3(c+dx)}{d}$$
$$-\frac{b^3\tanh^8(c+dx)}{8d}-\frac{b^3\tanh^6(c+dx)}{2d}-\frac{3b^3\tanh^4(c+dx)}{2d}$$

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-3*(a + b)*(a^2 + 23*a*b + 40*b^2)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a
- b)*(a^2 - 23*a*b + 40*b^2)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*a*(a^2 + 6
3*b^2)*Tanh[c + d*x])/(8*d) - (3*b*(3*a^2 + 5*b^2)*Tanh[c + d*x]^2)/(2*d) -
 (3*a*b^2*Tanh[c + d*x]^3)/d - (3*b^3*Tanh[c + d*x]^4)/(2*d) - (3*a*b^2*Tan
h[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^6)/(2*d) - (b^3*Tanh[c + d*x]^8)/(
8*d) + (Sinh[c + d*x]^4*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/
(4*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a*(a^2 + 39*b^2) + 4*b*(6*a^2 + 5*b
^2)*Tanh[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)^3}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d} + \frac{\text{Subst}\left(\int \frac{x^3(-4b(3a^2+b^2)-a(a^2+15b^2)x-4b(3a^2+b^2)x^2-12ab^2x^3-4b^3x^4-12ab^2x^5-4b^3x^6-4b^3x^8)}{(1-x^2)^2}\,dx, x, \tanh(c+dx\right)}{4d}$$

$$= \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d} - \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a(a^2+39b^2)+4b(6a^2+5b^2)\tanh(c+dx)\right)}{8d} + \frac{\text{Subst}\left(\int \frac{x^2(3a(a^2+39b^2)+72b(a^2+b^2)x+48ab^2x^2+24b^3x^3+24ab^2x^4+16b^3x^5+8b^3x^7)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d}$$
$$- \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a(a^2+39b^2)+4b(6a^2+5b^2)\tanh(c+dx)\right)}{8d}$$
$$+ \frac{\text{Subst}\Big(\int \Big(-3a(a^2+63b^2)-24b(3a^2+5b^2)x-72ab^2x^2-48b^3x^3-24ab^2x^4-24b^3x^5-8b^3x^7+}{8d}$$

$$= -\frac{3a(a^2+63b^2)\tanh(c+dx)}{8d} - \frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d} - \frac{3ab^2\tanh^3(c+dx)}{d}$$
$$- \frac{3b^3\tanh^4(c+dx)}{2d} - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^6(c+dx)}{2d}$$
$$- \frac{b^3\tanh^8(c+dx)}{8d} + \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d}$$
$$- \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a(a^2+39b^2)+4b(6a^2+5b^2)\tanh(c+dx)\right)}{8d}$$
$$+ \frac{3\text{Subst}\left(\int \frac{a^3+63ab^2+8b(3a^2+5b^2)x}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}$$

$$= -\frac{3a(a^2+63b^2)\tanh(c+dx)}{8d} - \frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d} - \frac{3ab^2\tanh^3(c+dx)}{d}$$
$$- \frac{3b^3\tanh^4(c+dx)}{2d} - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^6(c+dx)}{2d}$$
$$- \frac{b^3\tanh^8(c+dx)}{8d} + \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d}$$
$$- \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a(a^2+39b^2)+4b(6a^2+5b^2)\tanh(c+dx)\right)}{8d}$$
$$- \frac{\left(3(a-b)\left(a^2-23ab+40b^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{16d}$$
$$+ \frac{\left(3(a+b)\left(a^2+23ab+40b^2\right)\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{16d}$$

$$= -\frac{3(a+b)\left(a^2+23ab+40b^2\right)\log(1-\tanh(c+dx))}{16d}$$
$$+ \frac{3(a-b)\left(a^2-23ab+40b^2\right)\log(1+\tanh(c+dx))}{16d} - \frac{3a(a^2+63b^2)\tanh(c+dx)}{8d}$$
$$- \frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d} - \frac{3ab^2\tanh^3(c+dx)}{d} - \frac{3b^3\tanh^4(c+dx)}{2d}$$
$$- \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^6(c+dx)}{2d} - \frac{b^3\tanh^8(c+dx)}{8d}$$
$$+ \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d}$$
$$- \frac{\sinh^2(c+dx)\tanh(c+dx)\left(a(a^2+39b^2)+4b(6a^2+5b^2)\tanh(c+dx)\right)}{8d}$$

Mathematica [A] (verified)

Time = 7.37 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx\\&=\frac{3a(a^2+63b^2)(c+dx)}{8d}-\frac{b(15a^2+11b^2)\cosh(2(c+dx))}{8d}+\frac{b(3a^2+b^2)\cosh(4(c+dx))}{32d}\\&+\frac{3(3a^2b+5b^3)\log(\cosh(c+dx))}{d}+\frac{b(3a^2+20b^2)\operatorname{sech}^2(c+dx)}{2d}\\&-\frac{15b^3\operatorname{sech}^4(c+dx)}{4d}+\frac{b^3\operatorname{sech}^6(c+dx)}{d}-\frac{b^3\operatorname{sech}^8(c+dx)}{8d}\\&-\frac{a(a^2+12b^2)\sinh(2(c+dx))}{4d}+\frac{a(a^2+3b^2)\sinh(4(c+dx))}{32d}-\frac{108ab^2\tanh(c+dx)}{5d}\\&+\frac{21ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}-\frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}\end{aligned}$$

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a*(a^2 + 63*b^2)*(c + d*x))/(8*d) - (b*(15*a^2 + 11*b^2)*Cosh[2*(c + d*x
)])/(8*d) + (b*(3*a^2 + b^2)*Cosh[4*(c + d*x)])/(32*d) + (3*(3*a^2*b + 5*b^
3)*Log[Cosh[c + d*x]])/d + (b*(3*a^2 + 20*b^2)*Sech[c + d*x]^2)/(2*d) - (15
*b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/d - (b^3*Sech[c + d*x]^
8)/(8*d) - (a*(a^2 + 12*b^2)*Sinh[2*(c + d*x)])/(4*d) + (a*(a^2 + 3*b^2)*Si
nh[4*(c + d*x)])/(32*d) - (108*a*b^2*Tanh[c + d*x])/(5*d) + (21*a*b^2*Sech[
c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(
5*d)
```

Maple [A] (verified)

Time = 41.60 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tan}{}\right.}{}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tan}{}\right.}{}$
risch	$-\frac{e^{2dx+2c}a^3}{8d}-\frac{11\,e^{2dx+2c}b^3}{16d}+\frac{3a^3x}{8}-9b\,a^2x-\frac{18bc\,a^2}{d}+\frac{9b\ln\left(e^{2dx+2c}+1\right)a^2}{d}+\frac{15b^3\ln\left(e^{2dx+2c}+1\right)}{d}+$

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln
(cosh(d*x+c))-3/2*tanh(d*x+c)^2)+3*a*b^2*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9
```

```
/8*sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d
*x+c)^3-63/40*tanh(d*x+c)^5)+b^3*(1/4*sinh(d*x+c)^12/cosh(d*x+c)^8-3/2*sinh
(d*x+c)^10/cosh(d*x+c)^8+15*ln(cosh(d*x+c))-15/2*tanh(d*x+c)^2-15/4*tanh(d*
x+c)^4-5/2*tanh(d*x+c)^6-15/8*tanh(d*x+c)^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. $2(259) = 518$.

Time = 0.40 (sec) , antiderivative size = 12323, normalized size of antiderivative = 44.81

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(259) = 518$.

Time = 0.30 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.35

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx$$
$$= \frac{1}{64}a^3\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$+ \frac{3}{320}ab^2\left(\frac{2520(dx+c)}{d} + \frac{5\left(32e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{135e^{(-2dx-2c)} + 5358e^{(-4dx-4c)} + 18190e^{(-6}}{d(e^{(-4dx-4c)} + 5e^{(-6dx-6c)} + 10e}\right.$$
$$+ \frac{1}{64}b^3\left(\frac{960(dx+c)}{d} - \frac{44e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} + \frac{960\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{36e^{(-2dx-2c)} + 324e^{(-4d}}{d(e^{(-4dx-}}\right.$$
$$+ \frac{3}{64}a^2b\left(\frac{192(dx+c)}{d} - \frac{20e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} + \frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{18e^{(-2dx-2c)} + 39e^{(-4d}}{d(e^{(-4dx-4c)} + 2e^{(-}}\right.$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/320*a*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4
*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x -
10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6
*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) +
e^(-14*d*x - 14*c)))) + 1/64*b^3*(960*(d*x + c)/d - (44*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c))/d + 960*log(e^(-2*d*x - 2*c) + 1)/d - (36*e^(-2*d*x - 2*c
) + 324*e^(-4*d*x - 4*c) - 1384*e^(-6*d*x - 6*c) - 9126*e^(-8*d*x - 8*c) -
24112*e^(-10*d*x - 10*c) - 31868*e^(-12*d*x - 12*c) - 25912*e^(-14*d*x - 14
*c) - 11169*e^(-16*d*x - 16*c) - 2516*e^(-18*d*x - 18*c) - 1)/(d*(e^(-4*d*x
 - 4*c) + 8*e^(-6*d*x - 6*c) + 28*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c)
+ 70*e^(-12*d*x - 12*c) + 56*e^(-14*d*x - 14*c) + 28*e^(-16*d*x - 16*c) + 8
*e^(-18*d*x - 18*c) + e^(-20*d*x - 20*c)))) + 3/64*a^2*b*(192*(d*x + c)/d -
 (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)
/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)
/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(259) = 518$.

Time = 0.89 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.52

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{35\,a^3e^{(4\,dx+4\,c)}+105\,a^2be^{(4\,dx+4\,c)}+105\,ab^2e^{(4\,dx+4\,c)}+35\,b^3e^{(4\,dx+4\,c)}-280\,a^3e^{(2\,dx+2\,c)}-2100\,a^2be^{(2\,dx+2\,c)}}{}$$

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/2240*(35*a^3*e^(4*d*x + 4*c) + 105*a^2*b*e^(4*d*x + 4*c) + 105*a*b^2*e^(4
*d*x + 4*c) + 35*b^3*e^(4*d*x + 4*c) - 280*a^3*e^(2*d*x + 2*c) - 2100*a^2*b
*e^(2*d*x + 2*c) - 3360*a*b^2*e^(2*d*x + 2*c) - 1540*b^3*e^(2*d*x + 2*c) +
840*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*(d*x + c) - 35*(18*a^3*e^(4*d*x +
4*c) - 432*a^2*b*e^(4*d*x + 4*c) + 1134*a*b^2*e^(4*d*x + 4*c) - 720*b^3*e^(
4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) + 60*a^2*b*e^(2*d*x + 2*c) - 96*a*b^2*
e^(2*d*x + 2*c) + 44*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b + 3*a*b^2 - b^3)*e
^(-4*d*x - 4*c) + 6720*(3*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) + 1) - 8*(6849
*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 53112*a^2*b*e^(14*
d*x + 14*c) - 16800*a*b^2*e^(14*d*x + 14*c) + 80120*b^3*e^(14*d*x + 14*c) +
 181692*a^2*b*e^(12*d*x + 12*c) - 100800*a*b^2*e^(12*d*x + 12*c) + 269220*b
^3*e^(12*d*x + 12*c) + 358344*a^2*b*e^(10*d*x + 10*c) - 272160*a*b^2*e^(10*
d*x + 10*c) + 520520*b^3*e^(10*d*x + 10*c) + 445830*a^2*b*e^(8*d*x + 8*c) -
```

```
 423360*a*b^2*e^(8*d*x + 8*c) + 648970*b^3*e^(8*d*x + 8*c) + 358344*a^2*b*e
^(6*d*x + 6*c) - 405216*a*b^2*e^(6*d*x + 6*c) + 520520*b^3*e^(6*d*x + 6*c)
+ 181692*a^2*b*e^(4*d*x + 4*c) - 237888*a*b^2*e^(4*d*x + 4*c) + 269220*b^3*
e^(4*d*x + 4*c) + 53112*a^2*b*e^(2*d*x + 2*c) - 79968*a*b^2*e^(2*d*x + 2*c)
 + 80120*b^3*e^(2*d*x + 2*c) + 6849*a^2*b - 12096*a*b^2 + 11415*b^3)/(e^(2*
d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.48

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= x\left(\frac{3\,a^3}{8}-9\,a^2\,b+\frac{189\,a\,b^2}{8}-15\,b^3\right)-\frac{4\,(71\,b^3+12\,a\,b^2)}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{256\,b^3}{d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}$$
$$+\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)\,(9\,a^2\,b+15\,b^3)}{d}+\frac{2\,(3\,a^2\,b+30\,a\,b^2+20\,b^3)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$
$$+\frac{32\,(50\,b^3+3\,a\,b^2)}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{128\,b^3}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1\right)}$$
$$-\frac{2\,(3\,a^2\,b+30\,a\,b^2+50\,b^3)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$
$$-\frac{32\,b^3}{d\,\left(8\,\mathrm{e}^{2\,c+2\,d\,x}+28\,\mathrm{e}^{4\,c+4\,d\,x}+56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}+56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}+8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16\,c+\cdots}\right.}$$
$$+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,{(a+b)}^3}{64\,d}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,{(a-b)}^3}{64\,d}+\frac{8\,(23\,b^3+9\,a\,b^2)}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,{(a+b)}^2\,(2\,a+11\,b)}{16\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,{(a-b)}^2\,(2\,a-11\,b)}{16\,d}$$

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] x*((189*a*b^2)/8 - 9*a^2*b + (3*a^3)/8 - 15*b^3) - (4*(12*a*b^2 + 71*b^3))/
(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1)) - (256*b^3)/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 2
0*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) + 1)*(9*a^2*b + 15*b^3))/d + (2*
(30*a*b^2 + 3*a^2*b + 20*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (32*(3*a*b^2 +
50*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*
c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x
```

```
) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1))
 - (2*(30*a*b^2 + 3*a^2*b + 50*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1)) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(
6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c +
12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (exp(4*c + 4*d*
x)*(a + b)^3)/(64*d) - (exp(- 4*c - 4*d*x)*(a - b)^3)/(64*d) + (8*(9*a*b^2
+ 23*b^3))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
 1)) - (exp(2*c + 2*d*x)*(a + b)^2*(2*a + 11*b))/(16*d) + (exp(- 2*c - 2*d*
x)*(a - b)^2*(2*a - 11*b))/(16*d)
```

3.66 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 351

$$\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx\\
&=\frac{15a^2b\arctan(\sinh(c+dx))}{2d}+\frac{1155b^3\arctan(\sinh(c+dx))}{128d}-\frac{a^3\cosh(c+dx)}{d}\\
&\quad-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{18ab^2\operatorname{sech}(c+dx)}{d}\\
&\quad+\frac{4ab^2\operatorname{sech}^3(c+dx)}{d}-\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}-\frac{15a^2b\sinh(c+dx)}{2d}-\frac{1155b^3\sinh(c+dx)}{128d}\\
&\quad+\frac{5a^2b\sinh^3(c+dx)}{2d}+\frac{385b^3\sinh^3(c+dx)}{128d}-\frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}\\
&\quad-\frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d}-\frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d}\\
&\quad-\frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d}-\frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d}
\end{aligned}$$

```
[Out] 15/2*a^2*b*arctan(sinh(d*x+c))/d+1155/128*b^3*arctan(sinh(d*x+c))/d-a^3*cos
h(d*x+c)/d-12*a*b^2*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+a*b^2*cosh(d*x+c)
^3/d-18*a*b^2*sech(d*x+c)/d+4*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*sech(d*x+c)^5
/d-15/2*a^2*b*sinh(d*x+c)/d-1155/128*b^3*sinh(d*x+c)/d+5/2*a^2*b*sinh(d*x+c
)^3/d+385/128*b^3*sinh(d*x+c)^3/d-3/2*a^2*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d-2
31/128*b^3*sinh(d*x+c)^3*tanh(d*x+c)^2/d-33/64*b^3*sinh(d*x+c)^3*tanh(d*x+c
)^4/d-11/48*b^3*sinh(d*x+c)^3*tanh(d*x+c)^6/d-1/8*b^3*sinh(d*x+c)^3*tanh(d*
x+c)^8/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3747, 2713, 2672, 294, 308, 209, 2670, 276}

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{a^3\cosh^3(c+dx)}{3d}-\frac{a^3\cosh(c+dx)}{d}+\frac{15a^2b\arctan(\sinh(c+dx))}{2d}+\frac{5a^2b\sinh^3(c+dx)}{2d}-\frac{15a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}-\frac{3ab^2\text{sech}^5(c+dx)}{5d}+\frac{4ab^2\text{sech}^3(c+dx)}{d}-\frac{18ab^2\text{sech}(c+dx)}{d}+\frac{1155b^3\arctan(\sinh(c+dx))}{128d}+\frac{385b^3\sinh^3(c+dx)}{128d}-\frac{1155b^3\sinh(c+dx)}{128d}-\frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d}-\frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d}-\frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d}-\frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d}$$

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (15*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (1155*b^3*ArcTan[Sinh[c + d*x]])/(
128*d) - (a^3*Cosh[c + d*x])/d - (12*a*b^2*Cosh[c + d*x])/d + (a^3*Cosh[c +
 d*x]^3)/(3*d) + (a*b^2*Cosh[c + d*x]^3)/d - (18*a*b^2*Sech[c + d*x])/d + (
4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (15*a^2*b*Si
nh[c + d*x])/(2*d) - (1155*b^3*Sinh[c + d*x])/(128*d) + (5*a^2*b*Sinh[c + d
*x]^3)/(2*d) + (385*b^3*Sinh[c + d*x]^3)/(128*d) - (3*a^2*b*Sinh[c + d*x]^3
*Tanh[c + d*x]^2)/(2*d) - (231*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(128*d)
 - (33*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^4)/(64*d) - (11*b^3*Sinh[c + d*x]^
3*Tanh[c + d*x]^6)/(48*d) - (b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = i\int \left(-ia^3\sinh^3(c+dx)-3ia^2b\sinh^3(c+dx)\tanh^3(c+dx)\right.$$
$$\left.-3iab^2\sinh^3(c+dx)\tanh^6(c+dx)-ib^3\sinh^3(c+dx)\tanh^9(c+dx)\right)\,dx$$
$$=a^3\int \sinh^3(c+dx)\,dx+\left(3a^2b\right)\int \sinh^3(c+dx)\tanh^3(c+dx)\,dx$$
$$+\left(3ab^2\right)\int \sinh^3(c+dx)\tanh^6(c+dx)\,dx+b^3\int \sinh^3(c+dx)\tanh^9(c+dx)\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(c+dx)\right)}{d} + \frac{(3a^2b)\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{(3ab^2)\,\text{Subst}\left(\int \frac{(1-x^2)^4}{x^6}\,dx, x, \cosh(c+dx)\right)}{d} + \frac{b^3\text{Subst}\left(\int \frac{x^{12}}{(1+x^2)^5}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a^3\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d} + \frac{(15a^2b)\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d} + \frac{(3ab^2)\,\text{Subst}\left(\int \left(-4+\frac{1}{x^6}-\frac{4}{x^4}+\frac{6}{x^2}+x^2\right)\,dx, x, \cosh(c+dx)\right)}{d} + \frac{(11b^3)\,\text{Subst}\left(\int \frac{x^{10}}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{8d}$$

$$= -\frac{a^3\cosh(c+dx)}{d} - \frac{12ab^2\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} - \frac{18ab^2\text{sech}(c+dx)}{d} + \frac{4ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} - \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d} + \frac{(15a^2b)\,\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \sinh(c+dx)\right)}{2d} + \frac{(33b^3)\,\text{Subst}\left(\int \frac{x^8}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{16d}$$

$$= -\frac{a^3\cosh(c+dx)}{d} - \frac{12ab^2\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} - \frac{18ab^2\text{sech}(c+dx)}{d} + \frac{4ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{15a^2b\sinh(c+dx)}{2d} + \frac{5a^2b\sinh^3(c+dx)}{2d} - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} - \frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d} - \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d} + \frac{(15a^2b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d} + \frac{(231b^3)\,\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{64d}$$

$$
\begin{aligned}
&= \frac{15a^2b\arctan(\sinh(c+dx))}{2d} - \frac{a^3\cosh(c+dx)}{d} - \frac{12ab^2\cosh(c+dx)}{d} \\
&\quad + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} - \frac{18ab^2\operatorname{sech}(c+dx)}{d} + \frac{4ab^2\operatorname{sech}^3(c+dx)}{d} \\
&\quad - \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} - \frac{15a^2b\sinh(c+dx)}{2d} + \frac{5a^2b\sinh^3(c+dx)}{2d} \\
&\quad - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} - \frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d} \\
&\quad - \frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d} - \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} \\
&\quad - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d} + \frac{(1155b^3)\operatorname{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{128d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15a^2b\arctan(\sinh(c+dx))}{2d} - \frac{a^3\cosh(c+dx)}{d} - \frac{12ab^2\cosh(c+dx)}{d} \\
&\quad + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} - \frac{18ab^2\operatorname{sech}(c+dx)}{d} \\
&\quad + \frac{4ab^2\operatorname{sech}^3(c+dx)}{d} - \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} - \frac{15a^2b\sinh(c+dx)}{2d} \\
&\quad + \frac{5a^2b\sinh^3(c+dx)}{2d} - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} \\
&\quad - \frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d} - \frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d} \\
&\quad - \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d} \\
&\quad + \frac{(1155b^3)\operatorname{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \sinh(c+dx)\right)}{128d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15a^2b\arctan(\sinh(c+dx))}{2d} - \frac{a^3\cosh(c+dx)}{d} - \frac{12ab^2\cosh(c+dx)}{d} \\
&\quad + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} - \frac{18ab^2\operatorname{sech}(c+dx)}{d} \\
&\quad + \frac{4ab^2\operatorname{sech}^3(c+dx)}{d} - \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} - \frac{15a^2b\sinh(c+dx)}{2d} \\
&\quad - \frac{1155b^3\sinh(c+dx)}{128d} + \frac{5a^2b\sinh^3(c+dx)}{2d} + \frac{385b^3\sinh^3(c+dx)}{128d} \\
&\quad - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} - \frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d} \\
&\quad - \frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d} - \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} \\
&\quad - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d} + \frac{(1155b^3)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{128d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15a^2b\arctan(\sinh(c+dx))}{2d} + \frac{1155b^3\arctan(\sinh(c+dx))}{128d} - \frac{a^3\cosh(c+dx)}{d} \\
&- \frac{12ab^2\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} \\
&- \frac{18ab^2\text{sech}(c+dx)}{d} + \frac{4ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} \\
&- \frac{15a^2b\sinh(c+dx)}{2d} - \frac{1155b^3\sinh(c+dx)}{128d} + \frac{5a^2b\sinh^3(c+dx)}{2d} \\
&+ \frac{385b^3\sinh^3(c+dx)}{128d} - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} \\
&- \frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d} - \frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d} \\
&- \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.83

$$
\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{15(64a^2b+77b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3a(a^2+15b^2)\cosh(c+dx)}{4d} \\
&+ \frac{a(a^2+3b^2)\cosh(3(c+dx))}{12d} - \frac{18ab^2\text{sech}(c+dx)}{d} \\
&+ \frac{4ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{3b(9a^2+7b^2)\sinh(c+dx)}{4d} \\
&- \frac{3\text{sech}^2(c+dx)\left(64a^2b\sinh(c+dx)+255b^3\sinh(c+dx)\right)}{128d} \\
&+ \frac{b(3a^2+b^2)\sinh(3(c+dx))}{12d} + \frac{515b^3\text{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&- \frac{41b^3\text{sech}^5(c+dx)\tanh(c+dx)}{48d} + \frac{b^3\text{sech}^7(c+dx)\tanh(c+dx)}{8d}
\end{aligned}
$$

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (15*(64*a^2*b + 77*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (3*a*(a^2 + 15*
b^2)*Cosh[c + d*x])/(4*d) + (a*(a^2 + 3*b^2)*Cosh[3*(c + d*x)])/(12*d) - (1
8*a*b^2*Sech[c + d*x])/d + (4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c +
d*x]^5)/(5*d) - (3*b*(9*a^2 + 7*b^2)*Sinh[c + d*x])/(4*d) - (3*Sech[c + d*x
]^2*(64*a^2*b*Sinh[c + d*x] + 255*b^3*Sinh[c + d*x]))/(128*d) + (b*(3*a^2 +
 b^2)*Sinh[3*(c + d*x)])/(12*d) + (515*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(
192*d) - (41*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) + (b^3*Sech[c + d*x]
^7*Tanh[c + d*x])/(8*d)
```

Maple [A] (verified)

Time = 23.77 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\right.}{}$
default	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\right.}{}$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}a\,b^2}{8d}+\frac{b^3e^{3dx+3c}}{24d}-\frac{3\,e^{dx+c}a^3}{8d}-\frac{27\,e^{dx+c}a^2b}{8d}-\frac{45\,e^{dx+c}a\,b^2}{8d}-\frac{21b^3e^{dx+c}}{8d}$

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c)))+3*a*b^2*(1/3*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-64/3*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5)+b^3*(1/3*sinh(d*x+c)^11/cosh(d*x+c)^8-11/3*sinh(d*x+c)^9/cosh(d*x+c)^8-33*sinh(d*x+c)^7/cosh(d*x+c)^8-77*sinh(d*x+c)^5/cosh(d*x+c)^8-77*sinh(d*x+c)^3/cosh(d*x+c)^8-33*sinh(d*x+c)/cosh(d*x+c)^8+33*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+1155/64*arctan(exp(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8462 vs. $2(325) = 650$.

Time = 0.36 (sec) , antiderivative size = 8462, normalized size of antiderivative = 24.11

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.72

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{1}{192}b^3\left(\frac{8\left(63\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}-\frac{3465\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{440\,e^{(-2\,dx-2\,c)}+6103\,e^{(-4\,dx-4\,c)}+21019}{d(e^{(-3\,dx-3\,c)}+8\,e^{(-5\,dx-}}\right.$$
$$-\frac{1}{40}ab^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9}}\right.$$
$$+\frac{1}{8}a^2b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right.$$
$$+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
[Out] 1/192*b^3*(8*(63*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 3465*arctan(e^(-d*x -
 c))/d - (440*e^(-2*d*x - 2*c) + 6103*e^(-4*d*x - 4*c) + 21019*e^(-6*d*x -
6*c) + 41207*e^(-8*d*x - 8*c) + 40243*e^(-10*d*x - 10*c) + 22589*e^(-12*d*x
 - 12*c) + 505*e^(-14*d*x - 14*c) - 3331*e^(-16*d*x - 16*c) - 1791*e^(-18*d
*x - 18*c) - 8)/(d*(e^(-3*d*x - 3*c) + 8*e^(-5*d*x - 5*c) + 28*e^(-7*d*x -
7*c) + 56*e^(-9*d*x - 9*c) + 70*e^(-11*d*x - 11*c) + 56*e^(-13*d*x - 13*c)
+ 28*e^(-15*d*x - 15*c) + 8*e^(-17*d*x - 17*c) + e^(-19*d*x - 19*c)))) - 1/
40*a*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c)
+ 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) + 5
600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c)
+ 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-11
*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/8*a^2*b*((27*e^(-d*x - c) - e^(-3*
d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-
4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x -
 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d
 - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.65

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{40\,a^3e^{(3\,dx+3\,c)}+120\,a^2be^{(3\,dx+3\,c)}+120\,ab^2e^{(3\,dx+3\,c)}+40\,b^3e^{(3\,dx+3\,c)}-360\,a^3e^{(dx+c)}-3240\,a^2be^{(dx+c)}-540}{}$$

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/960*(40*a^3*e^(3*d*x + 3*c) + 120*a^2*b*e^(3*d*x + 3*c) + 120*a*b^2*e^(3*
d*x + 3*c) + 40*b^3*e^(3*d*x + 3*c) - 360*a^3*e^(d*x + c) - 3240*a^2*b*e^(d
*x + c) - 5400*a*b^2*e^(d*x + c) - 2520*b^3*e^(d*x + c) + 225*(64*a^2*b + 7
7*b^3)*arctan(e^(d*x + c)) - 40*(9*a^3*e^(2*d*x + 2*c) - 81*a^2*b*e^(2*d*x
+ 2*c) + 135*a*b^2*e^(2*d*x + 2*c) - 63*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b
 - 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) - (2880*a^2*b*e^(15*d*x + 15*c) + 34560*
a*b^2*e^(15*d*x + 15*c) + 11475*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d
*x + 13*c) + 211200*a*b^2*e^(13*d*x + 13*c) + 36775*b^3*e^(13*d*x + 13*c) +
 25920*a^2*b*e^(11*d*x + 11*c) + 590592*a*b^2*e^(11*d*x + 11*c) + 67715*b^3
*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 957696*a*b^2*e^(9*d*x +
9*c) + 27055*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 957696*a*b
^2*e^(7*d*x + 7*c) - 27055*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c
) + 590592*a*b^2*e^(5*d*x + 5*c) - 67715*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*
e^(3*d*x + 3*c) + 211200*a*b^2*e^(3*d*x + 3*c) - 36775*b^3*e^(3*d*x + 3*c)
- 2880*a^2*b*e^(d*x + c) + 34560*a*b^2*e^(d*x + c) - 11475*b^3*e^(d*x + c))
/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.16

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\frac{e^{3c+3dx}(a+b)^3}{24d}+\frac{e^{-3c-3dx}(a-b)^3}{24d}+\frac{15\,\mathrm{atan}\left(\frac{e^{dx}e^{c}\left(77b^3\sqrt{d^2}+64a^2b\sqrt{d^2}\right)}{d\sqrt{4096a^4b^2+9856a^2b^4+5929b^6}}\right)\sqrt{4096a^4b^2+9856a^2b^4+5929b^6}}{64\sqrt{d^2}}-\frac{3e^{-c-dx}(a-b)^2(a-7b)}{8d}-\frac{e^{c+dx}(11005b^3+6144ab^2)}{120d\,(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}+\frac{e^{c+dx}(3365b^3+768ab^2)}{20d\,(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1)}+\frac{596b^3e^{c+dx}}{3d\,(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1)}-\frac{3e^{c+dx}(a+b)^2(a+7b)}{8d}-\frac{3e^{c+dx}(64a^2b+768ab^2+255b^3)}{64d\,(e^{2c+2dx}+1)}-\frac{2e^{c+dx}(1625b^3+144ab^2)}{15d\,(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1)}-\frac{112b^3e^{c+dx}}{d\,(7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1)}+\frac{e^{c+dx}(576a^2b+3072ab^2+4355b^3)}{96d\,(2e^{2c+2dx}+e^{4c+4dx}+1)}+\frac{32b^3e^{c+dx}}{d\,(8e^{2c+2dx}+28e^{4c+4dx}+56e^{6c+6dx}+70e^{8c+8dx}+56e^{10c+10dx}+28e^{12c+12dx}+8e^{14c+14dx}+e^{16c}}$$

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b)^3)/(24*d) + (exp(- 3*c - 3*d*x)*(a - b)^3)/(24*d)
 + (15*atan((exp(d*x)*exp(c)*(77*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(
d*(5929*b^6 + 9856*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(5929*b^6 + 9856*a^2*b^4
 + 4096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - (3*exp(- c - d*x)*(a - b)^2*(a -
 7*b))/(8*d) - (exp(c + d*x)*(6144*a*b^2 + 11005*b^3))/(120*d*(3*exp(2*c +
2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(768*a
*b^2 + 3365*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*
c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (596*b^3*exp(c + d*x))/(3*d*(6*exp(2*
c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x
) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*exp(c + d*x)*(a +
b)^2*(a + 7*b))/(8*d) - (3*exp(c + d*x)*(768*a*b^2 + 64*a^2*b + 255*b^3))/(
64*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 1625*b^3))/(15*
d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8
*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2
*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*
```

```
x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)
) + (exp(c + d*x)*(3072*a*b^2 + 576*a^2*b + 4355*b^3))/(96*d*(2*exp(2*c + 2
*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*
x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*e
xp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c
 + 16*d*x) + 1))
```

3.67 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 220

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= -\frac{1}{2}a\left(a^2+21b^2\right)x-\frac{b(6a^2+5b^2)\log(\cosh(c+dx))}{d}+\frac{9ab^2\tanh(c+dx)}{d}\\ &\quad+\frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d}+\frac{2ab^2\tanh^3(c+dx)}{d}+\frac{3b^3\tanh^4(c+dx)}{4d}\\ &\quad+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^6(c+dx)}{3d}+\frac{b^3\tanh^8(c+dx)}{8d}\\ &\quad+\frac{\cosh(c+dx)\sinh(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\tanh(c+dx)\right)}{2d}\end{aligned}$$

```
[Out] -1/2*a*(a^2+21*b^2)*x-b*(6*a^2+5*b^2)*ln(cosh(d*x+c))/d+9*a*b^2*tanh(d*x+c)
/d+1/2*b*(3*a^2+4*b^2)*tanh(d*x+c)^2/d+2*a*b^2*tanh(d*x+c)^3/d+3/4*b^3*tanh
(d*x+c)^4/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/3*b^3*tanh(d*x+c)^6/d+1/8*b^3*tanh(
d*x+c)^8/d+1/2*cosh(d*x+c)*sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*tanh(d*
x+c))/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3744, 1818, 1816, 647, 31\}$

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{a(a^2+21b^2)\tanh(c+dx)}{2d} + \frac{\sinh^2(c+dx)\left(a(a^2+3b^2)\tanh(c+dx)+b(3a^2+b^2)\right)}{2d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{2ab^2\tanh^3(c+dx)}{d} + \frac{(a+b)^2(a+10b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-10b)(a-b)^2\log(\tanh(c+dx)+1)}{4d} + \frac{b^3\tanh^8(c+dx)}{8d} + \frac{b^3\tanh^6(c+dx)}{3d} + \frac{3b^3\tanh^4(c+dx)}{4d}$$

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] ((a + b)^2*(a + 10*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 10*b)*(a - b)^2
*Log[1 + Tanh[c + d*x]])/(4*d) + (a*(a^2 + 21*b^2)*Tanh[c + d*x])/(2*d) + (
b*(3*a^2 + 4*b^2)*Tanh[c + d*x]^2)/(2*d) + (2*a*b^2*Tanh[c + d*x]^3)/d + (3
*b^3*Tanh[c + d*x]^4)/(4*d) + (3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c
 + d*x]^6)/(3*d) + (b^3*Tanh[c + d*x]^8)/(8*d) + (Sinh[c + d*x]^2*(b*(3*a^2
 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)^3}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{x(-2b(3a^2+b^2)-a(a^2+9b^2)x-2b(3a^2+b^2)x^2-6ab^2x^3-2b^3x^4-6ab^2x^5-2b^3x^6-2b^3x^8)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d} \\
&\quad + \frac{\text{Subst}\left(\int \left(a^3+21ab^2+2b(3a^2+4b^2)x+12ab^2x^2+6b^3x^3+6ab^2x^4+4b^3x^5+2b^3x^7-\frac{a^3+21ab^2\cdots}{}\right.\right.}{2d} \\
&= \frac{a(a^2+21b^2)\tanh(c+dx)}{2d}+\frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d}+\frac{2ab^2\tanh^3(c+dx)}{d} \\
&\quad +\frac{3b^3\tanh^4(c+dx)}{4d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^6(c+dx)}{3d} \\
&\quad +\frac{b^3\tanh^8(c+dx)}{8d}+\frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{a^3+21ab^2+2b(6a^2+5b^2)x}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}
\end{aligned}$$

$$= \frac{a(a^2+21b^2)\tanh(c+dx)}{2d} + \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{2ab^2\tanh^3(c+dx)}{d} + \frac{3b^3\tanh^4(c+dx)}{4d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^6(c+dx)}{3d} + \frac{b^3\tanh^8(c+dx)}{8d} + \frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d} + \frac{((a-10b)(a-b)^2)\,\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{4d} - \frac{((a+b)^2(a+10b))\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{4d}$$

$$= \frac{(a+b)^2(a+10b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-10b)(a-b)^2\log(1+\tanh(c+dx))}{4d} + \frac{a(a^2+21b^2)\tanh(c+dx)}{2d} + \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{2ab^2\tanh^3(c+dx)}{d} + \frac{3b^3\tanh^4(c+dx)}{4d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^6(c+dx)}{3d} + \frac{b^3\tanh^8(c+dx)}{8d} + \frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d}$$

Mathematica [A] (verified)

Time = 7.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\frac{a(a^2+21b^2)(c+dx)}{2d} + \frac{b(3a^2+b^2)\cosh(2(c+dx))}{4d} + \frac{(-6a^2b-5b^3)\log(\cosh(c+dx))}{d} - \frac{b(3a^2+10b^2)\,\text{sech}^2(c+dx)}{2d} + \frac{5b^3\text{sech}^4(c+dx)}{2d} - \frac{5b^3\text{sech}^6(c+dx)}{6d} + \frac{b^3\text{sech}^8(c+dx)}{8d} + \frac{a(a^2+3b^2)\sinh(2(c+dx))}{4d} + \frac{58ab^2\tanh(c+dx)}{5d} - \frac{16ab^2\text{sech}^2(c+dx)\tanh(c+dx)}{5d} + \frac{3ab^2\text{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] -1/2*(a*(a^2 + 21*b^2)*(c + d*x))/d + (b*(3*a^2 + b^2)*Cosh[2*(c + d*x)])/(
4*d) + ((-6*a^2*b - 5*b^3)*Log[Cosh[c + d*x]])/d - (b*(3*a^2 + 10*b^2)*Sech
[c + d*x]^2)/(2*d) + (5*b^3*Sech[c + d*x]^4)/(2*d) - (5*b^3*Sech[c + d*x]^6
)/(6*d) + (b^3*Sech[c + d*x]^8)/(8*d) + (a*(a^2 + 3*b^2)*Sinh[2*(c + d*x)])
/(4*d) + (58*a*b^2*Tanh[c + d*x])/(5*d) - (16*a*b^2*Sech[c + d*x]^2*Tanh[c
+ d*x])/(5*d) + (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+3a\,b^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}\right.}{}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+3a\,b^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}\right.}{}$
risch	$-\frac{a^3x}{2}+6b\,a^2x-\frac{21a\,b^2x}{2}+5b^3x+\frac{e^{2dx+2c}a^3}{8d}+\frac{3\,e^{2dx+2c}a^2b}{8d}+\frac{3\,e^{2dx+2c}a\,b^2}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-}}{8d}$

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x+
c)^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+3*a*b^2*(1/2*sinh(d*x+c
)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*tanh
(d*x+c)^5)+b^3*(1/2*sinh(d*x+c)^10/cosh(d*x+c)^8-5*ln(cosh(d*x+c))+5/2*tanh
(d*x+c)^2+5/4*tanh(d*x+c)^4+5/6*tanh(d*x+c)^6+5/8*tanh(d*x+c)^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9862 vs. 2(206) = 412.

Time = 0.37 (sec) , antiderivative size = 9862, normalized size of antiderivative = 44.83

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\int\left(a+b\tanh^3(c+dx)\right)^3\sinh^2(c+dx)\,dx$$

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(206) = 412$.

Time = 0.32 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.47

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\frac{1}{8}\,a^3\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{40}\,ab^2\left(\frac{420\,(dx+c)}{d}+\frac{15\,e^{(-2\,dx-2\,c)}}{d}-\frac{1003\,e^{(-2\,dx-2\,c)}+3350\,e^{(-4\,dx-4\,c)}+5590\,e^{(-6\,dx-6\,c)}+3915\,e^{(-8\,c}}{d(e^{(-2\,dx-2\,c)}+5\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+10\,e^{(-8\,dx-8\,c)}+5}\right) - \frac{1}{24}\,b^3\left(\frac{120\,(dx+c)}{d}-\frac{3\,e^{(-2\,dx-2\,c)}}{d}+\frac{120\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{24\,e^{(-2\,dx-2\,c)}-396\,e^{(-4\,dx-4\,c)}-1752\,e}{d(e^{(-2\,dx-2\,c)}+8\,e^{(-4\,dx-4\,c)}+28\,e^{(-}}\right) - \frac{3}{8}\,a^2b\left(\frac{16\,(dx+c)}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}+\frac{16\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+1}{d(e^{(-2\,dx-2\,c)}+2\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)})}\right)$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/40*a*b^2*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*
x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x
- 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1
/24*b^3*(120*(d*x + c)/d - 3*e^(-2*d*x - 2*c)/d + 120*log(e^(-2*d*x - 2*c)
+ 1)/d - (24*e^(-2*d*x - 2*c) - 396*e^(-4*d*x - 4*c) - 1752*e^(-6*d*x - 6*c
) - 4430*e^(-8*d*x - 8*c) - 5464*e^(-10*d*x - 10*c) - 4556*e^(-12*d*x - 12*
c) - 1896*e^(-14*d*x - 14*c) - 477*e^(-16*d*x - 16*c) + 3)/(d*(e^(-2*d*x -
2*c) + 8*e^(-4*d*x - 4*c) + 28*e^(-6*d*x - 6*c) + 56*e^(-8*d*x - 8*c) + 70*
e^(-10*d*x - 10*c) + 56*e^(-12*d*x - 12*c) + 28*e^(-14*d*x - 14*c) + 8*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c)))) - 3/8*a^2*b*(16*(d*x + c)/d - e^(-2*
d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x - 2*c) - 15*e^
(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(206) = 412$.

Time = 0.71 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.62

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{105\,a^3e^{(2\,dx+2\,c)}+315\,a^2be^{(2\,dx+2\,c)}+315\,ab^2e^{(2\,dx+2\,c)}+105\,b^3e^{(2\,dx+2\,c)}-420\,(a^3-12\,a^2b+21\,ab^2-10\,b^3)}{}$$

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*a^3*e^(2*d*x + 2*c) + 315*a^2*b*e^(2*d*x + 2*c) + 315*a*b^2*e^(2
*d*x + 2*c) + 105*b^3*e^(2*d*x + 2*c) - 420*(a^3 - 12*a^2*b + 21*a*b^2 - 10
*b^3)*(d*x + c) + 105*(2*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(2*d*x + 2*c) + 4
2*a*b^2*e^(2*d*x + 2*c) - 20*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b - 3*a*b^2
+ b^3)*e^(-2*d*x - 2*c) - 840*(6*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) + 1) +
(13698*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 104544*a^2*b
*e^(14*d*x + 14*c) - 30240*a*b^2*e^(14*d*x + 14*c) + 74520*b^3*e^(14*d*x +
14*c) + 353304*a^2*b*e^(12*d*x + 12*c) - 171360*a*b^2*e^(12*d*x + 12*c) + 2
52420*b^3*e^(12*d*x + 12*c) + 691488*a^2*b*e^(10*d*x + 10*c) - 446880*a*b^2
*e^(10*d*x + 10*c) + 476840*b^3*e^(10*d*x + 10*c) + 858060*a^2*b*e^(8*d*x +
 8*c) - 682080*a*b^2*e^(8*d*x + 8*c) + 601930*b^3*e^(8*d*x + 8*c) + 691488*
a^2*b*e^(6*d*x + 6*c) - 644448*a*b^2*e^(6*d*x + 6*c) + 476840*b^3*e^(6*d*x
+ 6*c) + 353304*a^2*b*e^(4*d*x + 4*c) - 374304*a*b^2*e^(4*d*x + 4*c) + 2524
20*b^3*e^(4*d*x + 4*c) + 104544*a^2*b*e^(2*d*x + 2*c) - 125664*a*b^2*e^(2*d
*x + 2*c) + 74520*b^3*e^(2*d*x + 2*c) + 13698*a^2*b - 19488*a*b^2 + 11415*b
^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.80

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{8\,(29\,b^3+6\,a\,b^2)}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{736\,b^3}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}$$
$$-\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)\,\left(6\,a^2\,b+5\,b^3\right)}{d}-\frac{2\,(3\,a^2\,b+18\,a\,b^2+10\,b^3)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$
$$-\frac{96\,(15\,b^3+a\,b^2)}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}$$
$$-\frac{128\,b^3}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1\right)}$$
$$-\frac{x\,{(a-b)}^2\,(a-10\,b)}{2}+\frac{6\,(a^2\,b+8\,a\,b^2+10\,b^3)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$
$$+\frac{32\,b^3}{d\,\left(8\,\mathrm{e}^{2\,c+2\,d\,x}+28\,\mathrm{e}^{4\,c+4\,d\,x}+56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}+56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}+8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16\,c}\right.}$$
$$+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,{(a+b)}^3}{8\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,{(a-b)}^3}{8\,d}-\frac{16\,(25\,b^3+12\,a\,b^2)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] (8*(6*a*b^2 + 29*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(
6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (736*b^3)/(3*d*(6*exp(2*c + 2*d*x)
```

```
+ 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(1
0*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (log(exp(2*c)*exp(2*d*x) + 1)*(6
*a^2*b + 5*b^3))/d - (2*(18*a*b^2 + 3*a^2*b + 10*b^3))/(d*(exp(2*c + 2*d*x)
 + 1)) - (96*(a*b^2 + 15*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*
x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) -
(128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x
) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + ex
p(14*c + 14*d*x) + 1)) - (x*(a - b)^2*(a - 10*b))/2 + (6*(8*a*b^2 + a^2*b +
 10*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3)/(d*(8*
exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c +
 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d
*x) + exp(16*c + 16*d*x) + 1)) + (exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - (exp(
- 2*c - 2*d*x)*(a - b)^3)/(8*d) - (16*(12*a*b^2 + 25*b^3))/(3*d*(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.68 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 269

$$\begin{aligned}
&\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= -\frac{9a^2b\arctan(\sinh(c+dx))}{2d}-\frac{315b^3\arctan(\sinh(c+dx))}{128d}+\frac{a^3\cosh(c+dx)}{d} \\
&\quad+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9ab^2\operatorname{sech}(c+dx)}{d}-\frac{3ab^2\operatorname{sech}^3(c+dx)}{d}+\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} \\
&\quad+\frac{9a^2b\sinh(c+dx)}{2d}+\frac{315b^3\sinh(c+dx)}{128d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d} \\
&\quad-\frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d}-\frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d} \\
&\quad-\frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d}-\frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}
\end{aligned}$$

```
[Out] -9/2*a^2*b*arctan(sinh(d*x+c))/d-315/128*b^3*arctan(sinh(d*x+c))/d+a^3*cosh
(d*x+c)/d+3*a*b^2*cosh(d*x+c)/d+9*a*b^2*sech(d*x+c)/d-3*a*b^2*sech(d*x+c)^3
/d+3/5*a*b^2*sech(d*x+c)^5/d+9/2*a^2*b*sinh(d*x+c)/d+315/128*b^3*sinh(d*x+c
)/d-3/2*a^2*b*sinh(d*x+c)*tanh(d*x+c)^2/d-105/128*b^3*sinh(d*x+c)*tanh(d*x+
c)^2/d-21/64*b^3*sinh(d*x+c)*tanh(d*x+c)^4/d-3/16*b^3*sinh(d*x+c)*tanh(d*x+
c)^6/d-1/8*b^3*sinh(d*x+c)*tanh(d*x+c)^8/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3747, 2718, 2672, 294, 327, 209, 2670, 276}

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{a^3\cosh(c+dx)}{d}-\frac{9a^2b\arctan(\sinh(c+dx))}{2d}+\frac{9a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\text{sech}^5(c+dx)}{5d}-\frac{3ab^2\text{sech}^3(c+dx)}{d}+\frac{9ab^2\text{sech}(c+dx)}{d}-\frac{315b^3\arctan(\sinh(c+dx))}{128d}+\frac{315b^3\sinh(c+dx)}{128d}-\frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}-\frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d}-\frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d}-\frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d}$$

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-9*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) - (315*b^3*ArcTan[Sinh[c + d*x]])/(1
28*d) + (a^3*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x])/d + (9*a*b^2*Sech[c
 + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d)
+ (9*a^2*b*Sinh[c + d*x])/(2*d) + (315*b^3*Sinh[c + d*x])/(128*d) - (3*a^2*
b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d) - (105*b^3*Sinh[c + d*x]*Tanh[c + d*
x]^2)/(128*d) - (21*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/(64*d) - (3*b^3*Sinh
[c + d*x]*Tanh[c + d*x]^6)/(16*d) - (b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/(8*
d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int \left(ia^3\sinh(c+dx)+3ia^2b\sinh(c+dx)\tanh^3(c+dx)\right.\right.\\&\qquad\left.\left.+3iab^2\sinh(c+dx)\tanh^6(c+dx)+ib^3\sinh(c+dx)\tanh^9(c+dx)\right)\,dx\right)\\&= a^3\int\sinh(c+dx)\,dx+\left(3a^2b\right)\int\sinh(c+dx)\tanh^3(c+dx)\,dx\\&\quad+\left(3ab^2\right)\int\sinh(c+dx)\tanh^6(c+dx)\,dx+b^3\int\sinh(c+dx)\tanh^9(c+dx)\,dx\end{aligned}$$

$$=\frac{a^3\cosh(c+dx)}{d}+\frac{(3a^2b)\,\text{Subst}\left(\int\frac{x^4}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}-\frac{(3ab^2)\,\text{Subst}\left(\int\frac{(1-x^2)^3}{x^6}\,dx,x,\cosh(c+dx)\right)}{d}+\frac{b^3\text{Subst}\left(\int\frac{x^{10}}{(1+x^2)^5}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}-\frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}+\frac{(9a^2b)\,\text{Subst}\left(\int\frac{x^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{2d}-\frac{(3ab^2)\,\text{Subst}\left(\int\left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx,x,\cosh(c+dx)\right)}{d}+\frac{(9b^3)\,\text{Subst}\left(\int\frac{x^8}{(1+x^2)^4}\,dx,x,\sinh(c+dx)\right)}{8d}$$

$$=\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9ab^2\text{sech}(c+dx)}{d}-\frac{3ab^2\text{sech}^3(c+dx)}{d}+\frac{3ab^2\text{sech}^5(c+dx)}{5d}+\frac{9a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}-\frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d}-\frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}-\frac{(9a^2b)\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{2d}+\frac{(21b^3)\,\text{Subst}\left(\int\frac{x^6}{(1+x^2)^3}\,dx,x,\sinh(c+dx)\right)}{16d}$$

$$=-\frac{9a^2b\arctan(\sinh(c+dx))}{2d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9ab^2\text{sech}(c+dx)}{d}-\frac{3ab^2\text{sech}^3(c+dx)}{d}+\frac{3ab^2\text{sech}^5(c+dx)}{5d}+\frac{9a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}-\frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d}-\frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d}-\frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}+\frac{(105b^3)\,\text{Subst}\left(\int\frac{x^4}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{64d}$$

$$= -\frac{9a^2b\arctan(\sinh(c+dx))}{2d} + \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+dx)}{d} + \frac{9ab^2\text{sech}(c+dx)}{d} - \frac{3ab^2\text{sech}^3(c+dx)}{d} + \frac{3ab^2\text{sech}^5(c+dx)}{5d} + \frac{9a^2b\sinh(c+dx)}{2d} - \frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d} - \frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d} - \frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d} - \frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d} - \frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d} + \frac{(315b^3)\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{128d}$$

$$= -\frac{9a^2b\arctan(\sinh(c+dx))}{2d} + \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+dx)}{d} + \frac{9ab^2\text{sech}(c+dx)}{d} - \frac{3ab^2\text{sech}^3(c+dx)}{d} + \frac{3ab^2\text{sech}^5(c+dx)}{5d} + \frac{9a^2b\sinh(c+dx)}{2d} + \frac{315b^3\sinh(c+dx)}{128d} - \frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d} - \frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d} - \frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d} - \frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d} - \frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d} - \frac{(315b^3)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{128d}$$

$$= -\frac{9a^2b\arctan(\sinh(c+dx))}{2d} - \frac{315b^3\arctan(\sinh(c+dx))}{128d} + \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+dx)}{d} + \frac{9ab^2\text{sech}(c+dx)}{d} - \frac{3ab^2\text{sech}^3(c+dx)}{d} + \frac{3ab^2\text{sech}^5(c+dx)}{5d} + \frac{9a^2b\sinh(c+dx)}{2d} + \frac{315b^3\sinh(c+dx)}{128d} - \frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d} - \frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d} - \frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d} - \frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d} - \frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}$$

Mathematica [A] (verified)

Time = 7.69 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.87

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= -\frac{9(64a^2b+35b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{a(a^2+3b^2)\cosh(c+dx)}{d}$$
$$+\frac{9ab^2\text{sech}(c+dx)}{d}-\frac{3ab^2\text{sech}^3(c+dx)}{d}+\frac{3ab^2\text{sech}^5(c+dx)}{5d}$$
$$+\frac{b(3a^2+b^2)\sinh(c+dx)}{d}+\frac{\text{sech}^2(c+dx)\left(192a^2b\sinh(c+dx)+325b^3\sinh(c+dx)\right)}{128d}$$
$$-\frac{105b^3\text{sech}^3(c+dx)\tanh(c+dx)}{64d}$$
$$+\frac{11b^3\text{sech}^5(c+dx)\tanh(c+dx)}{16d}-\frac{b^3\text{sech}^7(c+dx)\tanh(c+dx)}{8d}$$

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-9*(64*a^2*b + 35*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) + (a*(a^2 + 3*b^2
)*Cosh[c + d*x])/d + (9*a*b^2*Sech[c + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/
d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Sinh[c + d*x])/d + (
Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*x] + 325*b^3*Sinh[c + d*x]))/(128*d)
- (105*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) + (11*b^3*Sech[c + d*x]^5*
Tanh[c + d*x])/(16*d) - (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)
```

Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.10

method	result
derivativedivides	$a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\text{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\text{e}^{dx+c}\right)\right)+3a\,b^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sinh}{\cosh(}\right.$
default	$a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\text{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\text{e}^{dx+c}\right)\right)+3a\,b^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sinh}{\cosh(}\right.$
risch	$\frac{\text{e}^{dx+c}a^3}{2d}+\frac{3\,\text{e}^{dx+c}a^2b}{2d}+\frac{3\,\text{e}^{dx+c}a\,b^2}{2d}+\frac{b^3\text{e}^{dx+c}}{2d}+\frac{\text{e}^{-dx-c}a^3}{2d}-\frac{3\,\text{e}^{-dx-c}a^2b}{2d}+\frac{3\,\text{e}^{-dx-c}a\,b^2}{2d}-\frac{\text{e}^{-dx-c}b^3}{2d}+$

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cos
h(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c)))+3*a*b^2*(sinh(
d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sinh(d*x+c)^2/cosh(d
*x+c)^5+16/5/cosh(d*x+c)^5)+b^3*(sinh(d*x+c)^9/cosh(d*x+c)^8+9*sinh(d*x+c)^
```

```
7/cosh(d*x+c)^8+21*sinh(d*x+c)^5/cosh(d*x+c)^8+21*sinh(d*x+c)^3/cosh(d*x+c)
^8+9*sinh(d*x+c)/cosh(d*x+c)^8-9*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/1
92*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)-315/64*arctan(exp(d*x+c)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6410 vs. $2(249) = 498$.

Time = 0.32 (sec) , antiderivative size = 6410, normalized size of antiderivative = 23.83

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3{(c+dx)}\right)^3\sinh{(c+dx)}\,dx$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.80

$$\begin{aligned}
&\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{1}{64}b^3\left(\frac{315\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{32\,e^{(-dx-c)}}{d} + \frac{581\,e^{(-2\,dx-2\,c)} + 1681\,e^{(-4\,dx-4\,c)} + 3605\,e^{(-6\,dx-6\,c)} + 2569\,e}{d(e^{(-dx-c)} + 8\,e^{(-3\,dx-3\,c)} + 28\,e^{(-5\,dx-5\,c)} + 56\,e^{(-7\,dx-7}}\right. \\
&\quad + \frac{3}{2}a^2b\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right) \\
&\quad + \frac{3}{10}ab^2\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c)}}{d(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx-}}\right. \\
&\quad + \frac{a^3\cosh(dx+c)}{d}
\end{aligned}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(315*arctan(e^(-d*x - c))/d - 32*e^(-d*x - c)/d + (581*e^(-2*d*x -
 2*c) + 1681*e^(-4*d*x - 4*c) + 3605*e^(-6*d*x - 6*c) + 2569*e^(-8*d*x - 8*
c) + 1463*e^(-10*d*x - 10*c) - 917*e^(-12*d*x - 12*c) - 529*e^(-14*d*x - 14
*c) - 293*e^(-16*d*x - 16*c) + 32)/(d*(e^(-d*x - c) + 8*e^(-3*d*x - 3*c) +
28*e^(-5*d*x - 5*c) + 56*e^(-7*d*x - 7*c) + 70*e^(-9*d*x - 9*c) + 56*e^(-11
*d*x - 11*c) + 28*e^(-13*d*x - 13*c) + 8*e^(-15*d*x - 15*c) + e^(-17*d*x -
17*c)))) + 3/2*a^2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) +
e^(-5*d*x - 5*c)))) + 3/10*a*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) +
 210*e^(-4*d*x - 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^
(-10*d*x - 10*c) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x
- 5*c) + 10*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) +
 a^3*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.72

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{160\,a^3e^{(dx+c)}+480\,a^2be^{(dx+c)}+480\,ab^2e^{(dx+c)}+160\,b^3e^{(dx+c)}-45\left(64\,a^2b+35\,b^3\right)\arctan\left(e^{(dx+c)}\right)+160\,(a}{}$$

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/320*(160*a^3*e^(d*x + c) + 480*a^2*b*e^(d*x + c) + 480*a*b^2*e^(d*x + c)
+ 160*b^3*e^(d*x + c) - 45*(64*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + 160*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*e^(-d*x - c) + (960*a^2*b*e^(15*d*x + 15*c) +
 5760*a*b^2*e^(15*d*x + 15*c) + 1625*b^3*e^(15*d*x + 15*c) + 4800*a^2*b*e^(
13*d*x + 13*c) + 32640*a*b^2*e^(13*d*x + 13*c) + 3925*b^3*e^(13*d*x + 13*c)
 + 8640*a^2*b*e^(11*d*x + 11*c) + 88704*a*b^2*e^(11*d*x + 11*c) + 9065*b^3*
e^(11*d*x + 11*c) + 4800*a^2*b*e^(9*d*x + 9*c) + 143232*a*b^2*e^(9*d*x + 9*
c) + 1645*b^3*e^(9*d*x + 9*c) - 4800*a^2*b*e^(7*d*x + 7*c) + 143232*a*b^2*e
^(7*d*x + 7*c) - 1645*b^3*e^(7*d*x + 7*c) - 8640*a^2*b*e^(5*d*x + 5*c) + 88
704*a*b^2*e^(5*d*x + 5*c) - 9065*b^3*e^(5*d*x + 5*c) - 4800*a^2*b*e^(3*d*x
+ 3*c) + 32640*a*b^2*e^(3*d*x + 3*c) - 3925*b^3*e^(3*d*x + 3*c) - 960*a^2*b
*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1625*b^3*e^(d*x + c))/(e^(2*d*x + 2
*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.63

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{e^{c+dx}\,(a+b)^3}{2\,d}+\frac{e^{-c-dx}\,(a-b)^3}{2\,d}$$
$$-\frac{9\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(35\,b^3\,\sqrt{d^2}+64\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4096\,a^4\,b^2+4480\,a^2\,b^4+1225\,b^6}}\right)\,\sqrt{4096\,a^4\,b^2+4480\,a^2\,b^4+1225\,b^6}}{64\,\sqrt{d^2}}$$
$$+\frac{e^{c+d\,x}\,\left(2455\,b^3+1728\,a\,b^2\right)}{40\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{e^{c+d\,x}\,\left(2605\,b^3+768\,a\,b^2\right)}{20\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{188\,b^3\,e^{c+d\,x}}{d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,\left(192\,a^2\,b+1152\,a\,b^2+325\,b^3\right)}{64\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$
$$+\frac{2\,e^{c+d\,x}\,\left(475\,b^3+48\,a\,b^2\right)}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{112\,b^3\,e^{c+d\,x}}{d\,\left(7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\,c+12\,d\,x}+e^{14\,c+14\,d\,x}+1\right)}$$
$$-\frac{e^{c+d\,x}\,\left(192\,a^2\,b+768\,a\,b^2+745\,b^3\right)}{32\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$-\frac{32\,b^3\,e^{c+d\,x}}{d\,\left(8\,e^{2\,c+2\,d\,x}+28\,e^{4\,c+4\,d\,x}+56\,e^{6\,c+6\,d\,x}+70\,e^{8\,c+8\,d\,x}+56\,e^{10\,c+10\,d\,x}+28\,e^{12\,c+12\,d\,x}+8\,e^{14\,c+14\,d\,x}+e^{16\,c}\right.}$$

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] (exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a - b)^3)/(2*d) - (9*atan
((exp(d*x)*exp(c)*(35*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(d*(1225*b^6
 + 4480*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(1225*b^6 + 4480*a^2*b^4 + 4096*a^4
*b^2)^(1/2))/(64*(d^2)^(1/2)) + (exp(c + d*x)*(1728*a*b^2 + 2455*b^3))/(40*
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(
c + d*x)*(768*a*b^2 + 2605*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*
d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (188*b^3*exp(c + d*x))
/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*ex
p(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c +
 d*x)*(1152*a*b^2 + 192*a^2*b + 325*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (
2*exp(c + d*x)*(48*a*b^2 + 475*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) +
 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(
12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 192*
```

```
a^2*b + 745*b^3))/(32*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*
b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c
 + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*
d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))
```

3.69 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= \frac{3a^2b\arctan(\sinh(c+dx))}{2d}+\frac{35b^3\arctan(\sinh(c+dx))}{128d} \\ &-\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3ab^2\operatorname{sech}(c+dx)}{d}+\frac{2ab^2\operatorname{sech}^3(c+dx)}{d} \\ &-\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}-\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} \\ &-\frac{35b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}-\frac{35b^3\operatorname{sech}(c+dx)\tanh^3(c+dx)}{192d} \\ &-\frac{7b^3\operatorname{sech}(c+dx)\tanh^5(c+dx)}{48d}-\frac{b^3\operatorname{sech}(c+dx)\tanh^7(c+dx)}{8d}\end{aligned}$$

```
[Out] 3/2*a^2*b*arctan(sinh(d*x+c))/d+35/128*b^3*arctan(sinh(d*x+c))/d-a^3*arctan
h(cosh(d*x+c))/d-3*a*b^2*sech(d*x+c)/d+2*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*se
ch(d*x+c)^5/d-3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d-35/128*b^3*sech(d*x+c)*ta
nh(d*x+c)/d-35/192*b^3*sech(d*x+c)*tanh(d*x+c)^3/d-7/48*b^3*sech(d*x+c)*tan
h(d*x+c)^5/d-1/8*b^3*sech(d*x+c)*tanh(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3747, 3855, 2691, 2686, 200}

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= -\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{3a^2b\arctan(\sinh(c+dx))}{2d} - \frac{3a^2b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}-\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} + \frac{2ab^2\operatorname{sech}^3(c+dx)}{d}-\frac{3ab^2\operatorname{sech}(c+dx)}{d}+\frac{35b^3\arctan(\sinh(c+dx))}{128d} - \frac{b^3\tanh^7(c+dx)\operatorname{sech}(c+dx)}{8d}-\frac{7b^3\tanh^5(c+dx)\operatorname{sech}(c+dx)}{48d} - \frac{35b^3\tanh^3(c+dx)\operatorname{sech}(c+dx)}{192d}-\frac{35b^3\tanh(c+dx)\operatorname{sech}(c+dx)}{128d}$$

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (35*b^3*ArcTan[Sinh[c + d*x]])/(128
*d) - (a^3*ArcTanh[Cosh[c + d*x]])/d - (3*a*b^2*Sech[c + d*x])/d + (2*a*b^2
*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (3*a^2*b*Sech[c + d
*x]*Tanh[c + d*x])/(2*d) - (35*b^3*Sech[c + d*x]*Tanh[c + d*x])/(128*d) - (
35*b^3*Sech[c + d*x]*Tanh[c + d*x]^3)/(192*d) - (7*b^3*Sech[c + d*x]*Tanh[c
 + d*x]^5)/(48*d) - (b^3*Sech[c + d*x]*Tanh[c + d*x]^7)/(8*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \left(-ia^3\text{csch}(c+dx)-3ia^2b\,\text{sech}(c+dx)\tanh^2(c+dx)\right.\\
&\qquad \left.-3iab^2\text{sech}(c+dx)\tanh^5(c+dx)-ib^3\text{sech}(c+dx)\tanh^8(c+dx)\right)\,dx\\
&= a^3\int \text{csch}(c+dx)\,dx+\left(3a^2b\right)\int \text{sech}(c+dx)\tanh^2(c+dx)\,dx\\
&\quad +\left(3ab^2\right)\int \text{sech}(c+dx)\tanh^5(c+dx)\,dx+b^3\int \text{sech}(c+dx)\tanh^8(c+dx)\,dx\\
&= -\frac{a^3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3a^2b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}\\
&\quad -\frac{b^3\text{sech}(c+dx)\tanh^7(c+dx)}{8d}+\frac{1}{2}\left(3a^2b\right)\int \text{sech}(c+dx)\,dx\\
&\quad +\frac{1}{8}\left(7b^3\right)\int \text{sech}(c+dx)\tanh^6(c+dx)\,dx\\
&\quad -\frac{\left(3ab^2\right)\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\text{sech}(c+dx)\right)}{d}\\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d}-\frac{a^3\text{arctanh}(\cosh(c+dx))}{d}\\
&\quad -\frac{3a^2b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}-\frac{7b^3\text{sech}(c+dx)\tanh^5(c+dx)}{48d}\\
&\quad -\frac{b^3\text{sech}(c+dx)\tanh^7(c+dx)}{8d}+\frac{1}{48}\left(35b^3\right)\int \text{sech}(c+dx)\tanh^4(c+dx)\,dx\\
&\quad -\frac{\left(3ab^2\right)\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\text{sech}(c+dx)\right)}{d}
\end{aligned}$$

$$= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} - \frac{a^3\text{arctanh}(\cosh(c+dx))}{d} - \frac{3ab^2\text{sech}(c+dx)}{d} + \frac{2ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{35b^3\text{sech}(c+dx)\tanh^3(c+dx)}{192d} - \frac{7b^3\text{sech}(c+dx)\tanh^5(c+dx)}{48d} - \frac{b^3\text{sech}(c+dx)\tanh^7(c+dx)}{8d} + \frac{1}{64}(35b^3)\int \text{sech}(c+dx)\tanh^2(c+dx)\,dx$$

$$= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} - \frac{a^3\text{arctanh}(\cosh(c+dx))}{d} - \frac{3ab^2\text{sech}(c+dx)}{d} + \frac{2ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{35b^3\text{sech}(c+dx)\tanh(c+dx)}{128d} - \frac{35b^3\text{sech}(c+dx)\tanh^3(c+dx)}{192d} - \frac{7b^3\text{sech}(c+dx)\tanh^5(c+dx)}{48d} - \frac{b^3\text{sech}(c+dx)\tanh^7(c+dx)}{8d} + \frac{1}{128}(35b^3)\int \text{sech}(c+dx)\,dx$$

$$= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{35b^3\arctan(\sinh(c+dx))}{128d} - \frac{a^3\text{arctanh}(\cosh(c+dx))}{d} - \frac{3ab^2\text{sech}(c+dx)}{d} + \frac{2ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{35b^3\text{sech}(c+dx)\tanh(c+dx)}{128d} - \frac{35b^3\text{sech}(c+dx)\tanh^3(c+dx)}{192d} - \frac{7b^3\text{sech}(c+dx)\tanh^5(c+dx)}{48d} - \frac{b^3\text{sech}(c+dx)\tanh^7(c+dx)}{8d}$$

Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

$$\int \text{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{30\left(b(192a^2+35b^2)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+64a^3\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{}$$

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (30*(b*(192*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 64*a^3*(-Log[Cosh[(c
+ d*x)/2]] + Log[Sinh[(c + d*x)/2]])) + 240*b^3*Sech[c + d*x]^7*Tanh[c + d*
x] - 8*b^2*Sech[c + d*x]^5*(144*a + 125*b*Tanh[c + d*x]) + 10*b^2*Sech[c +
d*x]^3*(384*a + 163*b*Tanh[c + d*x]) - 45*b*Sech[c + d*x]*(128*a*b + (64*a^
2 + 31*b^2)*Tanh[c + d*x]))/(1920*d)
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c}{3\cosh(dx+}\right.}{}$
default	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c}{3\cosh(dx+}\right.}{}$
risch	$-\frac{b\,e^{dx+c}(2880a^2e^{14dx+14c}+5760ab\,e^{14dx+14c}+1395b^2e^{14dx+14c}+14400a^2e^{12dx+12c}+24960ab\,e^{12dx+12c}+455b^2e^{12dx}}{}$

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sec
h(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a*b^2*(-sinh(d*x+c)^4/cosh(d*x+c
)^5-4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5)+b^3*(-sinh(d*x+c)^7
/cosh(d*x+c)^8-7/3*sinh(d*x+c)^5/cosh(d*x+c)^8-7/3*sinh(d*x+c)^3/cosh(d*x+c
)^8-sinh(d*x+c)/cosh(d*x+c)^8+(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/192*
sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+35/64*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7127 vs. 2(203) = 406.

Time = 0.34 (sec) , antiderivative size = 7127, normalized size of antiderivative = 32.54

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(203) = 406$.

Time = 0.29 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.99

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{1}{192}b^3\left(\frac{105\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{279\,e^{(-dx-c)}+91\,e^{(-3\,dx-3\,c)}+1799\,e^{(-5\,dx-5\,c)}-1085\,e^{(-7\,dx-7\,c)}+1085}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{}}\right.$$

$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

$$-\frac{2}{5}\,ab^2\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right.$$

$$+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d + (279*e^(-d*x - c) + 91*e^(-3*d*x -
 3*c) + 1799*e^(-5*d*x - 5*c) - 1085*e^(-7*d*x - 7*c) + 1085*e^(-9*d*x - 9*
c) - 1799*e^(-11*d*x - 11*c) - 91*e^(-13*d*x - 13*c) - 279*e^(-15*d*x - 15*
c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70
*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-1
4*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d
 + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4
*c) + 1))) - 2/5*a*b^2*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d
*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(
-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(203) = 406.

Time = 0.55 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.89

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{960\,a^3\log\left(e^{(dx+c)}+1\right)-960\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)-15\left(192\,a^2b+35\,b^3\right)\arctan\left(e^{(dx+c)}\right)+\frac{2880\,a^2be^{(15\,dx}}{}}{}$$

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/960*(960*a^3*log(e^(d*x + c) + 1) - 960*a^3*log(abs(e^(d*x + c) - 1)) -
15*(192*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + (2880*a^2*b*e^(15*d*x + 15*c)
 + 5760*a*b^2*e^(15*d*x + 15*c) + 1395*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*
e^(13*d*x + 13*c) + 24960*a*b^2*e^(13*d*x + 13*c) + 455*b^3*e^(13*d*x + 13*
c) + 25920*a^2*b*e^(11*d*x + 11*c) + 62592*a*b^2*e^(11*d*x + 11*c) + 8995*b
^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 103296*a*b^2*e^(9*d*x
+ 9*c) - 5425*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 103296*a*
b^2*e^(7*d*x + 7*c) + 5425*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c
) + 62592*a*b^2*e^(5*d*x + 5*c) - 8995*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^
(3*d*x + 3*c) + 24960*a*b^2*e^(3*d*x + 3*c) - 455*b^3*e^(3*d*x + 3*c) - 288
0*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1395*b^3*e^(d*x + c))/(e^(2*
d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{a^3\,\ln\left(\mathrm{e}^{c+d\,x}-1\right)}{d} - \frac{a^3\,\ln\left(\mathrm{e}^{c+d\,x}+1\right)}{d} - \frac{\mathrm{e}^{c+d\,x}\,\left(4445\,b^3+4224\,a\,b^2\right)}{120\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$
$$+ \frac{\mathrm{e}^{c+d\,x}\,\left(1925\,b^3+768\,a\,b^2\right)}{20\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$+ \frac{532\,b^3\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}$$
$$- \frac{3\,\mathrm{e}^{c+d\,x}\,\left(64\,a^2\,b+128\,a\,b^2+31\,b^3\right)}{64\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$
$$- \frac{2\,\mathrm{e}^{c+d\,x}\,\left(1225\,b^3+144\,a\,b^2\right)}{15\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}$$
$$- \frac{112\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1\right)}$$
$$+ \frac{\mathrm{e}^{c+d\,x}\,\left(576\,a^2\,b+1536\,a\,b^2+931\,b^3\right)}{96\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$
$$+ \frac{32\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(8\,\mathrm{e}^{2\,c+2\,d\,x}+28\,\mathrm{e}^{4\,c+4\,d\,x}+56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}+56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}+8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16\,c+\cdots}\right.}$$
$$- \frac{b\,\ln\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)\,\left(192\,a^2+35\,b^2\right)\,1\mathrm{i}}{128\,d} + \frac{b\,\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,\left(192\,a^2+35\,b^2\right)\,1\mathrm{i}}{128\,d}$$

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x),x)
```

```
[Out] (a^3*log(exp(c + d*x) - 1))/d - (a^3*log(exp(c + d*x) + 1))/d - (exp(c + d*
x)*(4224*a*b^2 + 4445*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 35*b^2)*1i
)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 35*b^2)*1i)/(128*d) + (exp
(c + d*x)*(768*a*b^2 + 1925*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (532*b^3*exp(c + d*x)
)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15
*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*ex
p(c + d*x)*(128*a*b^2 + 64*a^2*b + 31*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) -
 (2*exp(c + d*x)*(144*a*b^2 + 1225*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp
(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*
x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d
*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7
*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x)*(1536*a*b^2
+ 576*a^2*b + 931*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
+ (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*e
```

```
xp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c
 + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))
```

3.70 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

```
[Out] -a^3*coth(d*x+c)/d+3/2*a^2*b*tanh(d*x+c)^2/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/8*
b^3*tanh(d*x+c)^8/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 276}

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (3*a*b^2*Tanh[
c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^3)^3}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^2}+3a^2bx+3ab^2x^4+b^3x^7\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \text{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{-40a^3\coth(c+dx)+b\left(-20b^2\text{sech}^6(c+dx)+5b^2\text{sech}^8(c+dx)+24ab\tanh(c+dx)+6b\,\text{sech}^4(c+dx)(5\right.}{40d}$$

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
[Out] (-40*a^3*Coth[c + d*x] + b*(-20*b^2*Sech[c + d*x]^6 + 5*b^2*Sech[c + d*x]^8
 + 24*a*b*Tanh[c + d*x] + 6*b*Sech[c + d*x]^4*(5*b + 4*a*Tanh[c + d*x]) - 4
*Sech[c + d*x]^2*(15*a^2 + 5*b^2 + 12*a*b*Tanh[c + d*x])))/(40*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(65) = 130.

Time = 20.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.41

method	result
derivativedivides	$\frac{-a^3\coth(dx+c)-\frac{3a^2b}{2\cosh(dx+c)^2}+3a\,b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{-a^3\coth(dx+c)-\frac{3a^2b}{2\cosh(dx+c)^2}+3a\,b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
risch	$-\frac{2\left(-3a\,b^2+5a^3\mathrm{e}^{16dx+16c}+5b^3\mathrm{e}^{16dx+16c}+35b^3\mathrm{e}^{12dx+12c}+5a^3+5\,\mathrm{e}^{4dx+4c}b^3+40a^3\mathrm{e}^{14dx+14c}-5b^3\mathrm{e}^{14dx+14c}-6\,\mathrm{e}^{2dx+2c}\right.}{}$

[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^3*coth(d*x+c)-3/2*a^2*b/cosh(d*x+c)^2+3*a*b^2*(-1/2*sinh(d*x+c)^3/c
osh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15
*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^6/cosh(d*x+c)^8-3/4*sinh
(d*x+c)^4/cosh(d*x+c)^8-1/2*sinh(d*x+c)^2/cosh(d*x+c)^8-1/8/cosh(d*x+c)^8))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 1192, normalized size of antiderivative = 16.79

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] -2/5*((10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 8*(15*a^2*b
+ 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (10*a^3 + 15*a^2*b + 12
*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*
cosh(d*x + c)^6 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3 + 14*(10*a^3 + 15
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(15*a^2
*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x +
c))*sinh(d*x + c)^5 + 20*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^4 + 10*(7
*(10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 28*a^3 + 6*a^2*b
+ 4*b^3 + 3*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(15*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 45*(5*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^3 + 15*(7*a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c))*s
inh(d*x + c)^3 + 350*a^3 - 75*a^2*b - 12*a*b^2 + 35*b^3 + 2*(280*a^3 - 30*a
^2*b - 12*a*b^2 - 35*b^3)*cosh(d*x + c)^2 + 2*(14*(10*a^3 + 15*a^2*b + 12*a
*b^2 + 5*b^3)*cosh(d*x + c)^6 + 15*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*c
osh(d*x + c)^4 + 280*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3 + 60*(14*a^3 + 3*a^
2*b + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(15*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 30*(7*a^

```
2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^3 + 21*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 d*sinh(d*x + c)^10 + 6*d*cosh(d*x + c)^8 + 3*(15*d*cosh(d*x + c)^2 + 2*d)*
sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + 8*d*cosh(d*x + c))*sinh(d*x + c
)^7 + 13*d*cosh(d*x + c)^6 + (210*d*cosh(d*x + c)^4 + 168*d*cosh(d*x + c)^2
 + 13*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x + c)^5 + 224*d*cosh(d*x + c)^3
 + 81*d*cosh(d*x + c))*sinh(d*x + c)^5 + 8*d*cosh(d*x + c)^4 + (210*d*cosh(
d*x + c)^6 + 420*d*cosh(d*x + c)^4 + 195*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x
+ c)^4 + 4*(30*d*cosh(d*x + c)^7 + 112*d*cosh(d*x + c)^5 + 135*d*cosh(d*x +
 c)^3 + 48*d*cosh(d*x + c))*sinh(d*x + c)^3 - 14*d*cosh(d*x + c)^2 + (45*d*
cosh(d*x + c)^8 + 168*d*cosh(d*x + c)^6 + 195*d*cosh(d*x + c)^4 + 48*d*cosh
(d*x + c)^2 - 14*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 32*d*cosh(d*
x + c)^7 + 81*d*cosh(d*x + c)^5 + 96*d*cosh(d*x + c)^3 + 42*d*cosh(d*x + c)
)*sinh(d*x + c) - 14*d)
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\int\left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}^2\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(65) = 130$.

Time = 0.21 (sec) , antiderivative size = 679, normalized size of antiderivative = 9.56

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=$$

$$-2\,b^3\left(\frac{e^{(-2\,dx-2\,c)}}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{(-10\,dx-10\,c)}+28\,e^{(-12\,dx-12\,c)}+\cdots}\right.$$

$$+\frac{6}{5}\,ab^2\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{\cdots}{d(5\,e^{(-2\,dx-2\,c}\cdots}\right.$$

$$+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{6\,a^2b}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
[Out] -2*b^3*(e^(-2*d*x - 2*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*
e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*
d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-6*d*x
```

```
 - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c)
+ 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e
^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-10*d*x - 10*c)/(d*(8*e
^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x
- 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*
c) + e^(-16*d*x - 16*c) + 1)) + e^(-14*d*x - 14*c)/(d*(8*e^(-2*d*x - 2*c) +
 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-1
0*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x -
 16*c) + 1))) + 6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1
)) - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(65) = 130$.

Time = 0.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.38

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\frac{2\left(\frac{5a^3}{e^{(2dx+2c)}-1} + \frac{15a^2be^{(14dx+14c)}+15ab^2e^{(14dx+14c)}+5b^3e^{(14dx+14c)}+90a^2be^{(12dx+12c)}+45ab^2e^{(12dx+12c)}+225a^2be^{(10dx+10c)}+\cdots}{\cdots}\right)}{\cdots}$$

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(14*d*x + 14*c) + 15*a*b^2*
e^(14*d*x + 14*c) + 5*b^3*e^(14*d*x + 14*c) + 90*a^2*b*e^(12*d*x + 12*c) +
45*a*b^2*e^(12*d*x + 12*c) + 225*a^2*b*e^(10*d*x + 10*c) + 75*a*b^2*e^(10*d
*x + 10*c) + 35*b^3*e^(10*d*x + 10*c) + 300*a^2*b*e^(8*d*x + 8*c) + 105*a*b
^2*e^(8*d*x + 8*c) + 225*a^2*b*e^(6*d*x + 6*c) + 93*a*b^2*e^(6*d*x + 6*c) +
 35*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 39*a*b^2*e^(4*d*x + 4*
c) + 15*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x + 2*c) + 5*b^3*e^(2*d*x +
2*c) + 3*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 1515, normalized size of antiderivative = 21.34

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
[Out] ((3*a*b^2 - 15*a^2*b + 7*b^3)/(28*d) - (exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b
 + b^3))/(4*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((9*a*b^2 + 1
5*a^2*b - 35*b^3)/(140*d) + (exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4
*d) + (3*exp(2*c + 2*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(4*c
+ 4*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d))/(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((exp(10*c + 10
*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 9*a^2*b + 7*b^3)/(28*d)
 + (5*exp(6*c + 6*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (5*exp(8*c + 8
*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d) + (exp(2*c + 2*d*x)*(9*a*b^2 - 1
5*a^2*b + 35*b^3))/(28*d) + (exp(4*c + 4*d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3)
)/(14*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) +
 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((9
*a^2*b - 3*a*b^2 + 7*b^3)/(28*d) + (exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b
^3))/(4*d) - (exp(2*c + 2*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(14*d))/(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((9*a*b^2 - 15
*a^2*b + 35*b^3)/(140*d) + (exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*
d) + (3*exp(4*c + 4*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (exp(6*c + 6
*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(7*d) + (exp(2*c + 2*d*x)*(9*a*b^2 + 15
*a^2*b - 35*b^3))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((exp(12*c
+ 12*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 15*a^2*b - 7*b^3)/(
28*d) - (3*exp(2*c + 2*d*x)*(3*a*b^2 + 9*a^2*b + 7*b^3))/(14*d) + (15*exp(8
*c + 8*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(10*c + 10*d*x)*(3*
a*b^2 - 15*a^2*b + 7*b^3))/(14*d) + (3*exp(4*c + 4*d*x)*(9*a*b^2 - 15*a^2*b
 + 35*b^3))/(28*d) + (exp(6*c + 6*d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3))/(7*d)
)/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14
*d*x) + 1) + ((3*a^2*b - 3*a*b^2 + b^3)/(4*d) - (exp(14*c + 14*d*x)*(3*a*b^
2 + 3*a^2*b + b^3))/(4*d) + (3*exp(4*c + 4*d*x)*(3*a*b^2 + 9*a^2*b + 7*b^3)
)/(4*d) + (exp(2*c + 2*d*x)*(3*a*b^2 + 15*a^2*b - 7*b^3))/(4*d) - (3*exp(10
*c + 10*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(4*d) + (exp(12*c + 12*d*x)*(3*a*
b^2 - 15*a^2*b + 7*b^3))/(4*d) - (exp(6*c + 6*d*x)*(9*a*b^2 - 15*a^2*b + 35
*b^3))/(4*d) - (exp(8*c + 8*d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3))/(4*d))/(8*e
xp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c +
8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*
x) + exp(16*c + 16*d*x) + 1) - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) - (3*a*b^
2 + 3*a^2*b + b^3)/(4*d*(exp(2*c + 2*d*x) + 1))
```

3.71 $\int \mathbf{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= \frac{3a^2b\arctan(\sinh(c+dx))}{2d}+\frac{5b^3\arctan(\sinh(c+dx))}{128d}\\&\quad+\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\\&\quad-\frac{ab^2\operatorname{sech}^3(c+dx)}{d}+\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}+\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\\&\quad+\frac{5b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}-\frac{5b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{64d}\\&\quad-\frac{5b^3\operatorname{sech}^3(c+dx)\tanh^3(c+dx)}{48d}-\frac{b^3\operatorname{sech}^3(c+dx)\tanh^5(c+dx)}{8d}\end{aligned}$$

```
[Out] 3/2*a^2*b*arctan(sinh(d*x+c))/d+5/128*b^3*arctan(sinh(d*x+c))/d+1/2*a^3*arc
tanh(cosh(d*x+c))/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d-a*b^2*sech(d*x+c)^3/d
+3/5*a*b^2*sech(d*x+c)^5/d+3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+5/128*b^3*se
ch(d*x+c)*tanh(d*x+c)/d-5/64*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-5/48*b^3*sech(
d*x+c)^3*tanh(d*x+c)^3/d-1/8*b^3*sech(d*x+c)^3*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3747, 3853, 3855, 2686, 14, 2691}

$$\begin{aligned}&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx\\&=\frac{a^3\text{arctanh}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d}\\&\quad+\frac{3a^2b\arctan(\sinh(c+dx))}{2d}+\frac{3a^2b\tanh(c+dx)\text{sech}(c+dx)}{2d}\\&\quad+\frac{3ab^2\text{sech}^5(c+dx)}{5d}-\frac{ab^2\text{sech}^3(c+dx)}{d}+\frac{5b^3\arctan(\sinh(c+dx))}{128d}\\&\quad-\frac{b^3\tanh^5(c+dx)\text{sech}^3(c+dx)}{8d}-\frac{5b^3\tanh^3(c+dx)\text{sech}^3(c+dx)}{48d}\\&\quad-\frac{5b^3\tanh(c+dx)\text{sech}^3(c+dx)}{64d}+\frac{5b^3\tanh(c+dx)\text{sech}(c+dx)}{128d}\end{aligned}$$

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (5*b^3*ArcTan[Sinh[c + d*x]])/(128*
d) + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])
/(2*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (3*a
^2*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (5*b^3*Sech[c + d*x]*Tanh[c + d*x
])/(128*d) - (5*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) - (5*b^3*Sech[c +
 d*x]^3*Tanh[c + d*x]^3)/(48*d) - (b^3*Sech[c + d*x]^3*Tanh[c + d*x]^5)/(8*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\left(i \int \left(ia^3\operatorname{csch}^3(c+dx) + 3ia^2b\operatorname{sech}^3(c+dx) + 3iab^2\operatorname{sech}^3(c+dx)\tanh^3(c+dx) + ib^3\operatorname{sech}^3(c+dx)\tanh^6(c+dx)\right)\,dx\right)$$

$$= a^3\int \operatorname{csch}^3(c+dx)\,dx + \left(3a^2b\right)\int \operatorname{sech}^3(c+dx)\,dx + \left(3ab^2\right)\int \operatorname{sech}^3(c+dx)\tanh^3(c+dx)\,dx + b^3\int \operatorname{sech}^3(c+dx)\tanh^6(c+dx)\,dx$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{b^3\operatorname{sech}^3(c+dx)\tanh^5(c+dx)}{8d} - \frac{1}{2}a^3\int \operatorname{csch}(c+dx)\,dx + \frac{1}{2}\left(3a^2b\right)\int \operatorname{sech}(c+dx)\,dx + \frac{1}{8}\left(5b^3\right)\int \operatorname{sech}^3(c+dx)\tanh^4(c+dx)\,dx + \frac{\left(3ab^2\right)\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$
\begin{aligned}
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{a^3\text{arctanh}(\cosh(c+dx))}{2d} \\
&\quad - \frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d} + \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} \\
&\quad - \frac{5b^3\text{sech}^3(c+dx)\tanh^3(c+dx)}{48d} - \frac{b^3\text{sech}^3(c+dx)\tanh^5(c+dx)}{8d} \\
&\quad + \frac{1}{16}(5b^3)\int \text{sech}^3(c+dx)\tanh^2(c+dx)\,dx \\
&\quad + \frac{(3ab^2)\,\text{Subst}\left(\int (-x^2+x^4)\ dx, x, \text{sech}(c+dx)\right)}{d} \\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{a^3\text{arctanh}(\cosh(c+dx))}{2d} \\
&\quad - \frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{ab^2\text{sech}^3(c+dx)}{d} \\
&\quad + \frac{3ab^2\text{sech}^5(c+dx)}{5d} + \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} \\
&\quad - \frac{5b^3\text{sech}^3(c+dx)\tanh(c+dx)}{64d} - \frac{5b^3\text{sech}^3(c+dx)\tanh^3(c+dx)}{48d} \\
&\quad - \frac{b^3\text{sech}^3(c+dx)\tanh^5(c+dx)}{8d} + \frac{1}{64}(5b^3)\int \text{sech}^3(c+dx)\,dx \\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{a^3\text{arctanh}(\cosh(c+dx))}{2d} \\
&\quad - \frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{ab^2\text{sech}^3(c+dx)}{d} + \frac{3ab^2\text{sech}^5(c+dx)}{5d} \\
&\quad + \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{5b^3\text{sech}(c+dx)\tanh(c+dx)}{128d} \\
&\quad - \frac{5b^3\text{sech}^3(c+dx)\tanh(c+dx)}{64d} - \frac{5b^3\text{sech}^3(c+dx)\tanh^3(c+dx)}{48d} \\
&\quad - \frac{b^3\text{sech}^3(c+dx)\tanh^5(c+dx)}{8d} + \frac{1}{128}(5b^3)\int \text{sech}(c+dx)\,dx \\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{5b^3\arctan(\sinh(c+dx))}{128d} \\
&\quad + \frac{a^3\text{arctanh}(\cosh(c+dx))}{2d} - \frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d} \\
&\quad - \frac{ab^2\text{sech}^3(c+dx)}{d} + \frac{3ab^2\text{sech}^5(c+dx)}{5d} + \frac{3a^2b\text{sech}(c+dx)\tanh(c+dx)}{2d} \\
&\quad + \frac{5b^3\text{sech}(c+dx)\tanh(c+dx)}{128d} - \frac{5b^3\text{sech}^3(c+dx)\tanh(c+dx)}{64d} \\
&\quad - \frac{5b^3\text{sech}^3(c+dx)\tanh^3(c+dx)}{48d} - \frac{b^3\text{sech}^3(c+dx)\tanh^5(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 13.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.14

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{(192a^2b+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{ab^2\operatorname{sech}^3(c+dx)}{d}+\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}+\frac{\operatorname{sech}^2(c+dx)\left(192a^2b\sinh(c+dx)+5b^3\sinh(c+dx)\right)}{128d}-\frac{59b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d}+\frac{17b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{48d}-\frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}$$

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] ((192*a^2*b + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (a^3*Csch[(c + d*x
)/2]^2)/(8*d) + (a^3*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^3*Log[Sinh[(c + d*x
)/2]])/(2*d) - (a^3*Sech[(c + d*x)/2]^2)/(8*d) - (a*b^2*Sech[c + d*x]^3)/d
+ (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*
x] + 5*b^3*Sinh[c + d*x]))/(128*d) - (59*b^3*Sech[c + d*x]^3*Tanh[c + d*x])
/(192*d) + (17*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) - (b^3*Sech[c + d*
x]^7*Tanh[c + d*x])/(8*d)
```

Maple [A] (verified)

Time = 38.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15}\right.}{}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15}\right.}{}$
risch	$-\frac{e^{dx+c}\left(8640a^3e^{16dx+16c}+2135b^3e^{16dx+16c}+19760b^3e^{12dx+12c}+960a^3+8520\,e^{4dx+4c}b^3+34560a^3e^{14dx+14c}-8520b^3e^{1}\right.}{}$

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/2*se
ch(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a*b^2*(-1/3*sinh(d*x+c)^2/cosh(
d*x+c)^5-2/15/cosh(d*x+c)^5)+b^3*(-1/3*sinh(d*x+c)^5/cosh(d*x+c)^8-1/3*sinh
(d*x+c)^3/cosh(d*x+c)^8-1/7*sinh(d*x+c)/cosh(d*x+c)^8+1/7*(1/8*sech(d*x+c)^
7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+5
/64*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10985 vs. $2(212) = 424$.

Time = 0.38 (sec) , antiderivative size = 10985, normalized size of antiderivative = 47.35

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3\operatorname{csch}^3(c+dx)\,dx$$

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(212) = 424$.

Time = 0.29 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{1}{192}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}-397\,e^{(-3\,dx-3\,c)}+895\,e^{(-5\,dx-5\,c)}-1765\,e^{(-7\,dx-7\,c)}+1765\,e^{\cdots}}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{\cdots}}\right.$$
$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-\frac{8}{5}ab^2\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{\cdots}{d(5\,e^{(-2\,dx-2\,c)}\cdots}\right.$$

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] -1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x - 3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*c) - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/5*a*b^2*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(212) = 424$.

Time = 0.56 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.84

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{480\,a^3\log\left(e^{(dx+c)}+1\right) - 480\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right) + 15\left(192\,a^2b+5\,b^3\right)\arctan\left(e^{(dx+c)}\right) - \frac{960\,\left(a^3e^{(3\,dx+3\,c)}+a^3\cdots\right.}{\left(e^{(2\,dx+2\,c)}-1\right)\cdots}}{}$$

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out] 1/960*(480*a^3*log(e^(d*x + c) + 1) - 480*a^3*log(abs(e^(d*x + c) - 1)) + 15*(192*a^2*b + 5*b^3)*arctan(e^(d*x + c)) - 960*(a^3*e^(3*d*x + 3*c) + a^3*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + (2880*a^2*b*e^(15*d*x + 15*c) + 75*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) - 7680*a*b^2*e^(13*d*x + 13*c) - 1985*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) - 19968*a*b^2*e^(11*d*x + 11*c) + 4475*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) - 21504*a*b^2*e^(9*d*x + 9*c) - 8825*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) - 21504*a*b^2*e^(7*d*x + 7*c) + 8825*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c) - 19968*a*b^2*e^(5*d*x + 5*c) - 4475*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) - 7680*a*b^2*e^(3*d*x + 3*c) + 1985*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) - 75*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.15

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\frac{a^3\,\ln\left(e^{c+d\,x}+1\right)}{2\,d}-\frac{a^3\,\ln\left(e^{c+d\,x}-1\right)}{2\,d}$$
$$+\frac{e^{c+d\,x}\,\left(192\,a^2\,b+5\,b^3\right)}{64\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(2245\,b^3+3264\,a\,b^2\right)}{120\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{e^{c+d\,x}\,\left(1325\,b^3+768\,a\,b^2\right)}{20\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{500\,b^3\,e^{c+d\,x}}{3\,d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$
$$+\frac{2\,e^{c+d\,x}\,\left(1025\,b^3+144\,a\,b^2\right)}{15\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{112\,b^3\,e^{c+d\,x}}{d\,\left(7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\,c+12\,d\,x}+e^{14\,c+14\,d\,x}+1\right)}$$
$$-\frac{e^{c+d\,x}\,\left(576\,a^2\,b+768\,a\,b^2+251\,b^3\right)}{96\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$-\frac{32\,b^3\,e^{c+d\,x}}{d\,\left(8\,e^{2\,c+2\,d\,x}+28\,e^{4\,c+4\,d\,x}+56\,e^{6\,c+6\,d\,x}+70\,e^{8\,c+8\,d\,x}+56\,e^{10\,c+10\,d\,x}+28\,e^{12\,c+12\,d\,x}+8\,e^{14\,c+14\,d\,x}+e^{16\,c}\right.}$$
$$-\frac{a^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$
$$-\frac{b\,\ln\left(e^{c+d\,x}-\mathrm{i}\right)\,\left(192\,a^2+5\,b^2\right)\,1\mathrm{i}}{128\,d}+\frac{b\,\ln\left(e^{c+d\,x}+1\mathrm{i}\right)\,\left(192\,a^2+5\,b^2\right)\,1\mathrm{i}}{128\,d}$$

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^3,x)
```

```
[Out] (a^3*log(exp(c + d*x) + 1))/(2*d) - (a^3*log(exp(c + d*x) - 1))/(2*d) + (ex
p(c + d*x)*(192*a^2*b + 5*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*
x)*(3264*a*b^2 + 2245*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 5*b^2)*1i)
/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 5*b^2)*1i)/(128*d) - (exp(c
 + d*x)*(768*a*b^2 + 1325*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d
*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (500*b^3*exp(c + d*x))/
(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*e
xp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (2*exp(
c + d*x)*(144*a*b^2 + 1025*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4
*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
 + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35
*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*
c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 576*a^2
*b + 251*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*b^3
*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c +
```

```
6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x
) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (a^3*exp(c + d*x))/(d
*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*ex
p(2*c + 2*d*x) + 1))
```

3.72 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{ab^2\tanh^3(c+dx)}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^6(c+dx)}{6d}-\frac{b^3\tanh^8(c+dx)}{8d}$$

```
[Out] a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d+3*a^2*b*ln(tanh(d*x+c))/d-3/2*a^2
*b*tanh(d*x+c)^2/d+a*b^2*tanh(d*x+c)^3/d-3/5*a*b^2*tanh(d*x+c)^5/d+1/6*b^3*
tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 1816}

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{ab^2\tanh^3(c+dx)}{d}-\frac{b^3\tanh^8(c+dx)}{8d}+\frac{b^3\tanh^6(c+dx)}{6d}$$

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a^2*b*Log[Tanh[c +
 d*x]])/d - (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (a*b^2*Tanh[c + d*x]^3)/d - (
3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^6)/(6*d) - (b^3*Tanh[c
+ d*x]^8)/(8*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^3)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^2}+\frac{3a^2b}{x}-3a^2bx+3ab^2x^2-3ab^2x^4+b^3x^5-b^3x^7\right)dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}\\
&\quad+\frac{ab^2\tanh^3(c+dx)}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^6(c+dx)}{6d}-\frac{b^3\tanh^8(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\frac{2a^3\coth(c+dx)}{3d}-\frac{a^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{3a^2b\log(\cosh(c+dx))}{d}+\frac{3a^2b\log(\sinh(c+dx))}{d}+\frac{3a^2b\operatorname{sech}^2(c+dx)}{2d}-\frac{b^3\operatorname{sech}^4(c+dx)}{4d}+\frac{b^3\operatorname{sech}^6(c+dx)}{3d}-\frac{b^3\operatorname{sech}^8(c+dx)}{8d}+\frac{2ab^2\tanh(c+dx)}{5d}+\frac{ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}-\frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (2*a^3*Coth[c + d*x])/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (
3*a^2*b*Log[Cosh[c + d*x]])/d + (3*a^2*b*Log[Sinh[c + d*x]])/d + (3*a^2*b*S
ech[c + d*x]^2)/(2*d) - (b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)
/(3*d) - (b^3*Sech[c + d*x]^8)/(8*d) + (2*a*b^2*Tanh[c + d*x])/(5*d) + (a*b
^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c +
 d*x])/(5*d)
```

Maple [A] (verified)

Time = 62.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+3a\,b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\right.}{}\right.}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+3a\,b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\right.}{}\right.}{d}$
risch	$-\frac{2\left(-6a\,b^2+90a\,b^2\mathrm{e}^{18dx+18c}+230a^3\mathrm{e}^{16dx+16c}-130b^3\mathrm{e}^{16dx+16c}-490b^3\mathrm{e}^{12dx+12c}-10a^3-30\,\mathrm{e}^{4dx+4c}b^3+760a^3\mathrm{e}^{14dx+1}\right.}{}$

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(1/2/cosh(d*x+c)^2+ln(
tanh(d*x+c)))+3*a*b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*
x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/4*sinh(d*x+c)^4/cosh(d*x+c)
^8-1/6*sinh(d*x+c)^2/cosh(d*x+c)^8-1/24/cosh(d*x+c)^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9459 vs. $2(128) = 256$.

Time = 0.35 (sec) , antiderivative size = 9459, normalized size of antiderivative = 68.54

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}^4\left(c+dx\right)dx$$

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(128) = 256$.

Time = 0.29 (sec) , antiderivative size = 997, normalized size of antiderivative = 7.22

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x
- 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
+ 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(
-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
```

```
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c
) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) - 4/3*b^3*(3*e^(-4*d*x - 4*c
)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e
^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*
d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) - 4*e^(-6*d*x - 6*c)/(d*(8*e^(-2*d*x
 - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) +
 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(
-16*d*x - 16*c) + 1)) + 10*e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(
-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x -
 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c)
+ 1)) - 4*e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) +
 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(
-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 3*e^(-1
2*d*x - 12*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x -
 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c)
 + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(128) = 256$.

Time = 0.58 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.17

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx =$$

$$-\frac{2520\,a^2b\log\left(e^{(2\,dx+2\,c)}+1\right)-2520\,a^2b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{140\,(33\,a^2be^{(6\,dx+6\,c)}-99\,a^2be^{(4\,dx+4\,c)}+24\,a^3e^{(2\,dx+2\,c}}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{}$$

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/840*(2520*a^2*b*log(e^(2*d*x + 2*c) + 1) - 2520*a^2*b*log(abs(e^(2*d*x +
 2*c) - 1)) + 140*(33*a^2*b*e^(6*d*x + 6*c) - 99*a^2*b*e^(4*d*x + 4*c) + 24
*a^3*e^(2*d*x + 2*c) + 99*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 33*a^2*b)/(e^(2*d
*x + 2*c) - 1)^3 - (6849*a^2*b*e^(16*d*x + 16*c) + 59832*a^2*b*e^(14*d*x +
14*c) + 222012*a^2*b*e^(12*d*x + 12*c) - 10080*a*b^2*e^(12*d*x + 12*c) - 33
60*b^3*e^(12*d*x + 12*c) + 459144*a^2*b*e^(10*d*x + 10*c) - 26880*a*b^2*e^(
10*d*x + 10*c) + 4480*b^3*e^(10*d*x + 10*c) + 580230*a^2*b*e^(8*d*x + 8*c)
- 23520*a*b^2*e^(8*d*x + 8*c) - 11200*b^3*e^(8*d*x + 8*c) + 459144*a^2*b*e^
(6*d*x + 6*c) - 10752*a*b^2*e^(6*d*x + 6*c) + 4480*b^3*e^(6*d*x + 6*c) + 22
2012*a^2*b*e^(4*d*x + 4*c) - 8736*a*b^2*e^(4*d*x + 4*c) - 3360*b^3*e^(4*d*x
 + 4*c) + 59832*a^2*b*e^(2*d*x + 2*c) - 5376*a*b^2*e^(2*d*x + 2*c) + 6849*a
^2*b - 672*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.68

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{96\,(10\,b^3+a\,b^2)}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}$$

$$-\frac{640\,b^3}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}$$

$$-\frac{4\,(25\,b^3+12\,a\,b^2)}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$

$$+\frac{128\,b^3}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1\right)}$$

$$-\frac{2\,(3\,a^2\,b+6\,a\,b^2+2\,b^3)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}}$$

$$-\frac{32\,b^3}{d\,\left(8\,\mathrm{e}^{2\,c+2\,d\,x}+28\,\mathrm{e}^{4\,c+4\,d\,x}+56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}+56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}+8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16\,c+}\right.}$$

$$-\frac{4\,a^3}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a^3}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$

$$+\frac{8\,(11\,b^3+15\,a\,b^2)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}+\frac{6\,a^2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

```
[Out] (96*(a*b^2 + 10*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (640*b^3)
/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*
exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (4*(12
*a*b^2 + 25*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c +
 6*d*x) + exp(8*c + 8*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*ex
p(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c +
10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (2*(6*a*b^2 + 3
*a^2*b + 2*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*atan(
(a^2*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^(1/
2))/(-d^2)^(1/2) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) +
56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(
12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (4*a^3)/
(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^3)/(3*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) + (8*(15*a*b^2 + 11*b
^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
 + (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.73 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 491

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3 d}$$
$$-\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d}+\frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d}$$
$$-\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3 d}$$
$$+\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6\left(a^2-b^2\right)^3 d}$$
$$-\frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3 d}$$
$$+\frac{1}{16(a+b)d(1-\tanh(c+dx))^2}-\frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))}$$
$$-\frac{1}{16(a-b)d(1+\tanh(c+dx))^2}+\frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))}$$

```
[Out] -3/16*a*(a-5*b)*ln(1-tanh(d*x+c))/(a+b)^3/d+3/16*a*(a+5*b)*ln(1+tanh(d*x+c)
)/(a-b)^3/d-1/3*a^(2/3)*b^(1/3)*(a^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a^2
+b^2))*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/(a^2-b^2)^3/d+1/6*a^(2/3)*b^(1/3)*(a
^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a^2+b^2))*ln(a^(2/3)-a^(1/3)*b^(1/3)*
tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^3/d-a^2*b*(a^2+2*b^2)*ln(a+b*t
anh(d*x+c)^3)/(a^2-b^2)^3/d-1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(4/3)*b^(2/3)-b^2)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(1/2))/(a^(4/3)+a^(2/
3)*b^(2/3)+b^(4/3))^3/d*3^(1/2)+1/16/(a+b)/d/(1-tanh(d*x+c))^2+1/16*(-5*a+b
```

```
)/(a+b)^2/d/(1-tanh(d*x+c))-1/16/(a-b)/d/(1+tanh(d*x+c))^2+1/16*(5*a+b)/(a-
b)^2/d/(1+tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3744, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{d\left(a^2-b^2\right)^3} - \frac{a^{2/3}\sqrt[3]{b}\left(3a^{4/3}b^{2/3}+a^2-b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d\left(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3}\right)^3} + \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+3a^{2/3}b^{4/3}(2a^2+b^2)+b^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6d\left(a^2-b^2\right)^3} - \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+3a^{2/3}b^{4/3}(2a^2+b^2)+b^4\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3d\left(a^2-b^2\right)^3} - \frac{5a-b}{16d(a+b)^2(1-\tanh(c+dx))} + \frac{5a+b}{16d(a-b)^2(\tanh(c+dx)+1)} + \frac{1}{16d(a+b)(1-\tanh(c+dx))^2} - \frac{1}{16d(a-b)(\tanh(c+dx)+1)^2} - \frac{3a(a-5b)\log(1-\tanh(c+dx))}{16d(a+b)^3} + \frac{3a(a+5b)\log(\tanh(c+dx)+1)}{16d(a-b)^3}$$

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((a^(2/3)*b^(1/3)*(a^2 + 3*a^(4/3)*b^(2/3) - b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^(4/3) + a^(2/3)*b^(2/3)
+ b^(4/3))^3*d)) - (3*a*(a - 5*b)*Log[1 - Tanh[c + d*x]])/(16*(a + b)^3*d)
+ (3*a*(a + 5*b)*Log[1 + Tanh[c + d*x]])/(16*(a - b)^3*d) - (a^(2/3)*b^(1/3
)*(a^4 + 7*a^2*b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(1/3) + b
^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^3*d) + (a^(2/3)*b^(1/3)*(a^4 + 7*a^2*
b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tanh[c + d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^3*d) - (a^2*b*(a^2
 + 2*b^2)*Log[a + b*Tanh[c + d*x]^3])/((a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(
1 - Tanh[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*d*(1 - Tanh[c + d*x])) - 1/
(16*(a - b)*d*(1 + Tanh[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*d*(1 + Tanh[
c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+b)(-1+x)^3} + \frac{-5a+b}{16(a+b)^2(-1+x)^2} - \frac{3a(a-5b)}{16(a+b)^3(-1+x)} + \frac{1}{8(a-b)(1+x)^3} + \frac{-5a-b}{16(a-b)^2(1+x)^2} + \frac{3a(a+5b}{16(a-b)^3(1}\right.\right.}{d}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{1}{16(a+b)d(1-\tanh(c+dx))^2} - \frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))} - \frac{1}{16(a-b)d(1+\tanh(c+dx))^2} + \frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))} + \frac{(ab)\text{Subst}\left(\int \frac{-3ab(2a^2+b^2)+(a^4+7a^2b^2+b^4)x-3ab(a^2+2b^2)x^2}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{(a^2-b^2)^3\,d}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{1}{16(a+b)d(1-\tanh(c+dx))^2} - \frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))} - \frac{1}{16(a-b)d(1+\tanh(c+dx))^2} + \frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))} + \frac{(ab)\text{Subst}\left(\int \frac{-3ab(2a^2+b^2)+(a^4+7a^2b^2+b^4)x}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{(a^2-b^2)^3\,d} - \frac{(3a^2b^2(a^2+2b^2))\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{(a^2-b^2)^3\,d}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{(a^2-b^2)^3\,d} + \frac{1}{16(a+b)d(1-\tanh(c+dx))^2} - \frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))} - \frac{1}{16(a-b)d(1+\tanh(c+dx))^2} + \frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))} + \frac{\left(\sqrt[3]{a}b^{2/3}\right)\text{Subst}\Bigg(\int \frac{\sqrt[3]{a}\left(-6ab^{4/3}(2a^2+b^2)+\sqrt[3]{a}(a^4+7a^2b^2+b^4)\right)+\sqrt[3]{b}\left(3ab^{4/3}(2a^2+b^2)+\sqrt[3]{a}(a^4+7a^2b^2+b^4)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\left(a^2-b^2\right)^3 d} - \frac{\left(a^{2/3}b^{2/3}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3 d}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3d} - \frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3d} + \frac{1}{16(a+b)d(1-\tanh(c+dx))^2} - \frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))} - \frac{1}{16(a-b)d(1+\tanh(c+dx))^2} + \frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))} + \frac{\left(ab^{2/3}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\right)\text{Subst}\left(\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx,x,\tanh(c+dx)\right)}{2\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3d} + \frac{\left(a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\right)\text{Subst}\left(\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx,x,\tanh(c+dx\right.}{6\left(a^2-b^2\right)^3d}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3d} + \frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+\right.}{6\left(a^2-b^2\right)^3d} - \frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3d} + \frac{1}{16(a+b)d(1-\tanh(c+dx))^2} - \frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))} - \frac{1}{16(a-b)d(1+\tanh(c+dx))^2} + \frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))} + \frac{\left(a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)}{\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3d}$$

$$= -\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3 d}$$
$$-\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d}+\frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d}$$
$$-\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3 d}$$
$$+\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+\right.}{6\left(a^2-b^2\right)^3 d}$$
$$-\frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3 d}$$
$$+\frac{1}{16(a+b)d(1-\tanh(c+dx))^2}-\frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))}$$
$$-\frac{1}{16(a-b)d(1+\tanh(c+dx))^2}+\frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.15 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{-32ab\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\frac{-6a^3c-12ab^2c-6a^3dx-12ab^2dx+3a^3\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-32*a*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b
*#1^3 & , (-6*a^3*c - 12*a*b^2*c - 6*a^3*d*x - 12*a*b^2*d*x + 3*a^3*Log[E^(
2*(c + d*x)) - #1] + 6*a*b^2*Log[E^(2*(c + d*x)) - #1] - 8*a^3*c*#1 + 4*a^2
*b*c*#1 + 8*a*b^2*c*#1 - 4*b^3*c*#1 - 8*a^3*d*x*#1 + 4*a^2*b*d*x*#1 + 8*a*b
^2*d*x*#1 - 4*b^3*d*x*#1 + 4*a^3*Log[E^(2*(c + d*x)) - #1]*#1 - 2*a^2*b*Log
[E^(2*(c + d*x)) - #1]*#1 - 4*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^3*Lo
g[E^(2*(c + d*x)) - #1]*#1 - 10*a^3*c*#1^2 + 20*a^2*b*c*#1^2 - 20*a*b^2*c*#
1^2 + 4*b^3*c*#1^2 - 10*a^3*d*x*#1^2 + 20*a^2*b*d*x*#1^2 - 20*a*b^2*d*x*#1^
2 + 4*b^3*d*x*#1^2 + 5*a^3*Log[E^(2*(c + d*x)) - #1]*#1^2 - 10*a^2*b*Log[E^
(2*(c + d*x)) - #1]*#1^2 + 10*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1^2 - 2*b^3*
```

```
Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2)
 & ] + 3*(4*b*(5*a^3 + 5*a^2*b + a*b^2 + b^3)*Cosh[2*(c + d*x)] - (a - b)*b
*(a + b)^2*Cosh[4*(c + d*x)] - 8*a*(a^3 + a^2*b + 2*a*b^2 + 2*b^3)*Sinh[2*(
c + d*x)] + a*(a - b)*(12*(a^2 - 6*a*b + 5*b^2)*(c + d*x) + (a + b)^2*Sinh[
4*(c + d*x)]))))/(96*(a - b)^2*(a + b)^3*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 18.73 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.92

method	result
derivativedivides	$-\frac{8}{(32a-32b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{32}{(64a-64b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{-a-5b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3a+3b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{3a\cdots}{}$
default	$-\frac{8}{(32a-32b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{32}{(64a-64b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{-a-5b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3a+3b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{3a\cdots}{}$
risch	Expression too large to display

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(32*a-32*b)/(1+tanh(1/2*d*x+1/2*c))^4+32/(64*a-64*b)/(1+tanh(1/2*d*
x+1/2*c))^3-1/8*(-a-5*b)/(a-b)^2/(1+tanh(1/2*d*x+1/2*c))^2-1/8*(3*a+3*b)/(a
-b)^2/(1+tanh(1/2*d*x+1/2*c))+3/8*a*(a+5*b)/(a-b)^3*ln(1+tanh(1/2*d*x+1/2*c
))+8/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)-1)^4+32/(64*a+64*b)/(tanh(1/2*d*x+1/2
*c)-1)^3-1/8*(a-5*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a-3*b)/(a+b)^
2/(tanh(1/2*d*x+1/2*c)-1)-3/8*a*(a-5*b)/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)-1
/3*a*b/(a-b)^3/(a+b)^3*sum((3*a^2*(a^2+2*b^2)*_R^5+3*a*b*(-2*a^2-b^2)*_R^4+
2*(4*a^4+13*a^2*b^2+b^4)*_R^3+12*a*b*(a^2+2*b^2)*_R^2+(a^4-8*a^2*b^2-2*b^4)
*_R+6*a^3*b+3*a*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)
-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 17123, normalized size of antiderivative = 34.87

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^4}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] -6*a^4*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) +
3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x +
4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) - 12*a^2*b^3*(i
ntegrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*
e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a
 + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d
*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) + 10*a^4*b*integrate(e^(4*
d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)
*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
+ b^5) - 20*a^3*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*
(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 20*a^2*b^3*integrate(e^(4*d*x +
 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2
```

```
*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5
) - 4*a*b^4*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*
e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a
^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 8*a^4*b*integrate(e^(2*d*x + 2*c)/((a +
 b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c)
 + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 4*a^3*b
^2*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x
+ 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5) - 8*a^2*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(
6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a -
b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 4*a*b^4*integr
ate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3
 + a*b^4 + b^5) - 1/64*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 - 24*(a^4*d*e^(4*c) -
 7*a^3*b*d*e^(4*c) + 11*a^2*b^2*d*e^(4*c) - 5*a*b^3*d*e^(4*c))*x*e^(4*d*x)
- (a^4*e^(8*c) - 2*a^2*b^2*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) + 4*(2*a^4*e^(6
*c) - 3*a^3*b*e^(6*c) - a^2*b^2*e^(6*c) + 3*a*b^3*e^(6*c) - b^4*e^(6*c))*e^
(6*d*x) - 4*(2*a^4*e^(2*c) + 7*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) + 5*a*b^3*
e^(2*c) + b^4*e^(2*c))*e^(2*d*x))*e^(-4*d*x)/(a^5*d*e^(4*c) + a^4*b*d*e^(4*
c) - 2*a^3*b^2*d*e^(4*c) - 2*a^2*b^3*d*e^(4*c) + a*b^4*d*e^(4*c) + b^5*d*e^
(4*c))
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\sinh\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^3+a}\,dx$$

[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 3313, normalized size of antiderivative = 6.75

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\text{Too large to display}$$

[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^3),x)

```
[Out] symsum(log(- root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
- 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z
- a^2*b, z, k)*((96*(a^2*b^10*d + 20*a^3*b^9*d - 89*a^4*b^8*d + 270*a^5*b^7
```

*d - 417*a^6*b^6*d + 408*a^7*b^5*d - 190*a^8*b^4*d + 58*a^9*b^3*d - 7*a^10*
b^2*d - a^2*b^10*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*
b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*
a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 52*a^3*b^9*d*exp(2*root(81*a^4*b^2
*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b
^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) +
 59*a^4*b^8*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d
^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b
^2*d*z - a^2*b, z, k))*exp(2*d*x) - 218*a^5*b^7*d*exp(2*root(81*a^4*b^2*d^3
*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d
^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 241
*a^6*b^6*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*
z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*
d*z - a^2*b, z, k))*exp(2*d*x) + 220*a^7*b^5*d*exp(2*root(81*a^4*b^2*d^3*z^
3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*
z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 298*a^
8*b^4*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z
 - a^2*b, z, k))*exp(2*d*x) + 50*a^9*b^3*d*exp(2*root(81*a^4*b^2*d^3*z^3 -
81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2
- 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - a^10*b^2*d
*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a
^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*
b, z, k))*exp(2*d*x)))/((a + b)*(a^2 - b^2)*(a*b^2 - a^2*b - a^3 + b^3)*(3*
a*b^2 + 3*a^2*b + a^3 + b^3)*(a*b^4 + a^4*b + a^5 + b^5 - 2*a^2*b^3 - 2*a^3
*b^2)) - (288*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z
 - a^2*b, z, k)*(a^7*b*d^2 - 8*a^2*b^6*d^2 + 16*a^3*b^5*d^2 - 41*a^4*b^4*d^
2 + 37*a^5*b^3*d^2 - 5*a^6*b^2*d^2 + 18*a^2*b^6*d^2*exp(2*root(81*a^4*b^2*d
^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3
*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 1
4*a^3*b^5*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d
^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b
^2*d*z - a^2*b, z, k))*exp(2*d*x) + 79*a^4*b^4*d^2*exp(2*root(81*a^4*b^2*d^
3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*
d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 81
*a^5*b^3*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^
3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^
2*d*z - a^2*b, z, k))*exp(2*d*x) - a^6*b^2*d^2*exp(2*root(81*a^4*b^2*d^3*z^
3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*
z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + a^7*b*
d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 2
7*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a
^2*b, z, k))*exp(2*d*x)))/((a + b)^2*(a - b)*(a*b^2 - a^2*b - a^3 + b^3)*(3
*a*b^2 + 3*a^2*b + a^3 + b^3))) - (32*(22*a^4*b^7 - 4*a^3*b^8 - 68*a^5*b^6

+ 85*a^6*b^5 - 56*a^7*b^4 + 10*a^8*b^3 + 2*a^9*b^2 + 6*a^3*b^8*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 10*a^4*b^7*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 54*a^5*b^6*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 101*a^6*b^5*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 56*a^7*b^4*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 12*a^8*b^3*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 4*a^9*b^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x)))/((a + b)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a*b^4 + a^4*b + a^5 + b^5 - 2*a^2*b^3 - 2*a^3*b^2)^2))*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k), k, 1, 3) + exp(4*c + 4*d*x)/(64*d*(a + b)) - exp(- 4*c - 4*d*x)/(64*d*(a - b)) + (exp(- 2*c - 2*d*x)*(2*a + b))/(16*d*(a - b)^2) - (exp(2*c + 2*d*x)*(2*a - b))/(16*d*(a + b)^2) + (3*a*x*(a + 5*b))/(8*(a - b)^3)

3.74 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\text{Int}\left(-\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] I*Unintegrable(-I*sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] I*Defer[Int][((-I)*Sinh[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\text{integral} = i\int -\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(33) = 66.

Time = 1.06 (sec) , antiderivative size = 826, normalized size of antiderivative = 35.91

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{-9a(a^2+3b^2)\cosh(c+dx)+a^3\cosh(3(c+dx))-ab^2\cosh(3(c+dx))-2ab\text{RootSum}\Big[a-b+3a\#1^2+3}{}$$

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(c
 + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4
+ a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d*x
+ 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c
 + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log[-Cosh
[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2
]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1^2 + 4*a
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^4 - 3*a*b*c
*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*d*x*#1^4 + 6
*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] + 27*a^2*b
*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c + d*x)] + b^3*Sinh[3
*(c + d*x)])/(12*(a - b)^2*(a + b)^2*d)
```

Maple [N/A] (verified)

Time = 7.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 12.57

method	result
derivativedivides	$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a+b)^2(a-b)^2}$
default	$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a+b)^2(a-b)^2}$
risch	Expression too large to display

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a*b/(a+b)^2/(a-b)^2*sum(((2*a^2+b^2)*_R^4-6*a*b*_R^3+2*(4*a^2+5*b
^2)*_R^2-6*a*b*_R+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*
x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-16/3/(tanh(1/2
*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a
+b)^2*(-a+2*b)/(tanh(1/2*d*x+1/2*c)-1)-8/(16*a-16*b)/(1+tanh(1/2*d*x+1/2*c)
)^2+16/3/(1+tanh(1/2*d*x+1/2*c))^3/(16*a-16*b)-1/2*(a+2*b)/(a-b)^2/(1+tanh(
1/2*d*x+1/2*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 4.17 (sec) , antiderivative size = 62017, normalized size of antiderivative = 2696.39

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 533, normalized size of antiderivative = 23.17

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] 1/24*(a^3 + a^2*b - a*b^2 - b^3 + (a^3*e^(6*c) - a^2*b*e^(6*c) - a*b^2*e^(6
*c) + b^3*e^(6*c))*e^(6*d*x) - 9*(a^3*e^(4*c) - 3*a^2*b*e^(4*c) + 3*a*b^2*e
^(4*c) - b^3*e^(4*c))*e^(4*d*x) - 9*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^
2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x))*e^(-3*d*x)/(a^4*d*e^(3*c) - 2*a^2*b^2*d
*e^(3*c) + b^4*d*e^(3*c)) - 1/8*integrate(16*(3*(a^3*b*e^(5*c) - a^2*b^2*e^
(5*c) + a*b^3*e^(5*c))*e^(5*d*x) + 2*(a^3*b*e^(3*c) - a*b^3*e^(3*c))*e^(3*d
*x) + 3*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*e^(d*x))/(a^5 - a^4*b - 2*a^3
*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5*e^(6*c) + a^4*b*e^(6*c) - 2*a^3*b^2*e
^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c) + b^5*e^(6*c))*e^(6*d*x) + 3*(a^
5*e^(4*c) - a^4*b*e^(4*c) - 2*a^3*b^2*e^(4*c) + 2*a^2*b^3*e^(4*c) + a*b^4*e
^(4*c) - b^5*e^(4*c))*e^(4*d*x) + 3*(a^5*e^(2*c) + a^4*b*e^(2*c) - 2*a^3*b^
2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c) + b^5*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Hanged}$$

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^3),x)
[Out] \text{Hanged}
```

3.75 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 384

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx &= \frac{a^{2/3}\sqrt[3]{b}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,(a^2-b^2)^2\,d}\\
&\quad+\frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d}-\frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d}\\
&\quad+\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\,(a^2-b^2)^2\,d}\\
&\quad-\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6\,(a^2-b^2)^2\,d}\\
&\quad+\frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3\,(a^2-b^2)^2\,d}\\
&\quad+\frac{1}{4(a+b)d(1-\tanh(c+dx))}-\frac{1}{4(a-b)d(1+\tanh(c+dx))}\end{aligned}$$

```
[Out] 1/4*(a-2*b)*ln(1-tanh(d*x+c))/(a+b)^2/d-1/4*(a+2*b)*ln(1+tanh(d*x+c))/(a-b)
^2/d+1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(a^(1/3)+b^(1/3)*t
anh(d*x+c))/(a^2-b^2)^2/d-1/6*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^2/
d+1/3*b*(2*a^2+b^2)*ln(a+b*tanh(d*x+c)^3)/(a^2-b^2)^2/d+1/3*a^(2/3)*b^(1/3)
*(a^2-3*a^(2/3)*b^(4/3)+2*b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a
^(1/3)*3^(1/2))/(a^2-b^2)^2/d*3^(1/2)+1/4/(a+b)/d/(1-tanh(d*x+c))-1/4/(a-b)
/d/(1+tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3744, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$=\frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$
$$+\frac{a^{2/3}\sqrt[3]{b}\left(-3a^{2/3}b^{4/3}+a^2+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d\left(a^2-b^2\right)^2}$$
$$-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6d\left(a^2-b^2\right)^2}$$
$$+\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$
$$+\frac{1}{4d(a+b)(1-\tanh(c+dx))}-\frac{1}{4d(a-b)(\tanh(c+dx)+1)}$$
$$+\frac{(a-2b)\log(1-\tanh(c+dx))}{4d(a+b)^2}-\frac{(a+2b)\log(\tanh(c+dx)+1)}{4d(a-b)^2}$$

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a^(2/3)*b^(1/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^2 - b^2)^2*d) + ((a - 2*
b)*Log[1 - Tanh[c + d*x]])/(4*(a + b)^2*d) - ((a + 2*b)*Log[1 + Tanh[c + d*
x]])/(4*(a - b)^2*d) + (a^(2/3)*b^(1/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*L
og[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^2*d) - (a^(2/3)*b^(1/3)
*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d
*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^2*d) + (b*(2*a^2 + b^2)*Log[
a + b*Tanh[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Tanh[c +
d*x])) - 1/(4*(a - b)*d*(1 + Tanh[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2}+\frac{a-2b}{4(a+b)^2(-1+x)}+\frac{1}{4(a-b)(1+x)^2}+\frac{-a-2b}{4(a-b)^2(1+x)}+\frac{b(3a^2b-a(a^2+2b^2)x+b(2a^2+b^2)x^2)}{(a^2-b^2)^2(a+bx^3)}\right)\right.}{d}\\
&= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d}-\frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d}\\
&\quad+\frac{1}{4(a+b)d(1-\tanh(c+dx))}-\frac{1}{4(a-b)d(1+\tanh(c+dx))}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{3a^2b-a(a^2+2b^2)x+b(2a^2+b^2)x^2}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{(a^2-b^2)^2\,d}\\
&= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d}-\frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d}\\
&\quad+\frac{1}{4(a+b)d(1-\tanh(c+dx))}-\frac{1}{4(a-b)d(1+\tanh(c+dx))}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{3a^2b-a(a^2+2b^2)x}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{(a^2-b^2)^2\,d}\\
&\quad+\frac{(b^2(2a^2+b^2))\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{(a^2-b^2)^2\,d}
\end{aligned}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d}$$
$$+ \frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$
$$+ \frac{1}{4(a+b)d(1-\tanh(c+dx))} - \frac{1}{4(a-b)d(1+\tanh(c+dx))}$$
$$+ \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{a}(6a^2b^{4/3}-a^{4/3}(a^2+2b^2))+\sqrt[3]{b}(-3a^2b^{4/3}-a^{4/3}(a^2+2b^2))x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{3a^{2/3}\left(a^2-b^2\right)^2 d}$$
$$+ \frac{\left(a^{2/3}b^{2/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \tanh(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d}$$
$$+ \frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$
$$+ \frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}$$
$$+ \frac{1}{4(a+b)d(1-\tanh(c+dx))} - \frac{1}{4(a-b)d(1+\tanh(c+dx))}$$
$$- \frac{\left(ab^{2/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{2\left(a^2-b^2\right)^2 d}$$
$$- \frac{\left(a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{6\left(a^2-b^2\right)^2 d}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3}\sqrt[3]{b}(a^2+3a^{2/3}b^{4/3}+2b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3(a^2-b^2)^2d} - \frac{a^{2/3}\sqrt[3]{b}(a^2+3a^{2/3}b^{4/3}+2b^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6(a^2-b^2)^2d} + \frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3(a^2-b^2)^2d} + \frac{1}{4(a+b)d(1-\tanh(c+dx))} - \frac{1}{4(a-b)d(1+\tanh(c+dx))} - \frac{\left(a^{2/3}\sqrt[3]{b}(a^2-3a^{2/3}b^{4/3}+2b^2)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)}{(a^2-b^2)^2d}$$

$$= \frac{a^{2/3}\sqrt[3]{b}(a^2-3a^{2/3}b^{4/3}+2b^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}(a^2-b^2)^2d} + \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3}\sqrt[3]{b}(a^2+3a^{2/3}b^{4/3}+2b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3(a^2-b^2)^2d} - \frac{a^{2/3}\sqrt[3]{b}(a^2+3a^{2/3}b^{4/3}+2b^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6(a^2-b^2)^2d} + \frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3(a^2-b^2)^2d} + \frac{1}{4(a+b)d(1-\tanh(c+dx))} - \frac{1}{4(a-b)d(1+\tanh(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{6(a^2-3ab+2b^2)(c+dx)+3b(a+b)\cosh(2(c+dx))+4b\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2\right.}{}$$

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/12*(6*(a^2 - 3*a*b + 2*b^2)*(c + d*x) + 3*b*(a + b)*Cosh[2*(c + d*x)] +
4*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1^3
 & , (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x - 2*a^2*Log[E^(2*(c + d*x))
 - #1] - b^2*Log[E^(2*(c + d*x)) - #1] + 4*a^2*c*#1 - 4*b^2*c*#1 + 4*a^2*d*
x*#1 - 4*b^2*d*x*#1 - 2*a^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^2*Log[E^(2*(
c + d*x)) - #1]*#1 + 8*a^2*c*#1^2 - 8*a*b*c*#1^2 + 2*b^2*c*#1^2 + 8*a^2*d*x
*#1^2 - 8*a*b*d*x*#1^2 + 2*b^2*d*x*#1^2 - 4*a^2*Log[E^(2*(c + d*x)) - #1]*#
1^2 + 4*a*b*Log[E^(2*(c + d*x)) - #1]*#1^2 - b^2*Log[E^(2*(c + d*x)) - #1]*
#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] - 3*a*(a + b)*Sinh[2*
(c + d*x)])/((a - b)*(a + b)^2*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.07 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{(16a-1}}{}$
default	$\frac{\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{(16a-1}}{}$
risch	$-\frac{ax}{2(a+b)^2}+\frac{xb}{(a+b)^2}+\frac{e^{2dx+2c}}{8d(a+b)}-\frac{e^{-2dx-2c}}{8(a-b)d}-\frac{4a^2b\,d^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{2b^3d^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{4a^2bc\,d^2}{a^4d^3-2a^2b^2d^3}$

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/(16*a+16*b)/(tanh(1/2*d*x+1/2*
c)-1)+1/2*(a-2*b)/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)-4/(8*a-8*b)/(1+tanh(1/2
*d*x+1/2*c))^2+8/(16*a-16*b)/(1+tanh(1/2*d*x+1/2*c))+1/2/(a-b)^2*(-a-2*b)*l
n(1+tanh(1/2*d*x+1/2*c))+1/3*b/(a-b)^2/(a+b)^2*sum((a*(2*a^2+b^2)*_R^5-3*a^
2*b*_R^4+6*a*(a^2+b^2)*_R^3+4*b*(2*a^2+b^2)*_R^2-3*a*_R*b^2+3*a^2*b)/(_R^5*
a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^
4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 10695, normalized size of antiderivative = 27.85

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^2}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] 4*a^2*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3
*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4
*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x
 + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) + 2*b^3*(integrate(((a + b)*e^(4*d*x + 4
*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*
d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b)
, x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) - 8*a
^2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*
x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^
3) + 8*a*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*
b^2 - b^3) - 2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(
a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*
b - a*b^2 - b^3) - 4*a^2*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*
```

```
c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^
3 + a^2*b - a*b^2 - b^3) + 4*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*
x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
x)/(a^3 + a^2*b - a*b^2 - b^3) - 1/8*(4*(a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) +
2*b^2*d*e^(2*c))*x*e^(2*d*x) + a^2 + 2*a*b + b^2 - (a^2*e^(4*c) - b^2*e^(4*
c))*e^(4*d*x))*e^(-2*d*x)/(a^3*d*e^(2*c) + a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c
) - b^3*d*e^(2*c))
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 2100, normalized size of antiderivative = 5.47

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^3),x)
```

```
[Out] symsum(log(root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a
^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k)*((2304*root(54*a^2*b^2
*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*
z^2 - 9*b^2*d*z + b, z, k)*(146*a^5*b^5*d^2 - 133*a^4*b^6*d^2 - 24*a^3*b^7*
d^2 - 12*a^6*b^4*d^2 + 22*a^7*b^3*d^2 + a^8*b^2*d^2 + 32*a^3*b^7*d^2*exp(2*
root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^
2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 577*a^4*b^6*d^2*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 548*a^5*b^5*d^2*
exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*
d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 70*a^6*b^4*d^
2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*
b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 68*a^7*b^3*
d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^
2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + a^8*b^2*d
^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2
*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)))/((a + b)^8
```

```
*(a - b)^2*(a^2 - 2*a*b + b^2)) + (1536*(24*a^3*b^8*d + 105*a^4*b^7*d - 156
*a^5*b^6*d + 51*a^6*b^5*d - 30*a^7*b^4*d + 6*a^8*b^3*d - 32*a^3*b^8*d*exp(2
*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z
^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) - 509*a^4*b^7*d*exp(
2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*
z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) - 350*a^5*b^6*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 64*a^6*b^5*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) - 50*a^7*b^4*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 13*a^8*b^3*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)))/((a + b)^3*(a^2
- b^2)*(a - b)*(a^2 - 2*a*b + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^2)) + (2
56*(72*a^5*b^5 - 45*a^4*b^6 - 24*a^3*b^7 - 9*a^6*b^4 + 6*a^7*b^3 + 32*a^3*b
^7*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2
*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 393*a^4*b^
6*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*
b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 86*a^5*b^5*
exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*
d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 57*a^6*b^4*ex
p(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^
2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 8*a^7*b^3*exp(2
*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z
^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)))/((a + b)^2*(a^2 -
b^2)*(a - b)*(a^2 - 2*a*b + b^2)*(2*a*b + a^2 + b^2)^2*(3*a*b^2 + 3*a^2*b +
 a^3 + b^3)))*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 5
4*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k), k, 1, 3) - (x*(a +
 2*b))/(2*(a - b)^2) + exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/
(8*d*(a - b))
```

3.76 $\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -i\mathrm{Int}\left(\frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] -I*Unintegrable(I*sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-I)*Defer[Int][(I*Sinh[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\text{integral} = -\left(i\int \frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(31) = 62$.

Time = 0.72 (sec) , antiderivative size = 409, normalized size of antiderivative = 19.48

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{6a\cosh(c+dx) + b\text{RootSum}\Big[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&, \frac{2ac+bc+2adx+bc}{}$$

[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]

[Out] (6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b *#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2] *#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*# 1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d*x*#1 ^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/ 2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a* #1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) &] - 6*b*Sinh[c + d*x])/(6*(a - b)*(a + b)*d)

Maple [N/A] (verified)

Time = 1.62 (sec) , antiderivative size = 159, normalized size of antiderivative = 7.57

method	result
derivativedivides	$\frac{-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right.}{\_R^5a+2\_R^3a+4\_R}\right.}{3(a-b)(a+b)}}{d}$
default	$\frac{-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right.}{\_R^5a+2\_R^3a+4\_R}\right.}{3(a-b)(a+b)}}{d}$
risch	Expression too large to display

[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/d*(-4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3*b/(a-b)/(a+b)*sum((_R^4*a-2*_R^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/

```
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+4/(4*a-4*b)/(1+tan
h(1/2*d*x+1/2*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.74 (sec) , antiderivative size = 40923, normalized size of antiderivative = 1948.71

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**3),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 11.90

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] 1/2*((a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-d*x)/(a^2*d*e^c - b^2*d
*e^c) + 1/2*integrate(4*((2*a*b*e^(5*c) - b^2*e^(5*c))*e^(5*d*x) + (2*a*b*e
^c + b^2*e^c)*e^(d*x))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3*e^(6*c) + a^2*b*e^
(6*c) - a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) + 3*(a^3*e^(4*c) - a^2*b*e^(
4*c) - a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 3*(a^3*e^(2*c) + a^2*b*e^(2
*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 83.41 (sec) , antiderivative size = 4474, normalized size of antiderivative = 213.05

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^3),x)
```

```
[Out] exp(- c - d*x)/(2*(a*d - b*d)) + symsum(log((81920*a^2*b^5*exp(d*x)*exp(roo
t(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a
^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^
2 - b^2, z, k)) + 221184*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 +
 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4
*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^8*d^3 - 3538944*root(218
7*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^
6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b
^2, z, k)^3*a^3*b^7*d^3 + 1990656*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*
d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 72
9*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b^6*d^3 + 3538944
*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 7
29*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^
2*z^2 - b^2, z, k)^3*a^5*b^5*d^3 - 2211840*root(2187*a^6*b^2*d^6*z^6 - 2187
*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4
*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^6*b^4*d^3
+ 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^
6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a
^2*b^2*d^2*z^2 - b^2, z, k)^5*a^3*b^9*d^5 + 15925248*root(2187*a^6*b^2*d^6*
z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a
^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^
4*b^8*d^5 - 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*
a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*
z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^5*b^7*d^5 - 31850496*root(2187*a^
```

6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6*b^6*d^5 - 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^7*b^5*d^5 + 15925248*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^8*b^4*d^5 + 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^9*b^3*d^5 + 98304*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)*a^2*b^6*d - 98304*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)*a^3*b^5*d + 24576*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)*a^4*b^4*d + 8192*a*b^6*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 368640*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^2*b^7*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) - 2285568*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^3*b^6*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) - 5013504*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^4*b^5*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) - 368640*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^5*b^4*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 8626176*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^3*b^8*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 40476

672*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^4*b^7*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 70336512*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^5*b^6*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 54411264*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^6*b^5*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 16588800*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^7*b^4*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 663552*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^8*b^3*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)))/(10*a*b^11 - 10*a^11*b - a^12 + b^12 + 44*a^2*b^10 + 110*a^3*b^9 + 165*a^4*b^8 + 132*a^5*b^7 - 132*a^7*b^5 - 165*a^8*b^4 - 110*a^9*b^3 - 44*a^10*b^2))*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k), k, 1, 6) + exp(c + d*x)/(2*(a*d + b*d))

3.77 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)} dx = i\text{Int}\left(-\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] I*Unintegrable(-I*csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)} dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)} dx$$

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
[Out] I*Defer[Int][((-I)*Csch[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\text{integral} = i\int -\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)} dx$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 331 vs. $2(31) = 62$.

Time = 0.63 (sec) , antiderivative size = 331, normalized size of antiderivative = 15.76

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{6\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3\right.}{}$$

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/6*(6*Log[Cosh[(c + d*x)/2]] - 6*Log[Sinh[(c + d*x)/2]] + b*RootSum[a - b
 + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c + d*x
 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^
4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 +
a*#1^5 + b*#1^5) & ])/(a*d)
```

Maple [N/A] (verified)

Time = 0.63 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.57

method	result
derivativedivides	$\frac{-\frac{4b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{4b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{\ln\left(e^{dx+c}+1\right)}{da}+2\left(\sum_{\_R=\text{RootOf}\left(\left(46656a^8d^6-46656a^6b^2d^6\right)\_Z^6+3888a^4b^2d^4\_Z^4-108a^2b^2d^2\_Z^2+b^2\right)}\_R\ln\right.$

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/3*b/a*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*
c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/a*ln(tanh(1/2*d*x+
1/2*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.73 (sec) , antiderivative size = 20085, normalized size of antiderivative = 956.43

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**3),x)
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 7.62

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^
2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4
*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{b\tanh\left(dx+c\right)^3+a}\,dx$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 3679, normalized size of antiderivative = 175.19

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\text{Too large to display}$$

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)),x)
[Out] symsum(log(-(1409286144*b^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8
*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 1342177
28*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^
2*b^2*d^2*z^2 - b^2, z, k)*b^7*d + 1879048192*root(729*a^6*b^2*d^6*z^6 - 72
9*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a*b^6
*d - 2818572288*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^
4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^7*d^3 - 40869298176*root(72
9*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*
z^2 - b^2, z, k)^3*a^3*b^6*d^3 + 28185722880*root(729*a^6*b^2*d^6*z^6 - 729
*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*
b^5*d^3 + 15502147584*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^4*d^3 + 18119393280*r
oot(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^
2*d^2*z^2 - b^2, z, k)^5*a^4*b^7*d^5 + 235552112640*root(729*a^6*b^2*d^6*z^
6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)
^5*a^5*b^6*d^5 + 14495514624*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 2
43*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6*b^5*d^5 - 219244
658688*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 2
7*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^7*b^4*d^5 - 48922361856*root(729*a^6*b^2
*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2
, z, k)^5*a^8*b^3*d^5 - 32614907904*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*
z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^6*b^7*d^7 -
 179381993472*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*
z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^7*b^6*d^7 - 16307453952*root(729*
```

a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^8*b^5*d^7 + 179381993472*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^9*b^4*d^7 + 48922361856*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^10*b^3*d^7 - 1912602624*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a^2*b^5*d - 100663296*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a^3*b^4*d + 738197504*a*b^5*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 268435456*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a*b^7*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 29158801408*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^2*b^6*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 29125246976*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^3*b^5*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 2113929216*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^4*b^4*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 4831838208*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^3*b^7*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 165490458624*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^4*b^6*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 283870494720*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^5*b^5*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 132573560832*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^6*b^4*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 2717908992*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^7*b^3*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 21743271936*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^5*b^7*d^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 154920812544*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^6*b^6*d^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 27

```
9944626176*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^7*b^5*d^6*exp(d*x)*exp(root(729*a^6*
b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -
b^2, z, k)) - 105998450688*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243
*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^8*b^4*d^6*exp(d*x)*e
xp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^
2*b^2*d^2*z^2 - b^2, z, k)) - 2717908992*root(729*a^6*b^2*d^6*z^6 - 729*a^8
*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^9*b^3*
d^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d
^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)))/(13*a*b^12 + 13*a^12*b + a^13 +
b^13 + 78*a^2*b^11 + 286*a^3*b^10 + 715*a^4*b^9 + 1287*a^5*b^8 + 1716*a^6*b
^7 + 1716*a^7*b^6 + 1287*a^8*b^5 + 715*a^9*b^4 + 286*a^10*b^3 + 78*a^11*b^2
))*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^
2*b^2*d^2*z^2 - b^2, z, k), k, 1, 6) + log(exp(d*x + 1/(a*d)) - 1)/(a*d) -
log(exp(d*x - 1/(a*d)) + 1)/(a*d)
```

3.78 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} - \frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}$$

```
[Out] -coth(d*x+c)/a/d+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^(4/3)/d-1/6*
b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/a^(4/
3)/d+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(1/2)
)/a^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{3744, 331, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\coth(c+dx)}{ad}$$

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*d) - Coth[c + d*x]/(a*d) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tan
h[c + d*x]])/(3*a^(4/3)*d) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c
+ d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)}{ad} - \frac{b\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \tanh(c+dx)\right)}{3a^{4/3}d}\\
&\quad - \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{3a^{4/3}d}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}\\
&\quad - \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{6a^{4/3}d}\\
&\quad - \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{2ad}
\end{aligned}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d} - \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)}{a^{4/3}d}$$

$$= \frac{\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

$$\int \frac{\text{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{3\coth(c+dx)+2b\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&, \frac{-c-dx-\log(-\text{co}}{}\right.}{}$$

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/3*(3*Coth[c + d*x] + 2*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*
b*#1^2 + a*#1^3 + b*#1^3 & , (-c - d*x - Log[-Cosh[c + d*x] - Sinh[c + d*x]
 + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1] + c*#1 + d*x*#1 + Log[-Cosh[c + d*x
] - Sinh[c + d*x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1]*#1)/(a + b + 2*a*#
1 - 2*b*#1 + a*#1^2 + b*#1^2) & ])/(a*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{1}{2a\tanh(\ldots}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{1}{2a\tanh(\ldots}}{d}$
risch	$-\frac{2}{d a\left(\mathrm{e}^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(1728a^4d^3\_Z^3-b\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\frac{288a^3d^2\_R^2}{(a+b)\left(\frac{b}{a+b}+\frac{a}{a+b}\right)}-\frac{24a^2d\_\ldots}{(a+b)\left(\frac{b}{a+b}+\ldots\right.}\right.\right.$

[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2/3*b/a*sum((_R^3-_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/2/a/tanh(1/2*d*x+1/2*c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(123) = 246.

Time = 0.28 (sec) , antiderivative size = 640, normalized size of antiderivative = 4.08

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{2\left(\sqrt{3}\cosh\left(dx+c\right)^2+2\sqrt{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{3}\sinh\left(dx+c\right)^2-\sqrt{3}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b\cos\ldots}{\ldots}\right.}{\ldots}$$

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] -1/6*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b/a)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - (sqrt(3)*a*cosh(d*x + c)^2 + 2*sqrt(3)*a*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*a*sinh(d*x + c)^2 + sqrt(3)*a)*(b/a)^(2/3) - (sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c

```
)^2 - sqrt(3)*b)*(b/a)^(1/3))/b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x + c)^4 + 4
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b
)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) - 2*(a*c
osh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a)*(
b/a)^(2/3) + 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sin
h(d*x + c)^2 + a)*(b/a)^(1/3) + a + b) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + 2*
a*(b/a)^(2/3) - 2*a*(b/a)^(1/3) + a - b) + 12)/(a*d*cosh(d*x + c)^2 + 2*a*d
*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*
x + 2*c))/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*
c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.13

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{2}{ad(e^{(2dx+2c)}-1)}$$

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.26

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{b^{1/3}\ln\left(a^{1/3}-b^{1/3}+a^{1/3}\,\mathrm{e}^{2\,c+2\,d\,x}+b^{1/3}\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{3\,a^{4/3}\,d}-\frac{2}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$
$$+\frac{b^{1/3}\ln\left(\frac{256\,b^3\,\left(19\,a^2\,b-24\,a\,b^2+6\,a^3-b^3+8\,a^3\,\mathrm{e}^{2\,c+2\,d\,x}+b^3\,\mathrm{e}^{2\,c+2\,d\,x}+70\,a\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+113\,a^2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^4\,(a+b)^6}+\frac{b^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1536\,b^3}{\cdots}\right.}{\cdots}\right.}{\cdots}$$
$$-\frac{b^{1/3}\ln\left(\frac{256\,b^3\,\left(19\,a^2\,b-24\,a\,b^2+6\,a^3-b^3+8\,a^3\,\mathrm{e}^{2\,c+2\,d\,x}+b^3\,\mathrm{e}^{2\,c+2\,d\,x}+70\,a\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+113\,a^2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^4\,(a+b)^6}-\frac{b^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1536\,b^3\,d}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)),x)
```

```
[Out] (b^(1/3)*log(a^(1/3) - b^(1/3) + a^(1/3)*exp(2*c + 2*d*x) + b^(1/3)*exp(2*c
 + 2*d*x)))/(3*a^(4/3)*d) - 2/(a*d*(exp(2*c + 2*d*x) - 1)) + (b^(1/3)*log((
256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^3*e
xp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/
(a^4*(a + b)^6) + (b^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((1536*b^3*d*(8*a^2 - 8*b
^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2*c + 2
*d*x)))/(a^2*(a + b)^6) + (768*b^(7/3)*d*((3^(1/2)*1i)/2 - 1/2)*(24*a^2*b -
 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*d*x) + 1
13*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(a + b)^6)
))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(4/3)*d) - (b^(1/3)*log((256
*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^3*exp(
2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/(a^
4*(a + b)^6) - (b^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((1536*b^3*d*(8*a^2 - 8*b^2
+ 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2*c + 2*d*
x)))/(a^2*(a + b)^6) - (768*b^(7/3)*d*((3^(1/2)*1i)/2 + 1/2)*(24*a^2*b - 19
*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*d*x) + 113*
a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(a + b)^6)))/
(3*a^(4/3)*d))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d)
```

3.79 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx=-i\text{Int}\left(\frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] -I*Unintegrable(I*csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
[Out] (-I)*Defer[Int][(I*Csch[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\text{integral}=-\left(i\int \frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 214 vs. $2(33) = 66$.

Time = 0.92 (sec) , antiderivative size = 214, normalized size of antiderivative = 9.30

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{16b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)\right.}{a+b+}\right.}{}$$

[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]

[Out] -1/24*(16*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(a + b + 2*a*#1^2 - 2*b*#1^2 + a*#1^4 + b*#1^4) &] + 3*(Csch[(c + d*x)/2]^2 - 4*Log[Cosh[(c + d*x)/2]] + 4*Log[Sinh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)

Maple [N/A] (verified)

Time = 1.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.91

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{}{8a\tan}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{}{8a\tan}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^2}+8\left(\sum_{\_R=\text{RootOf}\left(191102976d^6\_Z^6a^{10}+1728a^4b^2d^2\_Z^2+a^2b^2-b^4\right)}\_R\ln\left(e^{dx+c}+\frac{159}{a^3b}\right.\right.$

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/3*b/a*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh(1/2*d*x+1/2*c))
)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.94 (sec) , antiderivative size = 6846, normalized size of antiderivative = 297.65

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 188, normalized size of antiderivative = 8.17

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(
6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(
2*c))*e^(2*d*x)), x) - (e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c)
 - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) -
 1/2*log((e^(d*x + c) - 1)*e^(-c))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

Mupad [B] (verification not implemented)

Time = 27.78 (sec) , antiderivative size = 3643, normalized size of antiderivative = 158.39

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)),x)
```

```
[Out] exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*e
xp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsum(log((570425344*a^4*b^6*exp
(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)
) - 33554432*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z,
 k)*a*b^10*d - 553648128*a^2*b^8*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^
4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 167772160*a^3*b^7*exp(d*x)*exp(root
(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 16777216*b
^10*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4
, z, k)) + 192937984*a^5*b^5*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^
2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2617245696*root(729*a^10*d^6*z^6 + 27*a
^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^5*b^8*d^3 - 150994944*root(729*a^
10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^6*b^7*d^3 - 1384
120320*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*
a^7*b^6*d^3 + 2415919104*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b
^2 - b^4, z, k)^3*a^8*b^5*d^3 - 3498049536*root(729*a^10*d^6*z^6 + 27*a^4*b
^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^9*b^4*d^3 + 5435817984*root(729*a^10*
d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^8*b^7*d^5 + 6794772
48*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^9*
b^6*d^5 - 70665633792*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2
- b^4, z, k)^5*a^10*b^5*d^5 + 52319748096*root(729*a^10*d^6*z^6 + 27*a^4*b^
2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^11*b^4*d^5 + 12230590464*root(729*a^10
*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^12*b^3*d^5 + 32614
907904*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^7*
a^11*b^6*d^7 + 146767085568*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^
2*b^2 - b^4, z, k)^7*a^12*b^5*d^7 - 130459631616*root(729*a^10*d^6*z^6 + 27
```

*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^7*a^13*b^4*d^7 - 48922361856*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^7*a^14*b^3*d^7 + 67108864*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^2*b^9*d - 427819008*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^3*b^8*d - 822083584*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^4*b^7*d + 436207616*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^5*b^6*d + 754974720*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^6*b^5*d + 25165824*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^7*b^4*d - 25165824*a*b^9*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 234881024*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^3*b^9*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2592079872*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^4*b^8*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 2860515328*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^5*b^7*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2919235584*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^6*b^6*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 2357198848*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^7*b^5*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 528482304*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^8*b^4*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 301989888*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^6*b^8*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 9965666304*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^7*b^7*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 33671872512*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^8*b^6*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 6568280064*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^9*b^5*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 29293019136*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^10*b^4*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 679477248*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^11*b^3*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 72024588288*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^6*a^10*b^6*d^6*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 27179089920*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^6*a^11*b^5*d^6*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 96485769216*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^6*a^12*b^4*d^6*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) -

```
 2717908992*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z,
k)^6*a^13*b^3*d^6*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 +
 a^2*b^2 - b^4, z, k)))/(12*a^16*b + a^17 + a^5*b^12 + 12*a^6*b^11 + 66*a^7
*b^10 + 220*a^8*b^9 + 495*a^9*b^8 + 792*a^10*b^7 + 924*a^11*b^6 + 792*a^12*
b^5 + 495*a^13*b^4 + 220*a^14*b^3 + 66*a^15*b^2))*root(729*a^10*d^6*z^6 + 2
7*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k), k, 1, 6) - log(33554432*a*b^9 - 1
6777216*b^10 + 113246208*a^2*b^8 - 260046848*a^3*b^7 + 321126400*a^4*b^6 -
382205952*a^5*b^5 + 191102976*a^6*b^4 + 16777216*b^10*exp(-1/(2*a*d))*exp(d
*x) - 33554432*a*b^9*exp(-1/(2*a*d))*exp(d*x) - 113246208*a^2*b^8*exp(-1/(2
*a*d))*exp(d*x) + 260046848*a^3*b^7*exp(-1/(2*a*d))*exp(d*x) - 321126400*a^
4*b^6*exp(-1/(2*a*d))*exp(d*x) + 382205952*a^5*b^5*exp(-1/(2*a*d))*exp(d*x)
 - 191102976*a^6*b^4*exp(-1/(2*a*d))*exp(d*x))/(2*a*d) + log(33554432*a*b^9
 - 16777216*b^10 + 113246208*a^2*b^8 - 260046848*a^3*b^7 + 321126400*a^4*b^
6 - 382205952*a^5*b^5 + 191102976*a^6*b^4 - 16777216*b^10*exp(1/(2*a*d))*ex
p(d*x) + 33554432*a*b^9*exp(1/(2*a*d))*exp(d*x) + 113246208*a^2*b^8*exp(1/(
2*a*d))*exp(d*x) - 260046848*a^3*b^7*exp(1/(2*a*d))*exp(d*x) + 321126400*a^
4*b^6*exp(1/(2*a*d))*exp(d*x) - 382205952*a^5*b^5*exp(1/(2*a*d))*exp(d*x) +
 191102976*a^6*b^4*exp(1/(2*a*d))*exp(d*x))/(2*a*d)
```

3.80 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d}+\frac{\coth(c+dx)}{ad} -\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d} -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} +\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d} +\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}$$

```
[Out] coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d-b*ln(tanh(d*x+c))/a^2/d-1/3*b^(1/3)*l
n(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^(4/3)/d+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/a^(4/3)/d+1/3*b*ln(a+b*tanh(d*x+c)^
3)/a^2/d-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(
1/2))/a^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3744, 1848, 1885, 12, 298, 31, 648, 631, 210, 642, 266}

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} + \frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d} - \frac{b\log(\tanh(c+dx))}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{\coth(c+dx)}{ad}$$

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(4/3)*d)) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d) - (b*Lo
g[Tanh[c + d*x]])/(a^2*d) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/
(3*a^(4/3)*d) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2
/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d) + (b*Log[a + b*Tanh[c + d*x]^3])/(3*a^2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4} - \frac{1}{ax^2} - \frac{b}{a^2x} + \frac{bx(a+bx)}{a^2(a+bx^3)}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} + \frac{b\text{Subst}\left(\int \frac{x(a+bx)}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{a^2d}$$

$$= \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} + \frac{b\text{Subst}\left(\int \frac{ax}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{a^2d} + \frac{b^2\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{a^2d}$$

$$= \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} + \frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d} + \frac{b\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{ad}$$

$$= \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} + \frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d} - \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \tanh(c+dx)\right)}{3a^{4/3}d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{3a^{4/3}d}$$

$$= \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} + \frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d} + \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{6a^{4/3}d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d} + \frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d} + \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)}{a^{4/3}d}$$

$$= -\frac{\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} + \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{b\log(\tanh(c+dx))}{a^2d} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d} + \frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$= \frac{-a\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b(c+dx-\log(\sinh(c+dx)))+b\text{RootSum}\left[a-b+3a\#1+3\right.}{}$$

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-(a*Coth[c + d*x]*(-2 + Csch[c + d*x]^2)) + 3*b*(c + d*x - Log[Sinh[c + d*
x]]) + b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b
*#1^3 & , (-2*a*c + 2*b*c - 2*a*d*x + 2*b*d*x + a*Log[E^(2*(c + d*x)) - #1]
 - b*Log[E^(2*(c + d*x)) - #1] - 8*a*c*#1 - 4*b*c*#1 - 8*a*d*x*#1 - 4*b*d*x
*#1 + 4*a*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b*Log[E^(2*(c + d*x)) - #1]*#1 +
 2*a*c*#1^2 + 2*b*c*#1^2 + 2*a*d*x*#1^2 + 2*b*d*x*#1^2 - a*Log[E^(2*(c + d*
x)) - #1]*#1^2 - b*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1
 + a*#1^2 - b*#1^2) & ])/(3*a^2*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^5a+4\_R^2b+3\_Ra\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a^2}-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}}{}}{d}$
default	$\frac{\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^5a+4\_R^2b+3\_Ra\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a^2}-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}}{}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3da\left(\mathrm{e}^{2dx+2c}-1\right)^3}+16\left(\sum_{\_R=\mathrm{RootOf}\left(110592a^6d^3\_Z^3-6912a^4b\,d^2\_Z^2+144a^2b^2d\_Z+a^2b-b^3\right)}\_R\ln\left(\mathrm{e}^{2dx+}\right.\right.$

[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/3/a^2*b*sum((_R^5*a+4*_R^2*b+3*_R*a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/24/a/tanh(1/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 3726, normalized size of antiderivative = 17.33

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] -1/12*(48*a*cosh(d*x + c)^2 + 2*(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1

/3) - 2*b/(a^2*d))^2*a^4*d^2 - (a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d
))*d + (a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - 3*a*b + 2*b^2) + 96*a*cosh(d*x +
c)*sinh(d*x + c) + 48*a*sinh(d*x + c)^2 - (6*b*cosh(d*x + c)^6 + 36*b*cosh(
d*x + c)*sinh(d*x + c)^5 + 6*b*sinh(d*x + c)^6 - 18*b*cosh(d*x + c)^4 + 18*
(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 24*(5*b*cosh(d*x + c)^3 - 3*b*c
osh(d*x + c))*sinh(d*x + c)^3 + 18*b*cosh(d*x + c)^2 + 18*(5*b*cosh(d*x + c
)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + (a^2*d*cosh(d*x + c)^6 + 6
*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh
(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)
*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh
(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2
+ a^2*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)
^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a
^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + 3
sqrt(1/3)(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 +
a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2
+ 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x
+ c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(
d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x + c)^2 + 6*(a^2*d*co
sh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*
b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 + 4*((1/2)^(1/3)*(I*sqrt
(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*
b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2)) + 36*(b*cosh(d*x + c)^5 - 2*b*cosh(d
*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - 6*b)*log(-1/4*((1/2)^(1/3)*(I*
sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3)
- 2*b/(a^2*d))^2*a^4*d^2 + 1/2*(a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d
))*d + (a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - b^2 + 3/4*sqrt(1/3)*(((1/2)^(1/3)
*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1
/3) - 2*b/(a^2*d))*a^4*d^2 + 2*(a^3 + a^2*b)*d)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*
b/(a^2*d))^2*a^4*d^2 + 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a
^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a
^4*d^2))) - (6*b*cosh(d*x + c)^6 + 36*b*cosh(d*x + c)*sinh(d*x + c)^5 + 6*b
*sinh(d*x + c)^6 - 18*b*cosh(d*x + c)^4 + 18*(5*b*cosh(d*x + c)^2 - b)*sinh
(d*x + c)^4 + 24*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3
+ 18*b*cosh(d*x + c)^2 + 18*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + (a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x
+ c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 + 4*(5*a^2*d*
cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d + 3*(5*a^2

```
*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x + c)^2 + 6*(
a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x + c))*sinh
(d*x + c))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2
*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) - 3*sqrt(1/3)*(a^2*d*cosh(d*x + c
)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2
*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 -
 a^2*d)*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c
))*sinh(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x
+ c)^2 + a^2*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d
*x + c)^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*
b/(a^2*d))^2*a^4*d^2 + 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a
^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a
^4*d^2)) + 36*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*s
inh(d*x + c) - 6*b)*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^
3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 + 1/2
*(a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3)
- (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*d + (a^2 + a*b)*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c
)^2 + a^2 - b^2 - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
- b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*a^4*d^2 + 2
*(a^3 + a^2*b)*d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a
^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 + 4*((1/2
)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d
^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))) + 12*(b*cosh(d*x + c
)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x
+ c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)
^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh
(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)
^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(2*sinh(d
*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - 16*a)/(a^2*d*cosh(d*x + c)^6 + 6
*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh
(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)
*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh
(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2
+ a^2*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)
^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^4}{b\tanh(dx+c)^3+a}\,dx$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
[Out] 2*a*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a
 + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c
) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (d*x + c)/((a^3
- a^2*b)*d)) - 2*b^2*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d
*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (
d*x + c)/((a^3 - a^2*b)*d)) + 2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d
*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
 x)/a + 2*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b
)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a^2 - 8*b*integr
ate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a - 4*b^2*integrate(e^(2*d*x + 2*c)/
((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x +
 2*c) + a - b), x)/a^2 + 2/3*(3*b*d*x*e^(6*d*x + 6*c) - 9*b*d*x*e^(4*d*x +
4*c) - 3*b*d*x + 3*(3*b*d*x*e^(2*c) - 2*a*e^(2*c))*e^(2*d*x) + 2*a)/(a^2*d*
e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d
) - b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*x + c) - 1)*e^(-c
))/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\frac{2\,b\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2}-\frac{6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2}+\frac{11\,be^{(6\,dx+6\,c)}\cdots}{}}{6\,d}$$

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c
) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b
))/a^2 - 6*b*log(abs(e^(2*d*x + 2*c) - 1))/a^2 + (11*b*e^(6*d*x + 6*c) - 33
*b*e^(4*d*x + 4*c) - 24*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + 8*a - 11
*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 4563, normalized size of antiderivative = 21.22

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)),x)
```

```
[Out] 8/(3*(a*d - 3*a*d*exp(2*c + 2*d*x) + 3*a*d*exp(4*c + 4*d*x) - a*d*exp(6*c +
 6*d*x))) - 4/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsu
m(log((1507328*a*b^9 + 1572864*b^10 - 5242880*a^2*b^8 - 7479296*a^3*b^7 + 3
948544*a^4*b^6 + 5963776*a^5*b^5 - 278528*a^6*b^4 + 8192*a^7*b^3 - 1572864*
b^10*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
 b^3, z, k))*exp(2*d*x) - 1769472*a*b^9*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*
b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 42467328*root(
27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^
4*b^8*d^2 + 21626880*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z
 + a^2*b - b^3, z, k)^2*a^5*b^7*d^2 - 70189056*root(27*a^6*d^3*z^3 - 27*a^4
*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^6*b^6*d^2 + 18038784*ro
ot(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2
*a^7*b^5*d^2 - 11993088*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*
d*z + a^2*b - b^3, z, k)^2*a^8*b^4*d^2 + 147456*root(27*a^6*d^3*z^3 - 27*a^
4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^9*b^3*d^2 - 98304*root
(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a
^10*b^2*d^2 - 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d
```

*z + a^2*b - b^3, z, k)^3*a^6*b^7*d^3 - 12091392*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^7*b^6*d^3 + 22708224*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^8*b^5*d^3 + 12386304*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^9*b^4*d^3 + 19759104*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^10*b^3*d^3 - 294912*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^11*b^2*d^3 - 14155776*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^2*b^9*d - 10387456*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^3*b^8*d + 32407552*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^4*b^7*d + 16187392*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^5*b^6*d - 29818880*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^6*b^5*d + 6135808*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^7*b^4*d - 376832*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^8*b^3*d + 8192*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^9*b^2*d - 3571712*a^2*b^8*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 30990336*a^3*b^7*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 43139072*a^4*b^6*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 8519680*a^5*b^5*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 245760*a^6*b^4*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 8192*a^7*b^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^4*b^8*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 22413312*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^5*b^7*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 54853632*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^6*b^6*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 67977216*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^7*b^5*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 60014592*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^8*b^4*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 2211840*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^9*b^3*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 147456*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^10*b^2*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 42467328*root(27*a^6*d^3*z^3

 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^6*b^7*d^3*exp(
2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z,
k))*exp(2*d*x) + 9732096*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2
*d*z + a^2*b - b^3, z, k)^3*a^7*b^6*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*
b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 85377024*root(
27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^
8*b^5*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^
2*b - b^3, z, k))*exp(2*d*x) + 246398976*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2
*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^9*b^4*d^3*exp(2*root(27*a^6*d
^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x)
+ 12828672*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
 b^3, z, k)^3*a^10*b^3*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9
*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 442368*root(27*a^6*d^3*z^3
- 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^11*b^2*d^3*exp(
2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z,
k))*exp(2*d*x) + 14155776*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^
2*d*z + a^2*b - b^3, z, k)*a^2*b^9*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d
^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 11698176*root(27*
a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^3*b^8
*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b
^3, z, k))*exp(2*d*x) + 6111232*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*
a^2*b^2*d*z + a^2*b - b^3, z, k)*a^4*b^7*d*exp(2*root(27*a^6*d^3*z^3 - 27*a
^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 165445632*r
oot(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*
a^5*b^6*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^
2*b - b^3, z, k))*exp(2*d*x) - 27688960*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*
z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^6*b^5*d*exp(2*root(27*a^6*d^3*z^
3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 105
59488*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3,
 z, k)*a^7*b^4*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d
*z + a^2*b - b^3, z, k))*exp(2*d*x) - 393216*root(27*a^6*d^3*z^3 - 27*a^4*b
*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^8*b^3*d*exp(2*root(27*a^6*d
^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x)
+ 8192*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3
, z, k)*a^9*b^2*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*
d*z + a^2*b - b^3, z, k))*exp(2*d*x))/(24*a^14*b + 3*a^15 + 3*a^7*b^8 + 24*
a^8*b^7 + 84*a^9*b^6 + 168*a^10*b^5 + 210*a^11*b^4 + 168*a^12*b^3 + 84*a^13
*b^2))*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3
, z, k), k, 1, 3) - (b*log(45613056*a*b^9 + 100663296*b^10 - 130547712*a^2*
b^8 - 18014208*a^3*b^7 + 2015232*a^4*b^6 + 270336*a^5*b^5 - 100663296*b^10*
exp(2*d*x)*exp(-(2*b)/(a^2*d)) + 130547712*a^2*b^8*exp(2*d*x)*exp(-(2*b)/(a
^2*d)) + 18014208*a^3*b^7*exp(2*d*x)*exp(-(2*b)/(a^2*d)) - 2015232*a^4*b^6*
exp(2*d*x)*exp(-(2*b)/(a^2*d)) - 270336*a^5*b^5*exp(2*d*x)*exp(-(2*b)/(a^2*
d)) - 45613056*a*b^9*exp(2*d*x)*exp(-(2*b)/(a^2*d))))/(a^2*d)

3.81 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{1}{8}(3a-b)x+\frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

```
[Out] 1/8*(3*a-b)*x+1/8*(3*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)^3
*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3756, 393, 205, 212}

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{(3a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a-b)$$

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a - b)*x)/8 + ((3*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)*
Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(3a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}\\
&= \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}\\
&\quad + \frac{(3a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}\\
&= \frac{1}{8}(3a-b)x + \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-4bdx+12a(c+dx)+8a\sinh(2(c+dx))+(a+b)\sinh(4(c+dx))}{32d}$$

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
[Out] (-4*b*d*x + 12*a*(c + d*x) + 8*a*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d*
x)])/(32*d)
```

Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
default	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
risch	$\frac{3ax}{8}-\frac{bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}+\frac{a\,e^{2dx+2c}}{8d}-\frac{a\,e^{-2dx-2c}}{8d}-\frac{e^{-4dx-4c}a}{64d}-\frac{e^{-4dx-4c}b}{64d}$

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\cosh(dx+c)\sinh(dx+c)^3+(3a-b)dx+\left((a+b)\cosh(dx+c)^3+4a\cosh(dx+c)\right)\sinh(dx+}{8d}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/8*((a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a - b)*d*x + ((a + b)*cosh(
d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)\cosh^4\left(c+dx\right)dx$$

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\begin{aligned}\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\ =\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)\\ -\frac{1}{64}\,b\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-
4*d*x - 4*c)/d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\ =\frac{8\,(dx+c)(3\,a-b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}-6\,be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}+\right.}{64\,d}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/64*(8*(d*x + c)*(3*a - b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 8*a*e
^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x
 + 2*c) + a + b)*e^(-4*d*x - 4*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{3a}{8}-\frac{b}{8}\right)-\frac{e^{-4c-4dx}\,(a+b)}{64\,d}+\frac{e^{4c+4dx}\,(a+b)}{64\,d}-\frac{a\,e^{-2c-2dx}}{8\,d}+\frac{a\,e^{2c+2dx}}{8\,d}$$

```
[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 - b/8) - (exp(- 4*c - 4*d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x)
*(a + b))/(64*d) - (a*exp(- 2*c - 2*d*x))/(8*d) + (a*exp(2*c + 2*d*x))/(8*d
)
```

3.82 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}$$

```
[Out] a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3757}

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}$$

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d)
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int \left(a+(a+b)x^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\&=\frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)\, dx = \frac{a\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}+\frac{b\sinh^3(c+dx)}{3d}$$

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{b\sinh(dx+c)^3}{3}}{d}$	37
default	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{b\sinh(dx+c)^3}{3}}{d}$	37
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}+\frac{3\,e^{dx+c}a}{8d}-\frac{e^{dx+c}b}{8d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{8d}-\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}$	116

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+1/3*b*sinh(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)\, dx = \frac{(a+b)\sinh\left(dx+c\right)^3+3\left((a+b)\cosh\left(dx+c\right)^2+3\,a-b\right)\sinh\left(dx+c\right)}{12\,d}$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/12*((a + b)*sinh(d*x + c)^3 + 3*((a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(
d*x + c))/d
```

Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)\cosh^3{(c+dx)}\,dx$$

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(28) = 56$.

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{24\,d}+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/24*b*(e^(d*x + c) - e^(-d*x - c))^3/d + 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(
d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(28) = 56$.

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.80

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{ae^{(3\,dx+3\,c)}+be^{(3\,dx+3\,c)}+9\,ae^{(dx+c)}-3\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/24*(a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) - 3*b*e^(d*x
+ c) - (9*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x - 3*c)
)/d
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{e^{3c+3dx}\,(a+b)}{24\,d}-\frac{e^{-3c-3dx}\,(a+b)}{24\,d}+\frac{e^{c+dx}\,(3\,a-b)}{8\,d}-\frac{e^{-c-dx}\,(3\,a-b)}{8\,d}$$

```
[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b))/(24*d) - (exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (
exp(c + d*x)*(3*a - b))/(8*d) - (exp(- c - d*x)*(3*a - b))/(8*d)
```

3.83 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}(a-b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] 1/2*(a-b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3756, 393, 212}

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{1}{2}x(a-b)$$

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] ((a - b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{1}{2}(a-b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2(a-b)(c+dx)+(a+b)\sinh(2(c+dx))}{4d}$$

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] (2*(a - b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

method	result	size
derivativedivides	$\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
default	$\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
risch	$\frac{ax}{2}-\frac{bx}{2}+\frac{a\,\mathrm{e}^{2dx+2c}}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{a\,\mathrm{e}^{-2dx-2c}}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}$	70

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a*(1/2*cosh(d*x+c)*sinh(
d*x+c)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-b)dx+(a+b)\cosh(dx+c)\sinh(dx+c)}{2\,d}$$

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*d*x + (a + b)*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2(c+dx)\right)\cosh^2(c+dx)\,dx$$

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*b*(4*x - e^(2*d*
x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4\,(dx+c)(a-b)+ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/8*(4*(d*x + c)*(a - b) + a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - (2*a*e^(
2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c))/d
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{\sinh(2\,c+2\,d\,x)\,\left(a+b\right)}{4\,d}$$

```
[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
[Out] x*(a/2 - b/2) + (sinh(2*c + 2*d*x)*(a + b))/(4*d)
```

3.84 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{b\arctan(\sinh(c+dx))}{d}+\frac{(a+b)\sinh(c+dx)}{d}$$

```
[Out] -b*arctan(sinh(d*x+c))/d+(a+b)*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3757, 396, 209}

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\sinh(c+dx)}{d}-\frac{b\arctan(\sinh(c+dx))}{d}$$

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + ((a + b)*Sinh[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{(a+b)\sinh(c+dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= -\frac{b\arctan(\sinh(c+dx))}{d} + \frac{(a+b)\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{b\arctan(\sinh(c+dx))}{d} + \frac{a\cosh(dx)\sinh(c)}{d} + \frac{a\cosh(c)\sinh(dx)}{d} + \frac{b\sinh(c+dx)}{d}$$

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[
d*x])/d + (b*Sinh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{a\sinh(dx+c)+b\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)}{d}$	32
default	$\frac{a\sinh(dx+c)+b\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)}{d}$	32
risch	$\frac{e^{dx+c}a}{2d}+\frac{e^{dx+c}b}{2d}-\frac{e^{-dx-c}a}{2d}-\frac{e^{-dx-c}b}{2d}+\frac{ib\ln\left(e^{dx+c}-i\right)}{d}-\frac{ib\ln\left(e^{dx+c}+i\right)}{d}$	90

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*sinh(d*x+c)+b*(sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\cosh\left(dx+c\right)^2+2\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a+b\right)\sinh\left(dx+c\right)^2-4\left(b\cosh\left(dx+c\right)+\right.}{2\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a +
 b)*sinh(d*x + c)^2 - 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x
 + c) + sinh(d*x + c)) - a - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\cosh\left(c+dx\right)dx$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right) + \frac{a\sinh\left(dx+c\right)}{d}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*sinh(
d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = -\frac{4\, b\arctan\left(e^{(dx+c)}\right) - ae^{(dx+c)} - be^{(dx+c)} + (a+b)e^{(-dx-c)}}{2\, d}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(4*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) + (a + b)*e^(
-d*x - c))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{e^{c+d\,x}\,(a+b)}{2\,d} - \frac{2\,\mathrm{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}} - \frac{e^{-c-d\,x}\,(a+b)}{2\,d}$$

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(c + d*x)*(a + b))/(2*d) - (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(
b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) - (exp(- c - d*x)*(a + b))/(2*d)
```

3.85 $\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(2a+b)\arctan(\sinh(c+dx))}{2d}-\frac{b\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c))/d-1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3757, 393, 209}

$$\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(2a+b)\arctan(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} \\ &= -\frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{(2a+b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d} \\ &= \frac{(2a+b)\arctan(\sinh(c+dx))}{2d} - \frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\arctan(\sinh(c+dx))}{d} + \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{2a\arctan\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}}+\frac{\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	56
default	$\frac{2a\arctan\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}}+\frac{\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	56
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^{2}}+\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)a}{d}+\frac{ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{2d}-\frac{i\ln\left(\mathrm{e}^{dx+c}-i\right)a}{d}-\frac{ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{2d}$	106

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a*arctan(exp(d*x+c))+b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c)*t
anh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 8.08

$$\int \mathrm{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+b\sinh\left(dx+c\right)^{3}-\left(\left(2\,a+b\right)\cosh\left(dx+c\right)^{4}+4\,(2\,a-\right.}{}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 - ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*c
osh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 +
(2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + si
nh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(
d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*c
osh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \mathrm{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)\mathrm{sech}\left(c+dx\right)dx$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \mathrm{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \mathrm{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(2\,a+b)-\frac{4\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) - 4*
b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.12

$$\int \operatorname{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(2\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^2+4\,a\,b+b^2}}\right)\,\sqrt{4\,a^2+4\,a\,b+b^2}}{\sqrt{d^2}} - \frac{b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{2\,b\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x),x)
[Out] (atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^2
 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(c + d*x))
/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1))
```

3.86 $\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3756}

$$\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a + bx^2\right) \, dx, x, \tanh(c + dx)\right)}{d}$$
$$= \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(c + dx) \left(a + b \tanh^2(c + dx)\right) \, dx = \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b \tanh(dx+c)^3}{3}+a \tanh(dx+c)}{d}$	25
default	$\frac{\frac{b \tanh(dx+c)^3}{3}+a \tanh(dx+c)}{d}$	25
risch	$-\frac{2\left(3a\, e^{4dx+4c}+3b\, e^{4dx+4c}+6\, e^{2dx+2c}a+3a+b\right)}{3d\left(e^{2dx+2c}+1\right)^3}$	60

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*b*tanh(d*x+c)^3+a*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.68

$$\int \operatorname{sech}^2(c + dx) \left(a + b \tanh^2(c + dx)\right) \, dx =$$
$$-\frac{4 \left((3\,a + 2\,b) \cosh\left(dx + c\right)^2 + 2\,b \cosh\left(dx + c\right) \sinh\left(a\right.\right.}{3 \left(d \cosh\left(dx + c\right)^4 + 4\,d \cosh\left(dx + c\right) \sinh\left(dx + c\right)^3 + d \sinh\left(dx + c\right)^4 + 4\,d \cosh\left(dx + c\right)^2 + 2 \left(3\,d \cosh\right.\right.}$$

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((3*a + 2*b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a
+ 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + 3*d)
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}^2\left(c+dx\right)dx$$

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{b\tanh\left(dx+c\right)^3}{3\,d}+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}+1)}$$

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*b*tanh(d*x + c)^3/d + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{2\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a+b\right)}{3\,d(e^{(2\,dx+2\,c)}+1)^3}$$

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -2/3*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*a
 + b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\left(3\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,a\,\mathrm{e}^{4\,c+4\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

```
[Out] -(2*(3*a + b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 3*b*exp(4*c +
4*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

3.87 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(4a+b)\arctan(\sinh(c+dx))}{8d}+\frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}-\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+b)*arctan(sinh(d*x+c))/d+1/8*(4*a+b)*sech(d*x+c)*tanh(d*x+c)/d-1/4
*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3757, 393, 205, 209}

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(4a+b)\arctan(\sinh(c+dx))}{8d}+\frac{(4a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((4*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) - (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b\text{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(4a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4d} \\
&= \frac{(4a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\text{sech}^3(c+dx)\tanh(c+dx)}{4d} \\
&\quad + \frac{(4a+b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{8d} \\
&= \frac{(4a+b)\arctan(\sinh(c+dx))}{8d} + \frac{(4a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d} \\
&\quad - \frac{b\text{sech}^3(c+dx)\tanh(c+dx)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \operatorname{sech}^3(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{a\arctan(\sinh(c+dx))}{2d} + \frac{b\arctan(\sinh(c+dx))}{8d} + \frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*Sech
[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d) - (b
*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+c}\right.}{4}\right.}{d}$
default	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+c}\right.}{4}\right.}{d}$
risch	$\frac{e^{dx+c}\left(4a\,e^{6dx+6c}+b\,e^{6dx+6c}+4a\,e^{4dx+4c}-7b\,e^{4dx+4c}-4\,e^{2dx+2c}a+7b\,e^{2dx+2c}-4a-b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2d}+\frac{ib\ln(e^{d}}{8}$

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b*(-1/3/cosh(d*x+c)
^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+1/4*arct
an(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 1046, normalized size of antiderivative = 15.85

$$\int \operatorname{sech}^3(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((4*a + b)*cosh(d*x + c)^7 + 7*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
+ (4*a + b)*sinh(d*x + c)^7 + (4*a - 7*b)*cosh(d*x + c)^5 + (21*(4*a + b)*c
osh(d*x + c)^2 + 4*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(4*a + b)*cosh(d*x + c)^
3 + (4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a - 7*b)*cosh(d*x + c)^
3 + (35*(4*a + b)*cosh(d*x + c)^4 + 10*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a +
7*b)*sinh(d*x + c)^3 + (21*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a - 7*b)*cosh(
d*x + c)^3 - 3*(4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a + b)*cosh
(d*x + c)^8 + 8*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + b)*sinh(d*
x + c)^8 + 4*(4*a + b)*cosh(d*x + c)^6 + 4*(7*(4*a + b)*cosh(d*x + c)^2 + 4
*a + b)*sinh(d*x + c)^6 + 8*(7*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh
(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + b)*cosh(d*x + c)^4 + 2*(35*(4*a + b)*
cosh(d*x + c)^4 + 30*(4*a + b)*cosh(d*x + c)^2 + 12*a + 3*b)*sinh(d*x + c)^
4 + 8*(7*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a
+ b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x + c)^2 + 4*(7*(4
*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(4*a + b)*cosh(d
*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*x + c)^7 + 3*(4*
a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a
+ b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4*a - 7*b)*cosh(d*x
+ c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c))/(d*cosh(d*
x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh
(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*co
sh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x
+ c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int\left(a+b\tanh^2(c+dx)\right)\operatorname{sech}^3(c+dx)\,dx$$

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.74

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=-\frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -1/4*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(
-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*
c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d*x - c))
/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(4\,a+b)+\frac{4\left(4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+16\,a\left(e^{(dx+c)}-e^{(\cdots}\right.\right.}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16\,d}$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + b) + 4
*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 + 1
6*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.24

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(4\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2+8\,a\,b+b^2}}\right)\,\sqrt{16\,a^2+8\,a\,b+b^2}}{4\,\sqrt{d^2}}-\frac{\frac{e^{5\,c+5\,d\,x}\,(a+b)}{d}+\frac{2\,e^{3\,c+3\,d\,x}\,(a-b)}{d}+\frac{e^{c+d\,x}\,(a+b)}{d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}-\frac{e^{c+d\,x}\,(2\,a+3\,b)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,(4\,a+b)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{2\,b\,e^{c+d\,x}}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(8*a*b + 16*a^
2 + b^2)^(1/2)))*(8*a*b + 16*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((exp(5*c
+ 5*d*x)*(a + b))/d + (2*exp(3*c + 3*d*x)*(a - b))/d + (exp(c + d*x)*(a + b
))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a + 3*b))/(2*d*(2*exp(2*c + 2*d*x) + ex
p(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(4*a + b))/(4*d*(exp(2*c + 2*d*x) + 1)
) + (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1))
```

3.88 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3756, 380}

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)
/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (1-x^2)\left(a+bx^2\right)\, dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(a-(a-b)x^2-bx^4\right)\, dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{a\tanh(c+dx)}{d} - \frac{(a-b)\tanh^3(c+dx)}{3d} - \frac{b\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\, dx = \frac{a\tanh(c+dx)}{d} + \frac{2b\tanh(c+dx)}{15d} + \frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d} - \frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d} - \frac{a\tanh^3(c+dx)}{3d}$$

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[
c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (a*Tanh[c + d*
x]^3)/(3*d)
```

Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\tanh(dx+c)^5 b}{5}+\frac{(a-b)\tanh(dx+c)^3}{3}-a\tanh(dx+c)}{d}$	42
default	$-\frac{\frac{\tanh(dx+c)^5 b}{5}+\frac{(a-b)\tanh(dx+c)^3}{3}-a\tanh(dx+c)}{d}$	42
risch	$-\frac{4\left(15a\,e^{6dx+6c}+15b\,e^{6dx+6c}+35a\,e^{4dx+4c}-5b\,e^{4dx+4c}+25\,e^{2dx+2c}a+5b\,e^{2dx+2c}+5a+b\right)}{15d\left(e^{2dx+2c}+1\right)^5}$	96

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/5*tanh(d*x+c)^5*b+1/3*(a-b)*tanh(d*x+c)^3-a*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 345, normalized size of antiderivative = 7.19

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{}{15\left(d\cosh(dx+c)^7+7\,d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+5\,d\cosh(dx+c)^5+\left(21\,d\,\mathrm{co}\right.\right.}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(5*a + 4*b)*cosh(d*x + c)^3 + 6*(5*a + 4*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + (5*a + 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) + (3*(5*a + 7*b)
*cosh(d*x + c)^2 + 5*a - 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(
d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d*
cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cosh(
d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 +
50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50*d*
cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x + c)
+ (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)
*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}^4\left(c+dx\right)dx$$

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(44) = 88$.

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.73

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4}{15}\,b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c)}+1}\right.$$
$$+\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-
4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)}+15\,be^{(6\,dx+6\,c)}+35\,ae^{(4\,dx+4\,c)}-5\,be^{(4\,dx+4\,c)}+25\,ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+5\,a+b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*a*e^(6*d*x + 6*c) + 15*b*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) -
 5*b*e^(4*d*x + 4*c) + 25*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 5*a + b
)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 6.33

$$\int \operatorname{sech}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{\frac{8\,(a-b)}{15\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(a+b)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} - \frac{\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(a+b)}{5\,d}+\frac{16\,e^{4\,c+4\,d\,x}\,(a-b)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1} - \frac{\frac{2\,(a+b)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a+b)}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a-b)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1} - \frac{2\,(a+b)}{5\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

```
[Out] - ((8*(a - b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d))/(3*exp(2*c + 2*
d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a
 + b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (16*exp(4*c + 4*d*x)*(a
 - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b))/(5*d) + (6
*exp(4*c + 4*d*x)*(a + b))/(5*d) + (8*exp(2*c + 2*d*x)*(a - b))/(5*d))/(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1) - (2*(a + b))/(5*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.89 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{8}\left(3a^2-2ab+3b^2\right)x+\frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*x+3/8*(a^2-b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)
*cosh(d*x+c)^3*sinh(d*x+c)*(a+b*tanh(d*x+c)^2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 424, 393, 212}

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3(a^2-b^2)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x\left(3a^2-2ab+3b^2\right) + \frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}$$

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + ((a + b)*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(4
*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}\\ &\quad - \frac{\text{Subst}\left(\int \frac{-a(3a-b)-(a-3b)bx^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}\end{aligned}$$

$$
\begin{aligned}
&= \frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d} \\
&\quad + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d} \\
&\quad + \frac{(3a^2-2ab+3b^2)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d} \\
&= \frac{1}{8}\left(3a^2-2ab+3b^2\right)x + \frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d} \\
&\quad + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$
\begin{aligned}
&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{4(3a^2-2ab+3b^2)(c+dx)+8(a^2-b^2)\sinh(2(c+dx))+(a+b)^2\sinh(4(c+dx))}{32d}
\end{aligned}
$$

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sinh[2*(c + d*x)] + (a
 + b)^2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A] (verified)

Time = 13.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+}{d}$
default	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+}{d}$
risch	$\frac{3a^2x}{8}-\frac{abx}{4}+\frac{3b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+$ e^{-}

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*
c)+a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)\cosh{(dx+c)}\sinh{(dx+c)}^3+(3\,a^2-2\,ab+3\,b^2)dx+\left((a^2+2\,ab+b^2)\cosh{(dx+c)}^3+\right.}{8\,d}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] 1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 - 2*a*b + 3
*b^2)*d*x + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c))/d
```

Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2\cosh^4{(c+dx)}\,dx$$

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(79) = 158$.

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.01

$$\begin{aligned}\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{1}{64}\,a^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)\\&\quad+\frac{1}{64}\,b^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)\\&\quad-\frac{1}{32}\,ab\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*
x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/32*a*b*(8*(d*x
+ c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(79) = 158$.

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.19

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a^2e^{(4\,dx+4\,c)} + 2\,abe^{(4\,dx+4\,c)} + b^2e^{(4\,dx+4\,c)} + 8\,a^2e^{(2\,dx+2\,c)} - 8\,b^2e^{(2\,dx+2\,c)} + 8\,(3\,a^2 - 2\,ab + 3\,b^2)(dx+c) - (}{64}$$

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/64*(a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + 8*(3*a^2 - 2*a*b + 3*b^2)*(d*x + c) - (18*a^2*e^(4*d*x + 4*c) - 12*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c))/d

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = x\left(\frac{3\,a^2}{8} - \frac{a\,b}{4} + \frac{3\,b^2}{8}\right) - \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(a^2-b^2\right)}{8\,d} + \frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{8\,d} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,{\left(a+b\right)}^2}{64\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{64\,d}$$

[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)

[Out] x*((3*a^2)/8 - (a*b)/4 + (3*b^2)/8) - (exp(- 2*c - 2*d*x)*(a^2 - b^2))/(8*d) + (exp(2*c + 2*d*x)*(a^2 - b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)/(64*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d)

3.90 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{b^2\arctan(\sinh(c+dx))}{d}+\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}$$

```
[Out] b^2*arctan(sinh(d*x+c))/d+(a^2-b^2)*sinh(d*x+c)/d+1/3*(a+b)^2*sinh(d*x+c)^3
/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 398, 209}

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}+\frac{b^2\arctan(\sinh(c+dx))}{d}$$

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + ((a^2 - b^2)*Sinh[c + d*x])/d + ((a + b)^2*
Sinh[c + d*x]^3)/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a^2-b^2+(a+b)^2x^2+\frac{b^2}{1+x^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}+\frac{b^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{b^2\arctan(\sinh(c+dx))}{d}+\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\sinh(c+dx)\left(\frac{3b^2\text{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}+(a+b)\left(3(a-b)+(a+b)\sinh^2(c+dx)\right)\right)}{3d}$$

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Sinh[c + d*x]*((3*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x]
^2] + (a + b)*(3*(a - b) + (a + b)*Sinh[c + d*x]^2)))/(3*d)
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{2ab\sinh(dx+c)^3}{3}+b^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{2ab\sinh(dx+c)^3}{3}+b^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)}{d}$
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}+\frac{3\,e^{dx+c}a^2}{8d}-\frac{e^{dx+c}ab}{4d}-\frac{5\,e^{dx+c}b^2}{8d}-\frac{3\,e^{-dx-c}a^2}{8d}+\frac{e^{-dx-c}ab}{4d}+5$

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+2/3*a*b*sinh(d*x+c)^3+b^2*(1/3
*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 519, normalized size of antiderivative = 9.61

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{(a^2+2\,ab+b^2)\cosh\left(dx+c\right)^6+6\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+\left(a^2+2\,ab+b^2\right)\sinh\left(dx\right.}{}$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(3*a^2 - 2*a
*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*
a^2 - 2*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^3 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2
- 2*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4
 + 6*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 - 3*a^2 + 2*a*b + 5*b^2)*sinh(
d*x + c)^2 - a^2 - 2*a*b - b^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x +
 c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x +
c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^5 + 2*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^3 - (3*a^2 - 2*a*b - 5*
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2
*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\cosh^3\left(c+dx\right)dx$$

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.98

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{12\,d}-\frac{1}{24}\,b^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right)+\frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/12*a*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/24*b^2*((15*e^(-2*d*x - 2*c)
- 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan
(e^(-d*x - c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d
*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(52) = 104$.

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.81

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+2\,abe^{(3\,dx+3\,c)}+b^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}-6\,abe^{(dx+c)}-15\,b^2e^{(dx+c)}-}{24\,d}$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 2*a*b*e^(3*d*x + 3
*c) + b^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c) - 6*a*b*e^(d*x + c) - 15*b^2*
e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) - 15*b^2*e^(2*
d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-3*d*x - 3*c))/d
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.41

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{e^{3c+3dx}\,(a+b)^2}{24\,d} - \frac{e^{-3c-3dx}\,(a+b)^2}{24\,d} - \frac{e^{c+dx}\,(-3\,a^2+2\,a\,b+5\,b^2)}{8\,d} + \frac{2\,\mathrm{atan}\left(\frac{b^2\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^4}}\right)\,\sqrt{b^4}}{\sqrt{d^2}} + \frac{e^{-c-d\,x}\,(-3\,a^2+2\,a\,b+5\,b^2)}{8\,d}$$

```
[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d)
 - (exp(c + d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d) + (2*atan((b^2*exp(d*x)*exp
(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^2)^(1/2) + (exp(- c - d*x
)*(2*a*b - 3*a^2 + 5*b^2))/(8*d)
```

3.91 $\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{1}{2}(a-3b)(a+b)x + \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d}$$

```
[Out] 1/2*(a-3*b)*(a+b)*x+1/2*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 398, 393, 212}

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{(a+b)^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{1}{2}x(a-3b)(a+b) + \frac{b^2\tanh(c+dx)}{d}$$

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 3*b)*(a + b)*x)/2 + ((a + b)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (
b^2*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b^2\tanh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}\\
&\quad+\frac{((a-3b)(a+b))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}\\
&= \frac{1}{2}(a-3b)(a+b)x+\frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a-3b)(a+b)(c+dx)}{2d}+\frac{(a+b)^2\sinh(2(c+dx))}{4d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] ((a - 3*b)*(a + b)*(c + d*x))/(2*d) + ((a + b)^2*Sinh[2*(c + d*x)])/(4*d) +
 (b^2*Tanh[c + d*x])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(47) = 94.

Time = 1.94 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
risch	$\frac{a^2x}{2}-abx-\frac{3b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}b^2}{8d}-$

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(1/2*cosh(d*x+c)
*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*
c+3/2*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.06

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{(a^2+2\,ab+b^2)\sinh(dx+c)^3+4\left((a^2-2\,ab-3\,b^2)dx-2\,b^2\right)\cosh(dx+c)+\left(3\,(a^2+2\,ab+b^2)\cosh(dx\right.}{8\,d\cosh(dx+c)}$$

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

[Out] 1/8*((a^2 + 2*a*b + b^2)*sinh(d*x + c)^3 + 4*((a^2 - 2*a*b - 3*b^2)*d*x - 2
*b^2)*cosh(d*x + c) + (3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
+ 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))

Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2(c+dx)\right)^2\cosh^2(c+dx)\,dx$$

[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**2, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(47) = 94$.

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.75

$$\begin{aligned}&\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{1}{8}\,a^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{4}\,ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)\\&\quad-\frac{1}{8}\,b^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)\end{aligned}$$

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/4*a*b*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b^2*(12*(d*x + c)/d + e^(-2*d*x
- 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
))))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.27

$$\begin{aligned}&\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{a^2e^{(2\,dx+2\,c)}+2\,abe^{(2\,dx+2\,c)}+b^2e^{(2\,dx+2\,c)}+4\,(a^2-2\,ab-3\,b^2)(dx+c)-\frac{a^2e^{(4\,dx+4\,c)}-2\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}}{e^{(4\,dx+4\,c}}}{8\,d}\end{aligned}$$

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

```
[Out] 1/8*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + 4*
(a^2 - 2*a*b - 3*b^2)*(d*x + c) - (a^2*e^(4*d*x + 4*c) - 2*a*b*e^(4*d*x + 4
*c) - 3*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c
) + a^2 + 2*a*b + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{e^{2c+2dx}\,(a+b)^2}{8d}-\frac{2b^2}{d\,(e^{2c+2dx}+1)}-\frac{e^{-2c-2dx}\,(a+b)^2}{8d}-x\left(-\frac{a^2}{2}+a\,b+\frac{3\,b^2}{2}\right)$$

```
[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - (
exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - x*(a*b - a^2/2 + (3*b^2)/2)
```

3.92 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{b(4a+3b)\arctan(\sinh(c+dx))}{2d}+\frac{(a+b)^2\sinh(c+dx)}{d}+\frac{b^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] -1/2*b*(4*a+3*b)*arctan(sinh(d*x+c))/d+(a+b)^2*sinh(d*x+c)/d+1/2*b^2*sech(d
*x+c)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3757, 398, 393, 209}

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{b(4a+3b)\arctan(\sinh(c+dx))}{2d}+\frac{(a+b)^2\sinh(c+dx)}{d}+\frac{b^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*ArcTan[Sinh[c + d*x]])/d + ((a + b)^2*Sinh[c + d*x])/d
+ (b^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left((a+b)^2-\frac{b(2a+b)+2b(a+b)x^2}{(1+x^2)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a+b)^2\sinh(c+dx)}{d}-\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a+b)^2\sinh(c+dx)}{d}+\frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d}\\
&\quad-\frac{(b(4a+3b))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d}\\
&= -\frac{b(4a+3b)\arctan(\sinh(c+dx))}{2d}+\frac{(a+b)^2\sinh(c+dx)}{d}+\frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2(a+b)^2\sinh(c+dx)+b(-((4a+3b)\arctan(\sinh(c+dx)))+b\operatorname{sech}(c+dx)\tanh(c+dx))}{2d}$$

[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (2*(a + b)^2*Sinh[c + d*x] + b*(-((4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + b*Se
ch[c + d*x]*Tanh[c + d*x]))/(2*d)

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{a^2\sinh(dx+c)+2ab(\sinh(dx+c)-2\arctan(e^{dx+c}))+b^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\operatorname{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan(\right.}{d}$
default	$\frac{a^2\sinh(dx+c)+2ab(\sinh(dx+c)-2\arctan(e^{dx+c}))+b^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\operatorname{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan(\right.}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{e^{dx+c}b^2}{2d}-\frac{e^{-dx-c}a^2}{2d}-\frac{e^{-dx-c}ab}{d}-\frac{e^{-dx-c}b^2}{2d}+\frac{b^2e^{dx+c}(e^{2dx+2c}-1)}{d(e^{2dx+2c}+1)^2}+\frac{2ib\ln(e^{dx+}}{d}$

[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*sinh(d*x+c)+2*a*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+b^2*(sinh(d*x
+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)
-3*arctan(exp(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(56) = 112.

Time = 0.28 (sec) , antiderivative size = 774, normalized size of antiderivative = 12.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)\cosh(dx+c)^6+6\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^5+(a^2+2\,ab+b^2)\sinh(dx}{}$$

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 + 2*a*b +
3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*
a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*a*b + 3*b^2)
*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b
 + 3*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*
a*b - b^2 - 2*((4*a*b + 3*b^2)*cosh(d*x + c)^5 + 5*(4*a*b + 3*b^2)*cosh(d*x
 + c)*sinh(d*x + c)^4 + (4*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(4*a*b + 3*b^2)
*cosh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)*si
nh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + 3*b^2)*cosh(d*x + c) + (5*(4*a*b +
3*b^2)*cosh(d*x + c)^4 + 6*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)
*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 - (a^2 + 2*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x
 + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2\cosh{(c+dx)}\,dx$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.53

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{1}{2}b^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d(e^{(-dx-c)}+2e^{(-3dx-3c)}+e^{(-5dx-5c)})}\right)\\&\quad+ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\sinh{(dx+c)}}{d}\end{aligned}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) + a*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) +
a^2*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(56) = 112$.

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.67

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}\right.\right.\right.}{4\,d}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + 4*a*b*(e^(d*x + c) - e^(-d*x - c)
) + 2*b^2*(e^(d*x + c) - e^(-d*x - c)) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c
) - 1)*e^(-d*x - c)))*(4*a*b + 3*b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c))/
((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.03

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\mathrm{e}^{c+d\,x}\,{\left(a+b\right)}^2}{2\,d}-\frac{\mathrm{e}^{-c-d\,x}\,{\left(a+b\right)}^2}{2\,d} - \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2\,b^2+24\,a\,b^3+9\,b^4}}\right)\,\sqrt{16\,a^2\,b^2+24\,a\,b^3+9\,b^4}}{\sqrt{d^2}} + \frac{b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(c + d*x)*(a + b)^2)/(2*d) - (exp(- c - d*x)*(a + b)^2)/(2*d) - (atan((
exp(d*x)*exp(c)*(3*b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(24*a*b^3 + 9*b
^4 + 16*a^2*b^2)^(1/2)))*(24*a*b^3 + 9*b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1/2)
 + (b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))/(d*
(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.93 $\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))}{8d} - \frac{3b(2a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\text{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d-3/8*b*(2*a+b)*sech(d*x+c)*tan
h(d*x+c)/d-1/4*b*sech(d*x+c)^3*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 424, 393, 209}

$$\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))}{8d} - \frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d} - \frac{b\tanh(c+dx)\text{sech}^3(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4d}$$

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) - (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) - (b*Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b\text{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{a(4a+b)+(a+b)(4a+3b)x^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4d} \\
&= -\frac{3b(2a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d} \\
&\quad - \frac{b\text{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} \\
&\quad + \frac{(8a^2+8ab+3b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{8d}
\end{aligned}$$

$$= \frac{(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))}{8d} - \frac{3b(2a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\text{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.31 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.69

$$\int \text{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{\text{csch}^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+a\sinh^2(c+dx)+b\sinh\right.\right.}{}$$

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1
, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + a*Sinh[c + d*x]^2 + b*Sinh[c
 + d*x]^2)^2 + 128*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^6*(5*b^2*Sinh[c + d*x]^4 + 2*a*b*Sinh[c + d*x]^2*(6
+ 5*Sinh[c + d*x]^2) + a^2*(7 + 12*Sinh[c + d*x]^2 + 5*Sinh[c + d*x]^4)) +
35*(b^2*Sinh[c + d*x]^4*(1947 + 485*Sinh[c + d*x]^2) + 2*a*b*Sinh[c + d*x]^
2*(2625 + 2554*Sinh[c + d*x]^2 + 485*Sinh[c + d*x]^4) + a^2*(3375 + 5907*Si
nh[c + d*x]^2 + 3161*Sinh[c + d*x]^4 + 485*Sinh[c + d*x]^6)) - (105*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]]*(b^2*Sinh[c + d*x]^4*(649 + 378*Sinh[c + d*x]^2 +
9*Sinh[c + d*x]^4) + 2*a*b*Sinh[c + d*x]^2*(875 + 1143*Sinh[c + d*x]^2 + 38
9*Sinh[c + d*x]^4 + 9*Sinh[c + d*x]^6) + a^2*(1125 + 2344*Sinh[c + d*x]^2 +
 1674*Sinh[c + d*x]^4 + 400*Sinh[c + d*x]^6 + 9*Sinh[c + d*x]^8)))/Sqrt[-Si
nh[c + d*x]^2]))/d
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{2a^2\arctan(e^{dx+c})+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\mathrm{se}\right.\right.}{d}$
default	$\frac{2a^2\arctan(e^{dx+c})+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\mathrm{se}\right.\right.}{d}$
risch	$-\frac{b\,e^{dx+c}(8a\,e^{6dx+6c}+5b\,e^{6dx+6c}+8a\,e^{4dx+4c}-3b\,e^{4dx+4c}-8\,e^{2dx+2c}a+3b\,e^{2dx+2c}-8a-5b)}{4d(e^{2dx+2c}+1)^4}+\frac{i\ln(e^{dx+c}+i)a^2}{d}+$

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2*arctan(exp(d*x+c))+2*a*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*
x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-sinh(d*x+c)^3/cosh(d*x+c)^4-1/co
sh(d*x+c)^4*sinh(d*x+c)+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4
*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. 2(85) = 170.

Time = 0.26 (sec) , antiderivative size = 1373, normalized size of antiderivative = 15.09

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((8*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 5*b^2)*cosh(d*x + c)*sin
h(d*x + c)^6 + (8*a*b + 5*b^2)*sinh(d*x + c)^7 + (8*a*b - 3*b^2)*cosh(d*x +
 c)^5 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^
5 + 5*(7*(8*a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c))*s
inh(d*x + c)^4 - (8*a*b - 3*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 5*b^2)*cosh
(d*x + c)^4 + 10*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b + 3*b^2)*sinh(d*x
+ c)^3 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^5 + 10*(8*a*b - 3*b^2)*cosh(d*x
+ c)^3 - 3*(8*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^2 + 8*a*b
 + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*b + 3*b
^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2
+ 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(8*a^2
+ 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b + 9*b^
```

```
2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a
^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(8*a
^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*
sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)*cosh(d
*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8*a*b +
3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 5*b^2)*cosh(d*x + c) +
 (7*(8*a*b + 5*b^2)*cosh(d*x + c)^6 + 5*(8*a*b - 3*b^2)*cosh(d*x + c)^4 - 3
*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b - 5*b^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh
(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*co
sh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x
+ c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int\left(a+b\tanh^2{(c+dx)}\right)^2\operatorname{sech}{(c+dx)}\,dx$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=$$
$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5e^{(-dx-c)}-3e^{(-3dx-3c)}+3e^{(-5dx-5c)}-5e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)-2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)+\frac{a^2\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) +
 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan(e
^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.87

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(8a^2+8ab+3b^2)-\frac{4\left(8ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+5b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{\left(\left(e^{(dx+c)}-e^{(-}\right.\right.}}{16d}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a*
b + 3*b^2) - 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 5*b^2*(e^(d*x + c) -
 e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x + c
) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.33

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(8a^2\sqrt{d^2}+3b^2\sqrt{d^2}+8ab\sqrt{d^2}\right)}{d\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}\right)\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}{4\sqrt{d^2}} - \frac{6b^2e^{c+dx}}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)} + \frac{4b^2e^{c+dx}}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)} - \frac{e^{c+dx}\left(5b^2+8ab\right)}{4d\left(e^{2c+2dx}+1\right)} + \frac{e^{c+dx}\left(9b^2+8ab\right)}{2d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2)
^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))*(
```

```
48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/2))
 - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c
 + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp(c + d*x)*(8*
a*b + 5*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(8*a*b + 9*b^2))
/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.94 $\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*tanh(d*x+c)/d+2/3*a*b*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 200}

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^2\, dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\, dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$
\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\, dx = \frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}
$$

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(45) = 90.

Time = 6.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

method	result
derivativedivides	$\frac{a^2\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sec}}{}\right.}{}\right.}{d}$
default	$\frac{a^2\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sec}}{}\right.}{}\right.}{d}$
risch	$-\frac{2\left(15a^2 e^{8dx+8c}+30ab\, e^{8dx+8c}+15b^2 e^{8dx+8c}+60a^2 e^{6dx+6c}+60ab\, e^{6dx+6c}+90a^2 e^{4dx+4c}+40ab\, e^{4dx+4c}+30\, e^{4dx+4c}b\right.}{15d\left(e^{2dx+2c}+1\right)^5}$

[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sec
h(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x
+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+
c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 7.98

$$\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left((15\,a^2+20\,ab+9\,b^2)\cosh(dx+c)^4+8\,(5\,ab+3\,b^2)\cosh(dx+c)\sinh(dx+\right.}{15\left(d\cosh(dx+c)^6+6\,d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6+6\,d\cosh(dx+c)^4+3\left(5\,d\,\mathrm{co}\right.\right.}$$

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -4/15*((15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^4 + 8*(5*a*b + 3*b^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + (15*a^2 + 20*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*(3
*a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c
)^2 + 30*a^2 + 20*a*b)*sinh(d*x + c)^2 + 45*a^2 + 20*a*b + 15*b^2 + 8*((5*a
*b + 3*b^2)*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6*d*cos
h(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(
d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*
(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d
*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) +
10*d)

Sympy [F]

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2 \operatorname{sech}^2{(c+dx)}\, dx$$

[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{b^2\tanh{(dx+c)}^5}{5\,d} + \frac{2\,ab\tanh{(dx+c)}^3}{3\,d} + \frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/5*b^2*tanh(d*x + c)^5/d + 2/3*a*b*tanh(d*x + c)^3/d + 2*a^2/(d*(e^(-2*d*x - 2*c) + 1))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(45) = 90.

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.45

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+60\,abe^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+40\right.}{15\,d(e^{(2\,dx+2\,c)}+1)^5}$$

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 40*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 15*a^2 + 10*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 482, normalized size of antiderivative = 9.84

$$\int \mathrm{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= -\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1} - \frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,{\left(a+b\right)}^2}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1} - \frac{\frac{2\,{\left(a+b\right)}^2}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,{\left(a+b\right)}^2}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1} - \frac{\frac{2\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{15\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1} - \frac{2\,{\left(a+b\right)}^2}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*(a^2 - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(a + b)^2)/(5*d))/(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c + 4
*d*x)*(a^2 - b^2))/(5*d) + (2*exp(6*c + 6*d*x)*(a + b)^2)/(5*d) + (2*exp(2*
c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c
+ 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a + b)^2)/(5*d
) + (8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a^2 - b^2
))/(5*d) + (2*exp(8*c + 8*d*x)*(a + b)^2)/(5*d) + (4*exp(4*c + 4*d*x)*(3*a^
2 - 2*a*b + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(3*a^2
 - 2*a*b + 3*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) + (2*exp
(4*c + 4*d*x)*(a + b)^2)/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) + 1) - (2*(a + b)^2)/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.95 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\begin{aligned}\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{(8a^2+4ab+b^2)\arctan(\sinh(c+dx))}{16d}+\frac{(8a^2+4ab+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d}\\&\quad-\frac{b(8a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d}\\&\quad-\frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d}\end{aligned}$$

```
[Out] 1/16*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+4*a*b+b^2)*sech(d*
x+c)*tanh(d*x+c)/d-1/24*b*(8*a+3*b)*sech(d*x+c)^3*tanh(d*x+c)/d-1/6*b*sech(
d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3757, 424, 393, 205, 209}

$$\begin{aligned}\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{(8a^2+4ab+b^2)\arctan(\sinh(c+dx))}{16d}+\frac{(8a^2+4ab+b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{16d}\\&\quad-\frac{b(8a+3b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{24d}\\&\quad-\frac{b\tanh(c+dx)\operatorname{sech}^5(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6d}\end{aligned}$$

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 4*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) - (b*(8*a + 3*b)*Sech[c + d*x]^3*Tan
h[c + d*x])/(24*d) - (b*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[
c + d*x])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{b\,\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\text{Subst}\left(\int \frac{a(6a+b)+3(a+b)(2a+b)x^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{6d}$$
$$= -\frac{b(8a+3b)\text{sech}^3(c+dx)\tanh(c+dx)}{24d} - \frac{b\,\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{(8a^2+4ab+b^2)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{8d}$$
$$= \frac{(8a^2+4ab+b^2)\,\text{sech}(c+dx)\tanh(c+dx)}{16d} - \frac{b(8a+3b)\text{sech}^3(c+dx)\tanh(c+dx)}{24d} - \frac{b\,\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{(8a^2+4ab+b^2)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{16d}$$
$$= \frac{(8a^2+4ab+b^2)\arctan(\sinh(c+dx))}{16d} + \frac{(8a^2+4ab+b^2)\,\text{sech}(c+dx)\tanh(c+dx)}{16d} - \frac{b(8a+3b)\text{sech}^3(c+dx)\tanh(c+dx)}{24d} - \frac{b\,\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.28 (sec) , antiderivative size = 792, normalized size of antiderivative = 6.34

$$\int \text{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{a^2\sinh(c+dx)\left(-\frac{23555(a+b)}{a} - \frac{32970(a+b)^2}{a^2} - 14980\,\text{csch}^2(c+dx) - \frac{91875(a+b)\text{csch}^2(c+dx)}{a} - 65625\,\text{csch}^4(c+d\right.}{}$$

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

[Out] (a^2*Sinh[c + d*x]*((-23555*(a + b))/a - (32970*(a + b)^2)/a^2 - 14980*Csch [c + d*x]^2 - (91875*(a + b)*Csch[c + d*x]^2)/a - 65625*Csch[c + d*x]^4 - (8855*(a + b)^2*Sinh[c + d*x]^2)/a^2 - 620*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 160*HypergeometricPFQ[{3/ 2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 16*Hype rgeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Si nh[c + d*x]^2 - (968*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (288*(a + b)*HypergeometricPFQ[{3/2 , 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (32*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (380*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (128*(a + b)^2*Hy pergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (16*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 + (65625*ArcTanh[Sqrt [-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) + (1680*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(5/2) - (36855*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(3/2) - (91875*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (54180*(a + b)*ArcTanh[S qrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]^2]) + (32970*(a + b)^2*ArcTa nh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*Sqrt[-Sinh[c + d*x]^2]) + (525*(a + b)^2*A rcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(a^2*Sqrt[-Sinh[c + d*x]^2]) - (1365*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2])/a - (19845*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2]) /a^2))/(2520*d)

Maple [A] (verified)

Time = 14.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^{4}}+\frac{\left(\frac{\operatorname{sech}(dx+c)^{3}}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(\mathrm{e}^{dx}\right.}{4}\right.}{a}$
default	$\frac{a^{2}\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^{4}}+\frac{\left(\frac{\operatorname{sech}(dx+c)^{3}}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(\mathrm{e}^{dx}\right.}{4}\right.}{a}$
risch	$\frac{\mathrm{e}^{dx+c}\left(24a^{2}\mathrm{e}^{10dx+10c}+12ab\,\mathrm{e}^{10dx+10c}+3b^{2}\mathrm{e}^{10dx+10c}+72a^{2}\mathrm{e}^{8dx+8c}-60ab\,\mathrm{e}^{8dx+8c}-47b^{2}\mathrm{e}^{8dx+8c}+48a^{2}\mathrm{e}^{6dx+6c}-72ab\,\mathrm{e}\right.}{24d}$

[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+2*a*b*(-1/3/cosh(d*x+c)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+1/

```
4*arctan(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*sinh(d*x+c)
/cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*
tanh(d*x+c)+1/8*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2824 vs. 2(117) = 234.

Time = 0.28 (sec) , antiderivative size = 2824, normalized size of antiderivative = 22.59

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 4*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^11 +
(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 4*a*b + b^2)*cos
h(d*x + c)^2 + 72*a^2 - 60*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^3 + (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c))*sin
h(d*x + c)^8 + 6*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(8*a^2
+ 4*a*b + b^2)*cosh(d*x + c)^4 + 6*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^2 + 8*a^2 - 12*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(8*a^2 + 4*a*b + b^2
)*cosh(d*x + c)^5 + 2*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^3 + (8*a^2 -
 12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(8*a^2 - 12*a*b + 13*b
^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 21*(72
*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^2 - 8*a^2 + 12*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(8*a^2 + 4*
a*b + b^2)*cosh(d*x + c)^7 + 21*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^5
+ 35*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(8*a^2 - 12*a*b + 13*b^2
)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^3 + (495*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(72*a^2 - 60*a*b - 47*
b^2)*cosh(d*x + c)^6 + 210*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^4 - 60*(
8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^2 - 72*a^2 + 60*a*b + 47*b^2)*sinh(d
*x + c)^3 + 3*(55*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^9 + 12*(72*a^2 - 60*a
*b - 47*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^5
 - 20*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - (72*a^2 - 60*a*b - 47*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^1
2 + 12*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 4*a*
b + b^2)*sinh(d*x + c)^12 + 6*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 6*(1
1*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c
)^10 + 20*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^
8 + 15*(33*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 4*a*b + b^2)
*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*
```

(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 315*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 105*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 40*a^2 + 20*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 70*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^9 + 36*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 42*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 45*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 50*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2 + 12*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^9 + 10*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 10*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^5 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + (8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c) + 3*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 3*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^8 + 14*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^6 - 10*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^2 - 8*a^2 - 4*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\operatorname{sech}^3(c+dx)\,dx$$

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(117) = 234$.

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.76

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{1}{24}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3e^{(-dx-c)} - 47e^{(-3dx-3c)} + 78e^{(-5dx-5c)} - 78e^{(-7dx-7c)} + 47e^{(-9dx-9}}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)} + 6e^{(-10dx-1}}\right.$$
$$-\frac{1}{2}ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - 7e^{(-3dx-3c)} + 7e^{(-5dx-5c)} - e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right)$$
$$-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right)$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c)
 + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-
11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x
 - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) +
 1))) - 1/2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c
) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4
*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^2*(arctan(e^
(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(117) = 234$.

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(8\,a^2+4\,ab+b^2)+\frac{4\left(24\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+12\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right.}{}}{}$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 4*
a*b + b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 12*a*b*(e^(d*x + c)
 - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^(d*x
 + c) - e^(-d*x - c))^3 - 32*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(
e^(d*x + c) - e^(-d*x - c)) - 192*a*b*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2
*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.58

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(8\,a^2\,\sqrt{d^2}+b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+64\,a^3\,b+32\,a^2\,b^2+8\,a\,b^3+b^4}}\right)\,\sqrt{64\,a^4+64\,a^3\,b+32\,a^2\,b^2+8\,a\,b^3+b^4}}{8\,\sqrt{d^2}}$$
$$-\frac{\frac{2\,e^{c+d\,x}\,(a+b)^2}{3\,d}+\frac{8\,e^{3\,c+3\,d\,x}\,(a^2-b^2)}{3\,d}+\frac{8\,e^{7\,c+7\,d\,x}\,(a^2-b^2)}{3\,d}+\frac{2\,e^{9\,c+9\,d\,x}\,(a+b)^2}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{3\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$
$$-\frac{2\,e^{c+d\,x}\,(15\,b^2+4\,a\,b)}{3\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{16\,b^2\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,(8\,a^2+4\,a\,b+b^2)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{e^{c+d\,x}\,(16\,a^2+44\,a\,b+23\,b^2)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,(21\,b^2+20\,a\,b)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(
1/2)))/(d*(8*a*b^3 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2)))*(8*a*b^3
```

```
 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*exp(c
 + d*x)*(a + b)^2)/(3*d) + (8*exp(3*c + 3*d*x)*(a^2 - b^2))/(3*d) + (8*exp(
7*c + 7*d*x)*(a^2 - b^2))/(3*d) + (2*exp(9*c + 9*d*x)*(a + b)^2)/(3*d) + (4
*exp(5*c + 5*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(3*d))/(6*exp(2*c + 2*d*x) + 15*
exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c +
 10*d*x) + exp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(4*a*b + 15*b^2))/(3*d
*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
1)) + (exp(c + d*x)*(4*a*b + 8*a^2 + b^2))/(8*d*(exp(2*c + 2*d*x) + 1)) - (
exp(c + d*x)*(44*a*b + 16*a^2 + 23*b^2))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*
c + 4*d*x) + 1)) + (exp(c + d*x)*(20*a*b + 21*b^2))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.96 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a^2\tanh(c+dx)}{d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{(2a-b)b\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] a^2*tanh(d*x+c)/d-1/3*a*(a-2*b)*tanh(d*x+c)^3/d-1/5*(2*a-b)*b*tanh(d*x+c)^5
/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 380}

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a^2\tanh(c+dx)}{d}-\frac{b(2a-b)\tanh^5(c+dx)}{5d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d - (a*(a - 2*b)*Tanh[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)\left(a+bx^2\right)^2\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2 - a(a-2b)x^2 - (2a-b)bx^4 - b^2x^6\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2\tanh(c+dx)}{d} - \frac{a(a-2b)\tanh^3(c+dx)}{3d} - \frac{(2a-b)b\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\left(70a^2+28ab+6b^2+\left(35a^2+14ab+3b^2\right)\operatorname{sech}^2(c+dx)-6b(7a+4b)\operatorname{sech}^4(c+dx)+15b^2\operatorname{sech}^6(c+dx)\right)}{105d}$$

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((70*a^2 + 28*a*b + 6*b^2 + (35*a^2 + 14*a*b + 3*b^2)*Sech[c + d*x]^2 - 6*b
*(7*a + 4*b)*Sech[c + d*x]^4 + 15*b^2*Sech[c + d*x]^6)*Tanh[c + d*x])/(105*
d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(70) = 140.

Time = 29.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^2\left(-\frac{\sinh(}{4\cosh(}\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^2\left(-\frac{\sinh(}{4\cosh(}\right.}{d}$
risch	$-\frac{4(105a^2e^{10dx+10c}+210ab\,e^{10dx+10c}+105b^2e^{10dx+10c}+455a^2e^{8dx+8c}+350ab\,e^{8dx+8c}-105b^2e^{8dx+8c}+770a^2e^{6dx+6c}}{}$

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/4*sinh(d*x+c)/cosh(d
*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-
1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/
7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(70) = 140.

Time = 0.25 (sec) , antiderivative size = 677, normalized size of antiderivative = 8.91

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{8\,(2\,(35}{105\,\left(d\cosh\left(dx+c\right)^9+9\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8+d\sinh\left(dx+c\right)^9+7\,d\cosh\left(dx+c\right)^7+\left(36\,d\,c\right.\right.}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/105*(2*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 56*a*b
+ 27*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 + 98*a*b + 51*b^2)*sinh(d
*x + c)^5 + 14*(25*a^2 + 16*a*b - 3*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 + 98
*a*b + 51*b^2)*cosh(d*x + c)^2 + 105*a^2 + 126*a*b - 63*b^2)*sinh(d*x + c)^
3 + 2*(10*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 16*a*b
- 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 4*a*b + 3*b^2)*cosh(
d*x + c) + (5*(35*a^2 + 98*a*b + 51*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 + 6*a*
b - 3*b^2)*cosh(d*x + c)^2 + 70*a^2 + 28*a*b + 126*b^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 7*
d*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 7*(12*d*
```

```
cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(d*x + c)^5
 + (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d*x + c)^5 +
 (126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*x + c))*sinh
(d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 245*d*cosh(d*x
 + c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36*d*cosh(d*x +
c)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c))
*sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 + 49*d*cosh(d*
x + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x +
c))
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}^4\left(c+dx\right)dx$$

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(70)=140$.

Time = 0.21 (sec) , antiderivative size = 928, normalized size of antiderivative = 12.21

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-
12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)
) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e
^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*
x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 3
5*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-
6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x -
 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
```

```
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x
 - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
+ 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(70) = 140$.

Time = 0.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.13

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(105\,a^2e^{(10\,dx+10\,c)}+210\,abe^{(10\,dx+10\,c)}+105\,b^2e^{(10\,dx+10\,c)}+455\,a^2e^{(8\,dx+8\,c)}+350\,abe^{(8\,dx+8\,c)}-105\,b^2\right.}{}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/105*(105*a^2*e^(10*d*x + 10*c) + 210*a*b*e^(10*d*x + 10*c) + 105*b^2*e^(
10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 350*a*b*e^(8*d*x + 8*c) - 105*b^
2*e^(8*d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 140*a*b*e^(6*d*x + 6*c) + 210
*b^2*e^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 84*a*b*e^(4*d*x + 4*c) - 4
2*b^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 98*a*b*e^(2*d*x + 2*c) +
21*b^2*e^(2*d*x + 2*c) + 35*a^2 + 14*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c) + 1)^
7)
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 732, normalized size of antiderivative = 9.63

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{4\,(3\,a^2-2\,a\,b+3\,b^2)}{35\,d}+\frac{32\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{35\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(a+b)^2}{7\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}-\frac{\frac{32\,(a^2-b^2)}{105\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)^2}{21\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$
$$-\frac{\frac{32\,(a^2-b^2)}{105\,d}+\frac{64\,e^{4\,c+4\,d\,x}\,(a^2-b^2)}{35\,d}+\frac{16\,e^{6\,c+6\,d\,x}\,(a+b)^2}{21\,d}+\frac{16\,e^{2\,c+2\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{35\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{32\,e^{4\,c+4\,d\,x}\,(a^2-b^2)}{7\,d}+\frac{32\,e^{8\,c+8\,d\,x}\,(a^2-b^2)}{7\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)^2}{7\,d}+\frac{8\,e^{10\,c+10\,d\,x}\,(a+b)^2}{7\,d}+\frac{16\,e^{6\,c+6\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{7\,d}}{7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\,c+12\,d\,x}+e^{14\,c+14\,d\,x}+1}$$
$$-\frac{\frac{4\,(a+b)^2}{21\,d}+\frac{32\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{21\,d}+\frac{64\,e^{6\,c+6\,d\,x}\,(a^2-b^2)}{21\,d}+\frac{20\,e^{8\,c+8\,d\,x}\,(a+b)^2}{21\,d}+\frac{8\,e^{4\,c+4\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{7\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$
$$-\frac{4\,(a+b)^2}{21\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

```
[Out] - ((4*(3*a^2 - 2*a*b + 3*b^2))/(35*d) + (32*exp(2*c + 2*d*x)*(a^2 - b^2))/(
35*d) + (4*exp(4*c + 4*d*x)*(a + b)^2)/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4
*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((32*(a^2 - b^2)
)/(105*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(21*d))/(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((32*(a^2 - b^2))/(105*d) + (64*
exp(4*c + 4*d*x)*(a^2 - b^2))/(35*d) + (16*exp(6*c + 6*d*x)*(a + b)^2)/(21*
d) + (16*exp(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(35*d))/(5*exp(2*c + 2*d
*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(
10*c + 10*d*x) + 1) - ((32*exp(4*c + 4*d*x)*(a^2 - b^2))/(7*d) + (32*exp(8*
c + 8*d*x)*(a^2 - b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(7*d) + (8*e
xp(10*c + 10*d*x)*(a + b)^2)/(7*d) + (16*exp(6*c + 6*d*x)*(3*a^2 - 2*a*b +
3*b^2))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d
*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) +
exp(14*c + 14*d*x) + 1) - ((4*(a + b)^2)/(21*d) + (32*exp(2*c + 2*d*x)*(a^2
 - b^2))/(21*d) + (64*exp(6*c + 6*d*x)*(a^2 - b^2))/(21*d) + (20*exp(8*c +
8*d*x)*(a + b)^2)/(21*d) + (8*exp(4*c + 4*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(7*
d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*ex
p(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (4*(a + b
)^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.97 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x + \frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{b^3\tanh(c+dx)}{d}$$

```
[Out] 3/8*(a+b)*(a^2-2*a*b+5*b^2)*x+3/8*(a-3*b)*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d
+1/4*(a+b)^3*cosh(d*x+c)^3*sinh(d*x+c)/d-b^3*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3756, 398, 1171, 393, 212}

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3}{8}x(a+b)\left(a^2-2ab+5b^2\right) + \frac{(a+b)^3\sinh(c+dx)\cosh^3(c+dx)}{4d} + \frac{3(a-3b)(a+b)^2\sinh(c+dx)\cosh(c+dx)}{8d} - \frac{b^3\tanh(c+dx)}{d}$$

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + b)*(a^2 - 2*a*b + 5*b^2)*x)/8 + (3*(a - 3*b)*(a + b)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(8*d) + ((a + b)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) -
(b^3*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b^3+\frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{b^3\tanh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b^3\tanh(c+dx)}{d}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-3(a-b)^2(a+b)+12b^2(a+b)x^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{4d}\\
&= \frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}\\
&\quad -\frac{b^3\tanh(c+dx)}{d}+\frac{\left(3(a+b)\left(a^2-2ab+5b^2\right)\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8d}\\
&= \frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x+\frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d}\\
&\quad +\frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b^3\tanh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.86 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$
\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx
= \frac{12(a^3-a^2b+3ab^2+5b^3)(c+dx)+8(a-2b)(a+b)^2\sinh(2(c+dx))+(a+b)^3\sinh(4(c+dx))-32b^3}{32d}
$$

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(c + d*x) + 8*(a - 2*b)*(a + b)^2*Sinh[
2*(c + d*x)] + (a + b)^3*Sinh[4*(c + d*x)] - 32*b^3*Tanh[c + d*x])/(32*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(85) = 170$.

Time = 55.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{}$
default	$\frac{a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{}$
risch	$\frac{3a^3x}{8}-\frac{3b\,a^2x}{8}+\frac{9a\,b^2x}{8}+\frac{15b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3\,e^{4dx+4c}a^2b}{64d}+\frac{3\,e^{4dx+4c}a\,b^2}{64d}+\frac{e^{4dx+4c}b^3}{64d}+\frac{e^{2dx+2c}a^3}{8d}-$

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/
8*c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c
)+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.49

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\sinh\left(dx+c\right)^5+\left(9\,a^3+3\,a^2b-21\,ab^2-15\,b^3+10\,(a^3+3\,a^2b+3\,ab^2+b^3)\cosh\left(\right.\right.}{}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + (9*a^3 + 3*a^2*b -
21*a*b^2 - 15*b^3 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 8*(8*b^3 + 3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x)*cosh(d*x +
 c) + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^3 - 24*a*b^2
 - 80*b^3 + 9*(3*a^3 + a^2*b - 7*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c))/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \cosh^4\left(c+dx\right) dx$$

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.93

$$\begin{aligned}&\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx \\ &= \frac{1}{64}\,a^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right) \\ &\quad+\frac{3}{64}\,ab^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right) \\ &\quad+\frac{1}{64}\,b^3\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15\,e^{(-2\,dx-2\,c)}+144\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}\right) \\ &\quad-\frac{3}{64}\,a^2b\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*
d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^3*(120*(
d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2
*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
- 3/64*a^2*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(85) = 170$.

Time = 0.54 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.10

$$\begin{aligned}&\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx \\ &= \frac{a^3e^{(4\,dx+4\,c)}+3\,a^2be^{(4\,dx+4\,c)}+3\,ab^2e^{(4\,dx+4\,c)}+b^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}-24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2}}{}\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*e^(4*d*x + 4*c) + 3*a^2*b*e^(4*d*x + 4*c) + 3*a*b^2*e^(4*d*x + 4*
c) + b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x + 2*c)
 - 16*b^3*e^(2*d*x + 2*c) + 24*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(d*x + c) +
128*b^3/(e^(2*d*x + 2*c) + 1) - (18*a^3*e^(4*d*x + 4*c) - 18*a^2*b*e^(4*d*x
 + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*
x + 2*c) - 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*c))/d
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(\frac{3\,a^3}{8}-\frac{3\,a^2\,b}{8}+\frac{9\,a\,b^2}{8}+\frac{15\,b^3}{8}\right)+\frac{2\,b^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,{\left(a+b\right)}^3}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^3}{64\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-2\,b\right)}{8\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-2\,b\right)}{8\,d}$$

```
[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*((9*a*b^2)/8 - (3*a^2*b)/8 + (3*a^3)/8 + (15*b^3)/8) + (2*b^3)/(d*(exp(2*
c + 2*d*x) + 1)) - (exp(- 4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d*x
)*(a + b)^3)/(64*d) - (exp(- 2*c - 2*d*x)*(a + b)^2*(a - 2*b))/(8*d) + (exp
(2*c + 2*d*x)*(a + b)^2*(a - 2*b))/(8*d)
```

3.98 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{b^2(6a+5b)\arctan(\sinh(c+dx))}{2d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c))/d+(a-2*b)*(a+b)^2*sinh(d*x+c)/d+1/3*(
a+b)^3*sinh(d*x+c)^3/d-1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3757, 398, 393, 209}

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{b^2(6a+5b)\arctan(\sinh(c+dx))}{2d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}-\frac{b^3\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - 2*b)*(a + b)^2*Sinh[c
 + d*x])/d + ((a + b)^3*Sinh[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left((a-2b)(a+b)^2+(a+b)^3x^2+\frac{b^2(3a+2b)+3b^2(a+b)x^2}{\left(1+x^2\right)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{b^2(3a+2b)+3b^2(a+b)x^2}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}
\end{aligned}
$$

$$= \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d} + \frac{(a+b)^3\sinh^3(c+dx)}{3d} - \frac{b^3\text{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{(b^2(6a+5b))\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2d}$$

$$= \frac{b^2(6a+5b)\arctan(\sinh(c+dx))}{2d} + \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d} + \frac{(a+b)^3\sinh^3(c+dx)}{3d} - \frac{b^3\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.06 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.68

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\text{csch}^5(c+dx)\left(-256\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+a\sinh^2(c+dx)+b\,\text{si}\right.\right.}{}$$

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d*x
]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*Sinh[c + d*x]^6*(2161 +
1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c
+ d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]
^6) + 3*a^2*b*(Sinh[c + d*x] + Sinh[c + d*x]^3)^2*(2401 + 1875*Sinh[c + d*x
]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(240
1 + 4180*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 244*Sinh[c + d*x]^6 + Sin
h[c + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*Sinh[c + d*x]^6*(32415 + 1
7320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3*a*b^2*Sinh[c + d*x]^4*(3601
5 + 50695*Sinh[c + d*x]^2 + 18073*Sinh[c + d*x]^4 + 753*Sinh[c + d*x]^6) +
3*a^2*b*Sinh[c + d*x]^2*(36015 + 88150*Sinh[c + d*x]^2 + 69728*Sinh[c + d*x
]^4 + 18826*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(36015 + 124165*Si
nh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^6 + 19579*Sinh
[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(30240*d)
```

Maple [A] (verified)

Time = 22.77 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+a^2b\sinh(dx+c)^3+3ab^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)+b^3\left(\frac{\sinh(dx+c}{3\cosh(dx+c}\right.}{d}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+a^2b\sinh(dx+c)^3+3ab^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)+b^3\left(\frac{\sinh(dx+c}{3\cosh(dx+c}\right.}{d}$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}a\,b^2}{8d}+\frac{e^{3dx+3c}b^3}{24d}+\frac{3\,e^{dx+c}a^3}{8d}-\frac{3\,e^{dx+c}a^2b}{8d}-\frac{15\,e^{dx+c}a\,b^2}{8d}-\frac{9b^3e^{dx+c}}{8d}-$

[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+a^2*b*sinh(d*x+c)^3+3*a*b^2*(1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c)))+b^3*(1/3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. 2(81) = 162.

Time = 0.28 (sec) , antiderivative size = 1840, normalized size of antiderivative = 21.15

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^8 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3 + 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 5*a^3 + 3*a^2*b + 21*a*b^2 + 25*b^3 + 15*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^3 + 3*a^2*b +

```
3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*
cosh(d*x + c)^5 + 5*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 -
 (5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3
 - 3*a^2*b - 3*a*b^2 - b^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*
x + c)^2 + (45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(11*a^3
 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(5*a^3 - 3*a^2*b - 21*
a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 11*a^3 + 3*a^2*b + 39*a*b^2 + 25*b^3 - 12
*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2
4*((6*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + (6*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + 5*b^3)*cos
h(d*x + c)^5 + (12*a*b^2 + 10*b^3 + 21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^5 + 5*(7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + 5*b^
3)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35
*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 6*a*b^2 + 5*b^3 + 20*(6*a*b^2 + 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
 20*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^2 + (7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + 5*b^
3)*cosh(d*x + c)^4 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^9 + 4*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^7
+ 6*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 - 3*a^
2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 2
5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)
*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x
 + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d
*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)
^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(
d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2(c+dx)\right)^3\cosh^3(c+dx)\,dx$$

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.26

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{8\,d} - \frac{1}{8}\,ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d} - \frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d} + \frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right) + \frac{1}{24}\,b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d} - \frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-\cdots}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right) + \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/8*a*b^2*((15*e^(-2*d*x - 2*c
) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arct
an(e^(-d*x - c))/d) + 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 12
0*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e
^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x
 - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/
d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(81) = 162$.

Time = 0.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.03

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+\cdots}{\cdots}$$

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x -
 c))^3 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + b^3*(e^(d*x + c) - e^(-d*
x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x - c)) - 36*a*b^2*(e^(d*x + c) - e
^(-d*x - c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) - 24*b^3*(e^(d*x + c) -
e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(1/2*(
e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + 5*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(5\,b^3\,\sqrt{d^2}+6\,a\,b^2\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^2\,b^4+60\,a\,b^5+25\,b^6}}\right)\sqrt{36\,a^2\,b^4+60\,a\,b^5+25\,b^6}}{\sqrt{d^2}} - \frac{e^{-3\,c-3\,d\,x}\,(a+b)^3}{24\,d}$$
$$+ \frac{e^{3\,c+3\,d\,x}\,(a+b)^3}{24\,d} + \frac{3\,e^{c+d\,x}\,(a+b)^2\,(a-3\,b)}{8\,d} - \frac{b^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$
$$- \frac{3\,e^{-c-d\,x}\,(a+b)^2\,(a-3\,b)}{8\,d} + \frac{2\,b^3\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(60*a*
b^5 + 25*b^6 + 36*a^2*b^4)^(1/2)))*(60*a*b^5 + 25*b^6 + 36*a^2*b^4)^(1/2))/
(d^2)^(1/2) - (exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*(a
+ b)^3)/(24*d) + (3*exp(c + d*x)*(a + b)^2*(a - 3*b))/(8*d) - (b^3*exp(c +
d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x)*(a + b)^2*(a - 3*b))/(
8*d) + (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.99 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{1}{2}(a-5b)(a+b)^2x + \frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+2b)\tanh(c+dx)}{d} + \frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] 1/2*(a-5*b)*(a+b)^2*x+1/2*(a+b)^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+2*b)*t
anh(d*x+c)/d+1/3*b^3*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 398, 393, 212}

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{b^2(3a+2b)\tanh(c+dx)}{d} + \frac{(a+b)^3\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{1}{2}x(a-5b)(a+b)^2 + \frac{b^3\tanh^3(c+dx)}{3d}$$

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 5*b)*(a + b)^2*x)/2 + ((a + b)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) +
 (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(b^2(3a+2b)+b^3x^2+\frac{(a-2b)(a+b)^2+3b(a+b)^2x^2}{(1-x^2)^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{(a-2b)(a+b)^2+3b(a+b)^2x^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+2b)\tanh(c+dx)}{d}\\
&\quad+\frac{b^3\tanh^3(c+dx)}{3d}+\frac{((a-5b)(a+b)^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}
\end{aligned}
$$

$$=\frac{1}{2}(a-5b)(a+b)^2x+\frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int\cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{6(a-5b)(a+b)^2(c+dx)+3(a+b)^3\sinh(2(c+dx))+4b^2\left(9a+7b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (6*(a - 5*b)*(a + b)^2*(c + d*x) + 3*(a + b)^3*Sinh[2*(c + d*x)] + 4*b^2*(9
*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(72) = 144.

Time = 6.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)-}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)-}{d}$
risch	$\frac{a^3x}{2}-\frac{3b\,a^2x}{2}-\frac{9a\,b^2x}{2}-\frac{5b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3\,e^{2dx+2c}a^2b}{8d}+\frac{3\,e^{2dx+2c}a\,b^2}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}a^3}{8d}-$

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(1/2*cosh(d*x+
c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a*b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*
x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c
+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(72) = 144$.

Time = 0.26 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.73

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3\left(a^3+3a^2b+3ab^2+b^3\right)\sinh\left(dx+c\right)^5-4\left(18ab^2+14b^3-3\left(a^3-3a^2b-9ab^2-5b^3\right)dx\right)\cosh\left(dx+\right.}{}$$

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 - 4*(18*a*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c) + 3*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*a^3 + 6*a^2*b + 30*a*b^2 + 10*b^3 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh^2\left(c+dx\right)dx$$

[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**2, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(72) = 144$.

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.28

$$\begin{aligned}\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)\\&\quad-\frac{1}{24}b^3\left(\frac{60\left(dx+c\right)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d\left(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)}\right)}\right)\\&\quad-\frac{3}{8}ab^2\left(\frac{12\left(dx+c\right)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}\right)}\right)\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 3/8*a^2*b*(4*x - e
^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^3*(60*(d*x + c)/d + 3*e^(-2
*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*c) + 147*e^(-6*d*
x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c
) + e^(-8*d*x - 8*c)))) - 3/8*a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d -
(17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(72) = 144$.

Time = 0.47 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.40

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,a^3e^{(2\,dx+2\,c)}+9\,a^2be^{(2\,dx+2\,c)}+9\,ab^2e^{(2\,dx+2\,c)}+3\,b^3e^{(2\,dx+2\,c)}+12\,(a^3-3\,a^2b-9\,ab^2-5\,b^3)(dx+c)-3\,(2}{}$$

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*e^(2*d*x + 2*c) + 9*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x +
2*c) + 3*b^3*e^(2*d*x + 2*c) + 12*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*(d*x +
c) - 3*(2*a^3*e^(2*d*x + 2*c) - 6*a^2*b*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x
 + 2*c) - 10*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-2*d*x
 - 2*c) - 16*(9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^
(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 7*b^3)/(e^(2*d*x + 2*c)
+ 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.12

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\mathrm{e}^{2\,c+2\,d\,x}\,{(a+b)}^3}{8\,d}-\frac{\frac{2\,(b^3+3\,a\,b^2)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^3+a\,b^2)}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{2\,(b^3+a\,b^2)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,{(a+b)}^3}{8\,d}$$
$$-\frac{\frac{2\,(b^3+a\,b^2)}{d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^3+3\,a\,b^2)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(b^3+a\,b^2)}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}+\frac{x\,{(a+b)}^2\,(a-5\,b)}{2}$$

```
[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(3*a*b^2 + b^3))/(3*d) + (2*exp(2*
c + 2*d*x)*(a*b^2 + b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) -
```

```
(2*(a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a + b)^
3)/(8*d) - ((2*(a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + b^3))/(3*d
) + (2*exp(4*c + 4*d*x)*(a*b^2 + b^3))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) + 1) + (x*(a + b)^2*(a - 5*b))/2
```

3.100 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\arctan(\sinh(c+dx))}{8d} + \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c))/d+(a+b)^3*sinh(d*x+c)/d+3/
8*b^2*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d-1/4*b^3*sech(d*x+c)^3*tanh(d*x+c)
/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3757, 398, 1171, 393, 209}

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\arctan(\sinh(c+dx))}{8d} + \frac{3b^2(4a+3b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d} + \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{b^3\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + ((a + b)^3
*Sinh[c + d*x])/d + (3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) -
 (b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((a+b)^3 - \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(1+x^2)^3}\right)\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{b^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12b(a+b)^2x^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4d}$$
$$= \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d} - \frac{(3b(4(a+b)^2+(2a+b)^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{8d}$$
$$= -\frac{3b(4(a+b)^2+(2a+b)^2)\arctan(\sinh(c+dx))}{8d} + \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{-3b(8a^2+12ab+5b^2)\arctan(\sinh(c+dx)) + 8(a+b)^3\sinh(c+dx) + 3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d}$$

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 8*(a + b)^3*Sinh[c +
 d*x] + 3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] - 2*b^3*Sech[c + d*x]
^3*Tanh[c + d*x])/(8*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(93) = 186.

Time = 2.81 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{\sinh(dx+c)a^3+3a^2b(\sinh(dx+c)-2\arctan(e^{dx+c}))+3ab^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\,\mathrm{arcta}\right.}{}$
default	$\frac{\sinh(dx+c)a^3+3a^2b(\sinh(dx+c)-2\arctan(e^{dx+c}))+3ab^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\,\mathrm{arcta}\right.}{}$
risch	$\frac{e^{dx+c}a^3}{2d}+\frac{3\,e^{dx+c}a^2b}{2d}+\frac{3\,e^{dx+c}a\,b^2}{2d}+\frac{b^3e^{dx+c}}{2d}-\frac{e^{-dx-c}a^3}{2d}-\frac{3\,e^{-dx-c}a^2b}{2d}-\frac{3\,e^{-dx-c}a\,b^2}{2d}-\frac{e^{-dx-c}b^3}{2d}+$

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(sinh(d*x+c)*a^3+3*a^2*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+3*a*b^2*(si
nh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(
d*x+c)-3*arctan(exp(d*x+c)))+b^3*(sinh(d*x+c)^5/cosh(d*x+c)^4+5*sinh(d*x+c)
^3/cosh(d*x+c)^4+5/cosh(d*x+c)^4*sinh(d*x+c)-5*(1/4*sech(d*x+c)^3+3/8*sech(
d*x+c))*tanh(d*x+c)-15/4*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2411 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 2411, normalized size of antiderivative = 24.35

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 20*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sinh(d*x + c)^10 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x
 + c)^8 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(10*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^3 + (2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x +
c))*sinh(d*x + c)^7 + (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6
 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^3 + 12*a^2*b
+ 24*a*b^2 + 5*b^3 + 84*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 6*(84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5
+ 28*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (4*a^3 + 12*a^2
*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 12*a^2*b +
 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
```

```
osh(d*x + c)^6 + 210*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^4 -
 4*a^3 - 12*a^2*b - 24*a*b^2 - 5*b^3 + 15*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^7 + 42*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
5 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*a^3 - 6*a^2*b -
 6*a*b^2 - 2*b^3 - 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^2 +
 3*(30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(2*a^3 + 6*a^2*
b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^4 - 2*a^3 - 6*a^2*b - 10*a*b^2 - 5*b^3 - 2*(4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b + 1
2*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x
+ c)*sinh(d*x + c)^8 + (8*a^2*b + 12*a*b^2 + 5*b^3)*sinh(d*x + c)^9 + 4*(8*
a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3 +
 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)*c
osh(d*x + c))*sinh(d*x + c)^6 + 6*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
)^5 + 6*(21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b
^2 + 5*b^3 + 14*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 2*(63*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 70*(8*a^2*b + 12*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 4*(
21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b + 12*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 15*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 12*a*b^2 + 5
*b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*
(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
) + (9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^6 + 30*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^
4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 12*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 12*(2*a^3 + 6*a^2*b + 10*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^5 - 2*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(2*a^3
 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x
 + c)^7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cos
h(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x
 + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*
cosh(d*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cos
h(d*x + c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*
x + c) + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4
```

```
+ 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3\cosh(c+dx)\,dx$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(93) = 186$.

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.98

$$\begin{aligned}
&\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{1}{4}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{2e^{(-dx-c)}}{d} + \frac{17e^{(-2dx-2c)}+13e^{(-4dx-4c)}+7e^{(-6dx-6c)}-7e^{(-8dx-8c)}+\cdots}{d\left(e^{(-dx-c)}+4e^{(-3dx-3c)}+6e^{(-5dx-5c)}+4e^{(-7dx-7c)}+e^{(-9dx\cdots}\right.}\right. \\
&\quad + \frac{3}{2}ab^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d\left(e^{(-dx-c)}+2e^{(-3dx-3c)}+e^{(-5dx-5c)}\right)}\right) \\
&\quad + \frac{3}{2}a^2b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a^3\sinh(dx+c)}{d}
\end{aligned}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*c
) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d*(
e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7*c)
 + e^(-9*d*x - 9*c)))) + 3/2*a*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)
/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3
*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e
^(d*x + c)/d - e^(-d*x - c)/d) + a^3*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(93) = 186$.

Time = 0.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.75

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{8\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+24\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+24\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+8\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{}$$

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 24*a^2*b*(e^(d*x + c) - e^(-d*x
- c)) + 24*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 8*b^3*(e^(d*x + c) - e^(-d*
x - c)) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2*
b + 12*a*b^2 + 5*b^3) + 4*(12*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*b^3*
(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 28
*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/
d
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.59

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{e^{c+d\,x}\,(a+b)^3}{2\,d}-\frac{e^{-c-d\,x}\,(a+b)^3}{2\,d}$$
$$-\frac{3\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(5\,b^3\,\sqrt{d^2}+12\,a\,b^2\,\sqrt{d^2}+8\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4\,b^2+192\,a^3\,b^3+224\,a^2\,b^4+120\,a\,b^5+25\,b^6}}\right)\,\sqrt{64\,a^4\,b^2+192\,a^3\,b^3+224\,a^2\,b^4+120\,a\,b^5+25\,b^6}}{4\,\sqrt{d^2}}$$
$$+\frac{3\,e^{c+d\,x}\,\left(3\,b^3+4\,a\,b^2\right)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{6\,b^3\,e^{c+d\,x}}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{e^{c+d\,x}\,\left(13\,b^3+12\,a\,b^2\right)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{4\,b^3\,e^{c+d\,x}}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(c + d*x)*(a + b)^3)/(2*d) - (exp(- c - d*x)*(a + b)^3)/(2*d) - (3*atan
((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 12*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^2)
^(1/2)))/(d*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^(
1/2)))*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^(1/2))
/(4*(d^2)^(1/2)) + (3*exp(c + d*x)*(4*a*b^2 + 3*b^3))/(4*d*(exp(2*c + 2*d*x
) + 1)) + (6*b^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(12*a*b^2 + 13*b^3))/(2*d*(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3*exp(c + d*x))/(d*(4*exp(2*c +
 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.101 $\int \mathrm{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}
&\int \mathrm{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\
&=\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))}{16d}\\
&-\frac{b\left(44a^2+44ab+15b^2\right)\mathrm{sech}(c+dx)\tanh(c+dx)}{48d}\\
&-\frac{5b(2a+b)\mathrm{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{24d}\\
&-\frac{b\mathrm{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}
\end{aligned}$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d-1/48*b*(44*a^2+44*a*
b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d-5/24*b*(2*a+b)*sech(d*x+c)^3*(a+(a+b)*s
inh(d*x+c)^2)*tanh(d*x+c)/d-1/6*b*sech(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)^2*t
anh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

$= \{3757, 424, 540, 393, 209\}$

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))}{16d} - \frac{b(44a^2+44ab+15b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d} - \frac{b\tanh(c+dx)\operatorname{sech}^5(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{6d} - \frac{5b(2a+b)\tanh(c+dx)\operatorname{sech}^3(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{24d}$$

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) - (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) - (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) - (b*Se
ch[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= -\frac{b\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)(a(6a+b)+(a+b)(6a+5b)x^2)}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{6d}\\
&= -\frac{5b(2a+b)\text{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{24d}\\
&\quad-\frac{b\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-a(24a^2+14ab+5b^2)-(a+b)(24a^2+34ab+15b^2)x^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{24d}\\
&= -\frac{b(44a^2+44ab+15b^2)\,\text{sech}(c+dx)\tanh(c+dx)}{48d}\\
&\quad-\frac{5b(2a+b)\text{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{24d}\\
&\quad-\frac{b\text{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}\\
&\quad+\frac{((2a+b)(8a^2+8ab+5b^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{16d}
\end{aligned}
$$

$$= \frac{(2a+b)(8a^2+8ab+5b^2)\arctan(\sinh(c+dx))}{16d} - \frac{b(44a^2+44ab+15b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} - \frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} - \frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}$$

Mathematica [A] (verified)

Time = 11.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{6(16a^3+24a^2b+18ab^2+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 3b(24a^2+30ab+11b^2)\operatorname{sech}(c+dx)\tanh(c+}{48d}$$

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] - 3*b*(
24*a^2 + 30*a*b + 11*b^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*b^2*(18*a + 13*b)
*Sech[c + d*x]^3*Tanh[c + d*x] - 8*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d
)
```

Maple [A] (verified)

Time = 6.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{2a^3\arctan(e^{dx+c})+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\right.\right.}{}$
default	$\frac{2a^3\arctan(e^{dx+c})+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\right.\right.}{}$
risch	$-\frac{b\,e^{dx+c}(72a^2e^{10dx+10c}+90ab\,e^{10dx+10c}+33b^2e^{10dx+10c}+216a^2e^{8dx+8c}+126ab\,e^{8dx+8c}-5b^2e^{8dx+8c}+144a^2e^{6dx+6c}+}{}$

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^3*arctan(exp(d*x+c))+3*a^2*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(
d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a*b^2*(-sinh(d*x+c)^3/cosh(d*x+c)^
4-1/cosh(d*x+c)^4*sinh(d*x+c)+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+
c)+3/4*arctan(exp(d*x+c)))+b^3*(-sinh(d*x+c)^5/cosh(d*x+c)^6-5/3*sinh(d*x+c
```

```
)^3/cosh(d*x+c)^6-sinh(d*x+c)/cosh(d*x+c)^6+(1/6*sech(d*x+c)^5+5/24*sech(d*
x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+5/8*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 3465, normalized size of antiderivative = 23.26

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 30*a*b^2 +
 11*b^3)*sinh(d*x + c)^11 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^9
 + (216*a^2*b + 126*a*b^2 - 5*b^3 + 165*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*
x + c)^3 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 +
 18*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 30*a*b
^2 + 11*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 2*a*b^2 + 5*b^3 + 2*(216*a^2*b + 1
26*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 30*
a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*
x + c)^3 + 3*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 1
8*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 30*a*b^2
 + 11*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c
)^4 - 8*a^2*b - 2*a*b^2 - 5*b^3 + 21*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c
)^7 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 35*(8*a^2*b + 2*a
*b^2 + 5*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^4 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + (495*
(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^8 + 84*(216*a^2*b + 126*a*b^2
- 5*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^4
- 216*a^2*b - 126*a*b^2 + 5*b^3 - 180*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c
)^9 + 12*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 126*(8*a^2*b + 2
*a*b^2 + 5*b^3)*cosh(d*x + c)^5 - 60*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x +
 c)^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*
((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (16*a^3 + 24*a^2
*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^12 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 11*(16
*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*
(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(16*a^3 + 24
```

```
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 15*(33*(16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 18
*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 30*(16*a^3
+ 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(16*a^3 + 24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b
^3)*cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 105*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(33*(16*a^3 + 24*a^2
*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(16*a^3 + 24*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*
x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 15*(3
3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(16*a^3 + 24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 20*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(1
1*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 36*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 42*(16*a^3 + 24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh
(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(
d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 6*(16*a^3 + 24*a^2*b +
18*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5
*b^3)*cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x
 + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 50*(1
6*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b +
18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 10*(16*a
^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 10*(16*a^3 + 24*a^2*b +
 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^
3)*cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(24*a^2*b + 30*a*b
^2 + 11*b^3)*cosh(d*x + c) + 3*(11*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x
+ c)^10 + 3*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 42*(8*a^2*b +
 2*a*b^2 + 5*b^3)*cosh(d*x + c)^6 - 30*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x
 + c)^4 - 24*a^2*b - 30*a*b^2 - 11*b^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh
(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cos
h(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 +
4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 +
```

```
5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35
*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c
)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^
4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 +
36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*co
sh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^
10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 1
5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cos
h(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x +
 c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{sech}\left(c+dx\right)dx$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.43

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \\ &-\frac{1}{24}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}+90\,e^{(-5\,dx-5\,c)}-90\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx}}\right.\\ &-\frac{3}{4}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)\\ &-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}\end{aligned}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*c
) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*
```

```
x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c)
+ 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a^2
*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(141) = 282$.

Time = 0.40 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.08

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(16a^3+24a^2b+18ab^2+5b^3\right)-\frac{4\left(72a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+90ab^2\right.}{}}{}$$

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 + 2
4*a^2*b + 18*a*b^2 + 5*b^3) - 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 +
90*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 33*b^3*(e^(d*x + c) - e^(-d*x - c
))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c) -
e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*(e^(d
*x + c) - e^(-d*x - c)) + 864*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 240*b^3*
(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.59

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(16a^3\sqrt{d^2}+5b^3\sqrt{d^2}+18ab^2\sqrt{d^2}+24a^2b\sqrt{d^2}\right)}{d\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+564a^2b^4+180ab^5+25b^6}}\right)\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+}}{8\sqrt{d^2}}$$

$$-\frac{e^{c+dx}\left(55b^3+54ab^2\right)}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

$$-\frac{80b^3e^{c+dx}}{3d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$

$$+\frac{6e^{c+dx}\left(5b^3+2ab^2\right)}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

$$+\frac{32b^3e^{c+dx}}{3d\left(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$

$$-\frac{e^{c+dx}\left(24a^2b+30ab^2+11b^3\right)}{8d\left(e^{2c+2dx}+1\right)}+\frac{e^{c+dx}\left(72a^2b+162ab^2+85b^3\right)}{12d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x),x)

[Out] (atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*(d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 + 768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 + 55*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (80*b^3*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b^2 + 5*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)*(30*a*b^2 + 24*a^2*b + 11*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(162*a*b^2 + 72*a^2*b + 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

3.102 $\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{a^3\tanh(c+dx)}{d} + \frac{a^2b\tanh^3(c+dx)}{d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] a^3*tanh(d*x+c)/d+a^2*b*tanh(d*x+c)^3/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/7*b^3*t
anh(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 200}

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{a^3\tanh(c+dx)}{d} + \frac{a^2b\tanh^3(c+dx)}{d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^7(c+dx)}{7d}$$

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^3\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{a^3\tanh(c+dx)}{d}+\frac{a^2b\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^3\tanh(c+dx)}{d}+\frac{a^2b\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(63) = 126.

Time = 21.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.39

method	result
derivativedivides	$\frac{a^3\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\right.}{}\right.}{}$
default	$\frac{a^3\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\right.}{}\right.}{}$
risch	$-\frac{2(105a^2b\,e^{12dx+12c}+21a\,b^2+105a\,b^2e^{12dx+12c}+35b^3e^{12dx+12c}+35a^3+105\,e^{4dx+4c}b^3+35a^2b+42\,e^{2dx+2c}a\,b^2+140a^2b}{}$

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*s
ech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*si
nh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tan
h(d*x+c))+b^3*(-1/2*sinh(d*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(d*x+
c)^7-5/16*sinh(d*x+c)/cosh(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35*sech
(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(63) = 126.

Time = 0.27 (sec) , antiderivative size = 786, normalized size of antiderivative = 11.73

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{4\left(\left(35\,a^3+70\,a^2b+63\,ab^2+20\,b^3\right)\cosh\left(dx+c\right)^6+6\left(35\,a^2b+42\,ab^2+15\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right.\right.}{35\left(d\cosh\left(dx\right.\right.}$$

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*cosh(d*x + c)^6 + 6*(35*a^2*
b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 70*a^2*b +
 63*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 20*a^2*b + 9*a*b^2)*cosh
(d*x + c)^4 + (210*a^3 + 280*a^2*b + 126*a*b^2 + 15*(35*a^3 + 70*a^2*b + 63
*a*b^2 + 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(35*a^2*b + 42*a*b
^2 + 15*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d
*x + c)^3 + 350*a^3 + 280*a^2*b + 210*a*b^2 + 7*(75*a^3 + 70*a^2*b + 39*a*b
^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*
cosh(d*x + c)^4 + 525*a^3 + 490*a^2*b + 273*a*b^2 + 140*b^3 + 84*(15*a^3 +
20*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(35*a^2*b + 42*
a*b^2 + 15*b^3)*cosh(d*x + c)^5 + 56*(5*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 +
7*(25*a^2*b + 6*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
```

```
c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x
 + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x
+ c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*c
osh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42
*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*c
osh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) +
35*d)
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{sech}^2\left(c+dx\right)dx$$

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{b^3\tanh\left(dx+c\right)^7}{7\,d}+\frac{3\,ab^2\tanh\left(dx+c\right)^5}{5\,d}+\frac{a^2b\tanh\left(dx+c\right)^3}{d}+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/7*b^3*tanh(d*x + c)^7/d + 3/5*a*b^2*tanh(d*x + c)^5/d + a^2*b*tanh(d*x +
c)^3/d + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.

Time = 0.43 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.18

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=$$

$$-\frac{2\left(35\,a^3e^{(12\,dx+12\,c)}+105\,a^2be^{(12\,dx+12\,c)}+105\,ab^2e^{(12\,dx+12\,c)}+35\,b^3e^{(12\,dx+12\,c)}+210\,a^3e^{(10\,dx+10\,c)}+420\right.}{}$$

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 105*a^2*b*e^(12*d*x + 12*c) + 105*a*b^2*e
^(12*d*x + 12*c) + 35*b^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 4
20*a^2*b*e^(10*d*x + 10*c) + 210*a*b^2*e^(10*d*x + 10*c) + 525*a^3*e^(8*d*x
 + 8*c) + 665*a^2*b*e^(8*d*x + 8*c) + 315*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e
^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 560*a^2*b*e^(6*d*x + 6*c) + 420*
a*b^2*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x + 4*c) + 315*a^2*b*e^(4*d*x + 4*c)
 + 231*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x +
 2*c) + 140*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 35*a^3 + 35*
a^2*b + 21*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 1050, normalized size of antiderivative = 15.67

$$\int \operatorname{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (2*exp(6*c + 6*d*x)*(a
+ b)^3)/(7*d) - (6*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d)
 + (6*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a + b)^3
)/(7*d) + (2*exp(12*c + 12*d*x)*(a + b)^3)/(7*d) - (6*exp(4*c + 4*d*x)*(a*b
^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (6*exp(8*c + 8*d*x)*(a*b^2 + a^2*b - 5
*a^3 - 5*b^3))/(7*d) + (8*exp(6*c + 6*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b
^3))/(7*d) + (12*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d) + (12*exp(10*c +
 10*d*x)*(a + b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x
) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*e
xp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*exp(4*c + 4*d*x)*(a + b)^
3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (4*exp(2*c + 2*d*x)
*(a + b)^2*(a - b))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) + 1) - ((2*(a + b)^2*(a - b))/(7*d) + (2*exp(2*c + 2*d*x)*(a +
b)^3)/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*exp(8*c + 8*
d*x)*(a + b)^3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) - (12*ex
p(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (8*exp(2*c + 2*d*x
)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (8*exp(6*c + 6*d*x)*(a + b)
^2*(a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b)^2*(a -
 b))/(7*d) + (2*exp(10*c + 10*d*x)*(a + b)^3)/(7*d) - (2*exp(2*c + 2*d*x)*(
a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (4*exp(6*c + 6*d*x)*(a*b^2 + a^2*b
- 5*a^3 - 5*b^3))/(7*d) + (4*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 -
5*b^3))/(7*d) + (10*exp(8*c + 8*d*x)*(a + b)^2*(a - b))/(7*d))/(6*exp(2*c +
 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) +
```

```
 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (2*(a + b)^3)/(7*d*(exp(2
*c + 2*d*x) + 1))
```

3.103 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{(64a^3+48a^2b+24ab^2+5b^3)\arctan(\sinh(c+dx))}{128d}\\ &+\frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}\\ &-\frac{b(72a^2+52ab+15b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d}\\ &-\frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d}\\ &-\frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d}\end{aligned}$$

```
[Out] 1/128*(64*a^3+48*a^2*b+24*a*b^2+5*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a^3+
48*a^2*b+24*a*b^2+5*b^3)*sech(d*x+c)*tanh(d*x+c)/d-1/192*b*(72*a^2+52*a*b+1
5*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d-1/48*b*(12*a+5*b)*sech(d*x+c)^5*(a+(a+b)
*sinh(d*x+c)^2)*tanh(d*x+c)/d-1/8*b*sech(d*x+c)^7*(a+(a+b)*sinh(d*x+c)^2)^2
*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3757, 424, 540, 393, 205, 209}

$$\begin{aligned}&\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=-\frac{b(72a^2+52ab+15b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}\\&+\frac{(64a^3+48a^2b+24ab^2+5b^3)\arctan(\sinh(c+dx))}{128d}\\&+\frac{(64a^3+48a^2b+24ab^2+5b^3)\tanh(c+dx)\operatorname{sech}(c+dx)}{128d}\\&-\frac{b\tanh(c+dx)\operatorname{sech}^7(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{8d}\\&-\frac{b(12a+5b)\tanh(c+dx)\operatorname{sech}^5(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{48d}\end{aligned}$$

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) + ((
64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(128*d)
- (b*(72*a^2 + 52*a*b + 15*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) - (b
*(12*a + 5*b)*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/
(48*d) - (b*Sech[c + d*x]^7*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/
(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^5}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)(a(8a+b)+(a+b)(8a+5b)x^2)}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{8d} \\
&= -\frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} \\
&\quad - \frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-a(48a^2+18ab+5b^2)-3(a+b)(16a^2+14ab+5b^2)x^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{48d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b(72a^2+52ab+15b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad - \frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} \\
&\quad - \frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} \\
&\quad + \frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{Subst}\left(\int\frac{1}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{64d} \\
&= \frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} \\
&\quad - \frac{b(72a^2+52ab+15b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad - \frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} \\
&\quad - \frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} \\
&\quad + \frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{128d} \\
&= \frac{(64a^3+48a^2b+24ab^2+5b^3)\arctan(\sinh(c+dx))}{128d} \\
&\quad + \frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} \\
&\quad - \frac{b(72a^2+52ab+15b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad - \frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} \\
&\quad - \frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 13.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$
\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx
= \frac{6(64a^3+48a^2b+24ab^2+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+3(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\,\mathrm{t}}{}
$$

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (6*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] + 3*(64
*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x] - 2*b*(144*
```

```
a^2 + 168*a*b + 59*b^2)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*(24*a + 17*b)
*Sech[c + d*x]^5*Tanh[c + d*x] - 48*b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(384
*d)
```

Maple [A] (verified)

Time = 43.36 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx}\right.}{4}\right.}{}$
default	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx}\right.}{4}\right.}{}$
risch	$\frac{e^{dx+c}\left(-432a^2b\,e^{12dx+12c}-72a\,b^2-984a\,b^2e^{12dx+12c}-397b^3e^{12dx+12c}-192a^3-895\,e^{4dx+4c}b^3+72a\,b^2e^{14dx+14c}+192a^3e\right.}{}$

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a^2*b*(-1/3/cos
h(d*x+c)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+
1/4*arctan(exp(d*x+c)))+3*a*b^2*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*sinh(
d*x+c)/cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*
x+c))*tanh(d*x+c)+1/8*arctan(exp(d*x+c)))+b^3*(-1/3*sinh(d*x+c)^5/cosh(d*x+
c)^8-1/3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/7*sinh(d*x+c)/cosh(d*x+c)^8+1/7*(1/8
*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*
tanh(d*x+c)+5/64*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6114 vs. $2(188) = 376$.

Time = 0.32 (sec) , antiderivative size = 6114, normalized size of antiderivative = 30.88

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2(c+dx)\right)^3\operatorname{sech}^3(c+dx)\,dx$$

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(188) = 376$.

Time = 0.30 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.79

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=$$

$$-\frac{1}{192}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}-397\,e^{(-3\,dx-3\,c)}+895\,e^{(-5\,dx-5\,c)}-1765\,e^{(-7\,dx-7\,c)}+1765\,\ldots}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{\ldots}}\right.$$
$$-\frac{1}{8}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}-47\,e^{(-3\,dx-3\,c)}+78\,e^{(-5\,dx-5\,c)}-78\,e^{(-7\,dx-7\,c)}+47\,e^{(-9\,dx\ldots}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx\ldots}}\right.$$
$$-\frac{3}{4}a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*c)
 - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*c))
/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c))/
d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*
d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x -
2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*
e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(arctan(e^(-d*x
- c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d
*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c)
 + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) -
e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(188) = 376$.

Time = 0.41 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.45

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(64\,a^3+48\,a^2b+24\,ab^2+5\,b^3\right)+\frac{4\left(192\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+144\,a^2}{}}{}$$

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
48*a^2*b + 24*a*b^2 + 5*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7 +
144*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 72*a*b^2*(e^(d*x + c) - e^(-d*x
- c))^7 + 15*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x + c) - e
^(-d*x - c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 - 480*a*b^2*(e^(d
*x + c) - e^(-d*x - c))^5 - 292*b^3*(e^(d*x + c) - e^(-d*x - c))^5 + 9216*a
^3*(e^(d*x + c) - e^(-d*x - c))^3 - 2304*a^2*b*(e^(d*x + c) - e^(-d*x - c))
^3 - 4224*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 880*b^3*(e^(d*x + c) - e^(
-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) - 9216*a^2*b*(e^(d*x
+ c) - e^(-d*x - c)) - 4608*a*b^2*(e^(d*x + c) - e^(-d*x - c)) - 960*b^3*(e
^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.80

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(64\,a^3\,\sqrt{d^2}+5\,b^3\,\sqrt{d^2}+24\,a\,b^2\,\sqrt{d^2}+48\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4096\,a^6+6144\,a^5\,b+5376\,a^4\,b^2+2944\,a^3\,b^3+1056\,a^2\,b^4+240\,a\,b^5+25\,b^6}}\right)\sqrt{4096\,a^6+6144\,a^5\,b+5376\,a^4\,b^2+2944\,a}}{64\,\sqrt{d^2}}$$
$$-\frac{\frac{e^{c+dx}\,(a+b)^3}{2\,d}+\frac{e^{13\,c+13\,dx}\,(a+b)^3}{2\,d}-\frac{3\,e^{5\,c+5\,dx}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{2\,d}-\frac{3\,e^{9\,c+9\,dx}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{2\,d}+\frac{2\,e^{7\,c+7\,dx}\,\left(5\,a^3-3\right.}{d}}{8\,e^{2\,c+2\,dx}+28\,e^{4\,c+4\,dx}+56\,e^{6\,c+6\,dx}+70\,e^{8\,c+8\,dx}+56\,e^{10\,c+10\,dx}+28\,e^{12\,c+12\,c}}$$
$$+\frac{2\,e^{c+dx}\,\left(85\,b^3+48\,a\,b^2\right)}{3\,d\,\left(5\,e^{2\,c+2\,dx}+10\,e^{4\,c+4\,dx}+10\,e^{6\,c+6\,dx}+5\,e^{8\,c+8\,dx}+e^{10\,c+10\,dx}+1\right)}$$
$$+\frac{16\,b^3\,e^{c+dx}}{d\,\left(7\,e^{2\,c+2\,dx}+21\,e^{4\,c+4\,dx}+35\,e^{6\,c+6\,dx}+35\,e^{8\,c+8\,dx}+21\,e^{10\,c+10\,dx}+7\,e^{12\,c+12\,dx}+e^{14\,c+14\,dx}+1\right)}$$
$$+\frac{e^{c+dx}\,\left(64\,a^3+48\,a^2\,b+24\,a\,b^2+5\,b^3\right)}{64\,d\,\left(e^{2\,c+2\,dx}+1\right)}$$
$$-\frac{4\,e^{c+dx}\,\left(35\,b^3+6\,a\,b^2\right)}{3\,d\,\left(6\,e^{2\,c+2\,dx}+15\,e^{4\,c+4\,dx}+20\,e^{6\,c+6\,dx}+15\,e^{8\,c+8\,dx}+6\,e^{10\,c+10\,dx}+e^{12\,c+12\,dx}+1\right)}$$
$$-\frac{e^{c+dx}\,\left(144\,a^3+576\,a^2\,b+600\,a\,b^2+203\,b^3\right)}{96\,d\,\left(2\,e^{2\,c+2\,dx}+e^{4\,c+4\,dx}+1\right)}$$
$$+\frac{e^{c+dx}\,\left(288\,a^2\,b+600\,a\,b^2+305\,b^3\right)}{24\,d\,\left(3\,e^{2\,c+2\,dx}+3\,e^{4\,c+4\,dx}+e^{6\,c+6\,dx}+1\right)}$$
$$-\frac{e^{c+dx}\,\left(24\,a^2\,b+168\,a\,b^2+145\,b^3\right)}{4\,d\,\left(4\,e^{2\,c+2\,dx}+6\,e^{4\,c+4\,dx}+4\,e^{6\,c+6\,dx}+e^{8\,c+8\,dx}+1\right)}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 24*a*b^2*(
d^2)^(1/2) + 48*a^2*b*(d^2)^(1/2)))/(d*(240*a*b^5 + 6144*a^5*b + 4096*a^6 +
 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4*b^2)^(1/2)))*(240*a*b^5 +
6144*a^5*b + 4096*a^6 + 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4*b^2
)^(1/2))/(64*(d^2)^(1/2)) - ((exp(c + d*x)*(a + b)^3)/(2*d) + (exp(13*c + 1
3*d*x)*(a + b)^3)/(2*d) - (3*exp(5*c + 5*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^
3))/(2*d) - (3*exp(9*c + 9*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(2*d) + (2
*exp(7*c + 7*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/d + (3*exp(3*c + 3*d
*x)*(a + b)^2*(a - b))/d + (3*exp(11*c + 11*d*x)*(a + b)^2*(a - b))/d)/(8*e
xp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c +
8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*
x) + exp(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 + 85*b^3))/(3*d*(5
*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c +
 8*d*x) + exp(10*c + 10*d*x) + 1)) + (16*b^3*exp(c + d*x))/(d*(7*exp(2*c +
2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) +
```

```
21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (
exp(c + d*x)*(24*a*b^2 + 48*a^2*b + 64*a^3 + 5*b^3))/(64*d*(exp(2*c + 2*d*x
) + 1)) - (4*exp(c + d*x)*(6*a*b^2 + 35*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15
*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c
+ 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)*(600*a*b^2 + 576*a^2*b
 + 144*a^3 + 203*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) +
 (exp(c + d*x)*(600*a*b^2 + 288*a^2*b + 305*b^3))/(24*d*(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(168*a*b^2 +
 24*a^2*b + 145*b^3))/(4*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.104 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh^5(c+dx)}{5d}-\frac{(3a-b)b^2\tanh^7(c+dx)}{7d}-\frac{b^3\tanh^9(c+dx)}{9d}$$

```
[Out] a^3*tanh(d*x+c)/d-1/3*a^2*(a-3*b)*tanh(d*x+c)^3/d-3/5*a*(a-b)*b*tanh(d*x+c)
^5/d-1/7*(3*a-b)*b^2*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 380}

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}-\frac{b^2(3a-b)\tanh^7(c+dx)}{7d}-\frac{3ab(a-b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^9(c+dx)}{9d}$$

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - 3*b)*Tanh[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x]^5)/(5*d) - ((3*a - b)*b^2*Tanh[c + d*x]^7)/(7*d) - (b^3*T
anh[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (1-x^2)\,(a+bx^2)^3\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^3 - a^2(a-3b)x^2 - 3a(a-b)bx^4 - (3a-b)b^2x^6 - b^3x^8\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^3\tanh(c+dx)}{d} - \frac{a^2(a-3b)\tanh^3(c+dx)}{3d} - \frac{3a(a-b)b\tanh^5(c+dx)}{5d} - \frac{(3a-b)b^2\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(102) = 204.

Time = 2.89 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{(5775a^3 - 1071a^2b + 621ab^2 - 725b^3 + 10(903a^3 - 63a^2b - 27ab^2 + 107b^3)\cosh(2(c+dx)) + 8(525a^3 + 1\ldots}{}$$

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((5775*a^3 - 1071*a^2*b + 621*a*b^2 - 725*b^3 + 10*(903*a^3 - 63*a^2*b - 27
*a*b^2 + 107*b^3)*Cosh[2*(c + d*x)] + 8*(525*a^3 + 126*a^2*b - 81*a*b^2 - 5
0*b^3)*Cosh[4*(c + d*x)] + 1050*a^3*Cosh[6*(c + d*x)] + 630*a^2*b*Cosh[6*(c
 + d*x)] + 270*a*b^2*Cosh[6*(c + d*x)] + 50*b^3*Cosh[6*(c + d*x)] + 105*a^3
*Cosh[8*(c + d*x)] + 63*a^2*b*Cosh[8*(c + d*x)] + 27*a*b^2*Cosh[8*(c + d*x)
] + 5*b^3*Cosh[8*(c + d*x)])*Sech[c + d*x]^8*Tanh[c + d*x])/(20160*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(94) = 188$.

Time = 80.58 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3a\,b^2\left(-\frac{s}{4}\right.}{}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3a\,b^2\left(-\frac{s}{4}\right.}{}$
risch	$-\frac{4(3465a^2b\,\mathrm{e}^{12dx+12c}+27a\,b^2+945a\,b^2\mathrm{e}^{12dx+12c}-525b^3\mathrm{e}^{12dx+12c}+105a^3-135\,\mathrm{e}^{4dx+4c}b^3+945a\,b^2\mathrm{e}^{14dx+14c}+315a^3\mathrm{e}}{}$

[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/4*sinh(d*x+c)^5/cosh(d*x+c)^9-5/24*sinh(d*x+c)^3/cosh(d*x+c)^9-5/64*sinh(d*x+c)/cosh(d*x+c)^9+5/64*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(94) = 188$.

Time = 0.26 (sec) , antiderivative size = 1185, normalized size of antiderivative = 11.62

$$\int \mathrm{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -8/315*(2*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^7 + 14*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3 + 7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^3 + 3*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 18*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cosh(d*x + c)^3 + (35*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^4 + 94

```
5*a^3 + 1701*a^2*b + 459*a*b^2 + 855*b^3 + 30*(175*a^3 + 483*a^2*b + 117*a*
b^2 - 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 252*a^2*b
+ 243*a*b^2 + 80*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 336*a^2*b + 99*a*b^2
- 40*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 12*a^2*b + 9*a*b^2)*cosh(d*x +
c) + (7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^6 + 15*(1
75*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^4 + 525*a^3 + 693*a^
2*b + 567*a*b^2 - 945*b^3 + 27*(105*a^3 + 189*a^2*b + 51*a*b^2 + 95*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh
(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x +
c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x + c))
*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d*co
sh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*cos
h(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^5 +
 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^2 +
 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5 +
1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*cosh(
d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cosh(d
*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d*x
+ c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x + c
)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d*co
sh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*cosh(
d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2{(c+dx)}\right)^3\operatorname{sech}^4{(c+dx)}\,dx$$

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(94) = 188$.

Time = 0.22 (sec) , antiderivative size = 1847, normalized size of antiderivative = 18.11

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/63*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e
^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d
*x - 18*c) + 1)) - 27*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*
x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10
*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c)
+ e^(-18*d*x - 18*c) + 1)) + 189*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) +
36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-
10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d
*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 189*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*
x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
 + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 315*e^(-10*d*x - 10*c)/(
d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(
-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*
d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 105*e^(-12*
d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6
*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
 + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))
+ 63*e^(-14*d*x - 14*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12
*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x
 - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 1
2*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
1))) + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*
x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c
) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(
d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-
8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x -
14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4
*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7
*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*
e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21
*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d
*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/5*a^2*b*(
5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x -
4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^
(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x -
```

```
8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.

Time = 0.44 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.38

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{4\left(315\,a^3e^{(14\,dx+14\,c)}+945\,a^2be^{(14\,dx+14\,c)}+945\,ab^2e^{(14\,dx+14\,c)}+315\,b^3e^{(14\,dx+14\,c)}+1995\,a^3e^{(12\,dx+12\,c)}+3\right.}{}$$

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 945*a^2*b*e^(14*d*x + 14*c) + 945*a*b^2
*e^(14*d*x + 14*c) + 315*b^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c)
 + 3465*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) - 525*b^3*e^(
12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 4725*a^2*b*e^(10*d*x + 10*c)
+ 945*a*b^2*e^(10*d*x + 10*c) + 1575*b^3*e^(10*d*x + 10*c) + 7875*a^3*e^(8*
d*x + 8*c) + 3213*a^2*b*e^(8*d*x + 8*c) + 2457*a*b^2*e^(8*d*x + 8*c) - 945*
b^3*e^(8*d*x + 8*c) + 6825*a^3*e^(6*d*x + 6*c) + 1827*a^2*b*e^(6*d*x + 6*c)
 + 1323*a*b^2*e^(6*d*x + 6*c) + 945*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x
 + 4*c) + 1323*a^2*b*e^(4*d*x + 4*c) + 27*a*b^2*e^(4*d*x + 4*c) - 135*b^3*e
^(4*d*x + 4*c) + 945*a^3*e^(2*d*x + 2*c) + 567*a^2*b*e^(2*d*x + 2*c) + 243*
a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) + 105*a^3 + 63*a^2*b + 27*a*
b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 1424, normalized size of antiderivative = 13.96

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

```
[Out] - ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(2*c + 2*d*x)*(a + b)^3)/(9*d))/(3*
exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((5*exp(8*c
 + 8*d*x)*(a + b)^3)/(9*d) - (a*b^2 + a^2*b - 5*a^3 - 5*b^3)/(21*d) - (10*e
xp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(2*c + 2*d
```

```
*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(63*d) + (40*exp(6*c + 6*d*x)*(a +
 b)^2*(a - b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*
x) + 1) - ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(10*c + 10*d*x)*(a + b)^3)/
(3*d) - (2*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (20*ex
p(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(4*c + 4*d*
x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(21*d) + (20*exp(8*c + 8*d*x)*(a +
b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c
 + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d
*x) + exp(14*c + 14*d*x) + 1) - ((16*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(
315*d) + (4*exp(6*c + 6*d*x)*(a + b)^3)/(9*d) - (4*exp(2*c + 2*d*x)*(a*b^2
+ a^2*b - 5*a^3 - 5*b^3))/(21*d) + (8*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(
7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*e
xp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a + b)^3)
/(9*d) + (8*exp(14*c + 14*d*x)*(a + b)^3)/(9*d) - (8*exp(6*c + 6*d*x)*(a*b^
2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) - (8*exp(10*c + 10*d*x)*(a*b^2 + a^2*b -
5*a^3 - 5*b^3))/(3*d) + (32*exp(8*c + 8*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5
*b^3))/(9*d) + (16*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(3*d) + (16*exp(12*c
 + 12*d*x)*(a + b)^2*(a - b))/(3*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d
*x) + 84*exp(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) +
 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp
(18*c + 18*d*x) + 1) - ((a + b)^3/(9*d) + (7*exp(12*c + 12*d*x)*(a + b)^3)/
(9*d) - (exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/d - (5*exp(8*c +
 8*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) + (16*exp(6*c + 6*d*x)*(3*a*
b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(9*d) + (4*exp(2*c + 2*d*x)*(a + b)^2*(a -
b))/(3*d) + (4*exp(10*c + 10*d*x)*(a + b)^2*(a - b))/d)/(8*exp(2*c + 2*d*x)
 + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp
(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c +
 16*d*x) + 1) - ((exp(4*c + 4*d*x)*(a + b)^3)/(3*d) - (a*b^2 + a^2*b - 5*a^
3 - 5*b^3)/(21*d) + (4*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*
c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
) - (a + b)^3/(9*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.105 $\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2+10ab+15b^2)\,x}{8(a+b)^3}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d}+\frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d}+\frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

```
[Out] 1/8*(3*a^2+10*a*b+15*b^2)*x/(a+b)^3+1/8*(3*a+7*b)*cosh(d*x+c)*sinh(d*x+c)/(
a+b)^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+b^(5/2)*arctan(b^(1/2)*tanh(
d*x+c)/a^(1/2))/(a+b)^3/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 212, 211}

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x(3a^2+10ab+15b^2)}{8(a+b)^3}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}+\frac{(3a+7b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2}$$

```
[In] Int[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 10*a*b + 15*b^2)*x)/(8*(a + b)^3) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((3*a + 7*b)*Cosh[c + d*x]*Sin
h[c + d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}\\
&= \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^2d}\\
&= \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}\\
&\quad + \frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d}\\
&\quad + \frac{(3a^2+10ab+15b^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{8(a+b)^3d}\\
&= \frac{(3a^2+10ab+15b^2)\,x}{8(a+b)^3} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d}\\
&\quad + \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\begin{aligned}
\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{(3a^2+10ab+15b^2)\,(c+dx)}{8(a+b)^3d} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d}\\
&\quad + \frac{(a+2b)\sinh(2(c+dx))}{4(a+b)^2d} + \frac{\sinh(4(c+dx))}{32(a+b)d}
\end{aligned}$$

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 10*a*b + 15*b^2)*(c + d*x))/(8*(a + b)^3*d) + (b^(5/2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((a + 2*b)*Sinh[2*(c
 + d*x)])/(4*(a + b)^2*d) + Sinh[4*(c + d*x)]/(32*(a + b)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(106) = 212$.

Time = 11.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.28

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{5axb}{4(a+b)^3}+\frac{15x\,b^2}{8(a+b)^3}+\frac{e^{4dx+4c}}{64d(a+b)}+\frac{e^{2dx+2c}a}{8(a+b)^2d}+\frac{e^{2dx+2c}b}{4(a+b)^2d}-\frac{e^{-2dx-2c}a}{8(a^2+2ab+b^2)d}-\frac{e^{-2dx-2c}b}{4(a^2+2ab+b^2)d}$
derivativedivides	$\frac{1}{2(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-7a-11b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-5a-9b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-3a^2}{}$
default	$\frac{1}{2(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-7a-11b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-5a-9b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-3a^2}{}$

```
[In] int(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x/(a+b)^3+5/4*a*x/(a+b)^3*b+15/8*x/(a+b)^3*b^2+1/64/d/(a+b)*exp(4*d
*x+4*c)+1/8/(a+b)^2/d*exp(2*d*x+2*c)*a+1/4/(a+b)^2/d*exp(2*d*x+2*c)*b-1/8/(
a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)*a-1/4/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)*b-
1/64/d/(a+b)*exp(-4*d*x-4*c)+1/2/a*(-a*b)^(1/2)*b^2/(a+b)^3/d*ln(exp(2*d*x+
2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))-1/2/a*(-a*b)^(1/2)*b^2/(a+b)^3/d*ln(exp(2*
d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(106) = 212$.

Time = 0.30 (sec) , antiderivative size = 2180, normalized size of antiderivative = 18.17

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 + 10
*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*a*b + 4*b^2)*sinh(
d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6*(a^2 + 3*a*b + 2*
```

b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a
^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
 + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(7*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(
d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2*a^2 - 6*a*b - 4*b
^2)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh
(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b
^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*si
nh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d
x + c)^7 + 4(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 + 3*a*
b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*
x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)
^8 + 8*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b
^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*
a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6
*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b
+ 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d
x + c)^5 + 4(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b
 + 2*b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d
x + c)^2 + 4(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b +
 15*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2
*a^2 - 6*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*
cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*s
inh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^7 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 +
6*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x

```
+ c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*sinh(d*x + c)^4)]
```

Sympy [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(cosh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(cosh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(106) = 212.

Time = 0.33 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.28

$$\begin{aligned}\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{(ab-b^2)(dx+c)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{\left(8\,be^{(-2\,dx-2\,c)}+a+b\right)e^{(4\,dx+4\,c)}}{64\,(a^2+2\,ab+b^2)d}\\&+\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}\\&-\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}\\&-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d}\\&-\frac{(a^2b-6\,ab^2+b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}d}\\&+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d}\\&-\frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}(a+b)d}\\&-\frac{8\,be^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}}{64\,(a^2+2\,ab+b^2)d}\\&+\frac{3\,(dx+c)}{8\,(a+b)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b*
e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) + 1/4*b*l
og((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a
*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/8
*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) + 1/4*(a*b - b^2)*arct
an(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*s
qrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) + 1/8*e^(2*d*x +
2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Giac [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\cosh\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 967, normalized size of antiderivative = 8.06

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{x\,\left(3\,a^2+10\,a\,b+15\,b^2\right)}{8\,{\left(a+b\right)}^3}-\frac{{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d\,\left(a+b\right)}+\frac{{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d\,\left(a+b\right)}+\frac{\mathrm{atan}\left(\left(\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(\frac{4\,b^3}{d\,{\left(a+b\right)}^5\,\sqrt{b^5}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}+\frac{\left(a-b\right)\,\left(a^4\,d\,\sqrt{b^5}-b^4\,d\,\sqrt{b^5}-2\,a\,b^3\,d\,\sqrt{b^5}+2\,a\right)}{b^3\,{\left(a+b\right)}^2\,\sqrt{a\,d^2\,{\left(a+b\right)}^6}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)\,\sqrt{a^7\,d^2+6\,a^6\,b\,d^2+15\,a^5\,b^2\,d^2}}\right)\right)\right)\right)}{}-\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(a+2\,b\right)}{8\,d\,{\left(a+b\right)}^2}+\frac{{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{8\,d\,{\left(a+b\right)}^2}$$

```
[In] int(cosh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (x*(10*a*b + 3*a^2 + 15*b^2))/(8*(a + b)^3) - exp(- 4*c - 4*d*x)/(64*d*(a +
 b)) + exp(4*c + 4*d*x)/(64*d*(a + b)) + (atan((exp(2*c)*exp(2*d*x)*((4*b^3
)/(d*(a + b)^5*(b^5)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + ((a - b)*(a^4
*d*(b^5)^(1/2) - b^4*d*(b^5)^(1/2) - 2*a*b^3*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5
```

```
)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 +
 b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 +
 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1/2) + b
^4*d*(b^5)^(1/2) + 4*a*b^3*d*(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) + 6*a^2*b^
2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2 + 3*a^2*b
 + a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b
^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*((a^4*(a^7*d^2 + a*b^6*d^
2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*
b^2*d^2)^(1/2))/2 + (b^4*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2
 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/2 + 2*a*b^3*(a^
7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b
^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 2*a^3*b*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2
 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)
+ 3*a^2*b^2*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*(b^5)^(1/2))/(a^7*d^2 + a*b
^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15
*a^5*b^2*d^2)^(1/2) - (exp(- 2*c - 2*d*x)*(a + 2*b))/(8*d*(a + b)^2) + (exp
(2*c + 2*d*x)*(a + 2*b))/(8*d*(a + b)^2)
```

3.106 $\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}d}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2d}+\frac{\sinh^3(c+dx)}{3(a+b)d}$$

```
[Out] (a+2*b)*sinh(d*x+c)/(a+b)^2/d+1/3*sinh(d*x+c)^3/(a+b)/d+b^2*arctan(sinh(d*x
+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 398, 211}

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{5/2}}+\frac{\sinh^3(c+dx)}{3d(a+b)}+\frac{(a+2b)\sinh(c+dx)}{d(a+b)^2}$$

```
[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)*d)
 + ((a + 2*b)*Sinh[c + d*x])/((a + b)^2*d) + Sinh[c + d*x]^3/(3*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2(a+(a+b)x^2)}\right)dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d}+\frac{\sinh^3(c+dx)}{3(a+b)d}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^2 d}\\
&= \frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}d}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d}+\frac{\sinh^3(c+dx)}{3(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\frac{12b^2\arctan\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{3(3a+7b)\sinh(c+dx)}{(a+b)^2}+\frac{\sinh(3(c+dx))}{a+b}}{12d}$$

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((-12*b^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/
2)) + (3*(3*a + 7*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[3*(c + d*x)]/(a + b))/
(12*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.

Time = 3.97 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.88

method	result
risch	$\frac{e^{3dx+3c}}{24d(a+b)} + \frac{3\,e^{dx+c}a}{8(a+b)^2 d} + \frac{7\,e^{dx+c}b}{8(a+b)^2 d} - \frac{3\,e^{-dx-c}a}{8(a+b)^2 d} - \frac{7\,e^{-dx-c}b}{8(a+b)^2 d} - \frac{e^{-3dx-3c}}{24d(a+b)} - \frac{b^2 \ln\left(e^{2dx+2c} - \frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}} - 1\right)}{2\sqrt{-a^2-ab}\,(a+b)^2 d} +$
derivativedivides	$\frac{-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2a+2b)} - \frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{a+2b}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{2}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(2a+2b)} + \frac{\cdots}{(2a+2\cdots}}{}$
default	$\frac{-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2a+2b)} - \frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{a+2b}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{2}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(2a+2b)} + \frac{\cdots}{(2a+2\cdots}}{}$

```
[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d/(a+b)*exp(3*d*x+3*c)+3/8/(a+b)^2/d*exp(d*x+c)*a+7/8/(a+b)^2/d*exp(d*
x+c)*b-3/8/(a+b)^2/d*exp(-d*x-c)*a-7/8/(a+b)^2/d*exp(-d*x-c)*b-1/24/d/(a+b)
*exp(-3*d*x-3*c)-1/2/(-a^2-a*b)^(1/2)*b^2/(a+b)^2/d*ln(exp(2*d*x+2*c)-2*a/(
-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/2/(-a^2-a*b)^(1/2)*b^2/(a+b)^2/d*ln(exp(2*d
*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(70) = 140$.

Time = 0.30 (sec) , antiderivative size = 1850, normalized size of antiderivative = 23.12

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^6 + 3
*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b
^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3
 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d
```

```
*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*
a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 3*a
^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2
- a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*c
osh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3
*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*(
(a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) +
6*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)
*cosh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c
))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*d*sinh(d*x + c)^3), 1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*
a^2*b + a*b^2)*sinh(d*x + c)^6 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x +
c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b
- a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^
2*b + a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a
^2*b + 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3
+ 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2
 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3
 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a
 - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/
sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x
 + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2
 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + 6*(
(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)*co
sh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*sinh(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^3{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh{(dx+c)}^3}{b\tanh{(dx+c)}^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(3*a*e^(4*c) + 7*b*e^(4*c))*e^(
4*d*x) - 3*(3*a*e^(2*c) + 7*b*e^(2*c))*e^(2*d*x) - a - b)*e^(-3*d*x)/(a^2*d
*e^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) + 1/8*integrate(16*(b^2*e^(3*d*
x + 3*c) + b^2*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) +
 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c
) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh{(dx+c)}^3}{b\tanh{(dx+c)}^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 2194, normalized size of antiderivative = 27.42

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

[Out] exp(3*c + 3*d*x)/(24*d*(a + b)) - exp(- 3*c - 3*d*x)/(24*d*(a + b)) + ((b^4
)^(1/2)*(2*atan((exp(d*x)*exp(c)*((4*(10*a^2*d*(b^4)^(5/2) + 12*a^6*d*(b^4)
^(3/2) + 2*a*b^9*d*(b^4)^(1/2) + 10*a^3*b^3*d*(b^4)^(3/2) + 2*a^2*b^8*d*(b^
4)^(1/2) + 20*a^3*b^7*d*(b^4)^(1/2) + 40*a^4*b^6*d*(b^4)^(1/2) + 30*a^5*b^5
d(b^4)^(1/2) + 2*a^7*b^3*d*(b^4)^(1/2)))/(a*b^5*(a + b)^5*(a*d^2*(a + b)^
5)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4*b + a
^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*
a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)) - (2*(b^9*(a^6*d^2 +
a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^
(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3
*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 6*a^2*b^7*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b
*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 4*a^3*b^6*(
a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4
*b^2*d^2)^(1/2) + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))/(a^2*b^3*d*(a + b)^7*(b^4)^(1/
2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4*b + a^5 + b
^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^
4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))) + (2*exp(3*c)*exp(3*d*x)*(
b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 1
0*a^4*b^2*d^2)^(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b
^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 6*a^2*b^7*(a^6*d^2 + a*b^
5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2
) + 4*a^3*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b
^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2
 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))/(a^2*b^3*d*(a +
 b)^7*(b^4)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*
a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b
*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))*((a^11*(a^6
*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^
2*d^2)^(1/2))/4 + (a*b^10*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/4 + (5*a^10*b*(a^6*d^2 + a*b^5*
d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))
/2 + (5*a^2*b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3
*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/2 + (45*a^3*b^8*(a^6*d^2 + a*b^5*d^2 + 5*
a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/4 + 30*
a^4*b^7*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2
 + 10*a^4*b^2*d^2)^(1/2) + (105*a^5*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2
+ 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/2 + 63*a^6*b^5*(a
^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*
b^2*d^2)^(1/2) + (105*a^7*b^4*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^
4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/2 + 30*a^8*b^3*(a^6*d^2 + a
*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(
1/2) + (45*a^9*b^2*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*
a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/4)) + 2*atan((b^2*exp(d*x)*exp(c)*(a*d
^2*(a + b)^5)^(1/2))/(2*a*d*(a + b)^2*(b^4)^(1/2)))))/(2*(a^6*d^2 + a*b^5*d

```
^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))
- (exp(- c - d*x)*(3*a + 7*b))/(8*d*(a + b)^2) + (exp(c + d*x)*(3*a + 7*b))
/(8*d*(a + b)^2)
```

3.107 $\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+3b)x}{2(a+b)^2}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

```
[Out] 1/2*(a+3*b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d+b^(3/2)*arctan(b^
(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3756, 425, 536, 212, 211}

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}+\frac{x(a+3b)}{2(a+b)^2}$$

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + 3*b)*x)/(2*(a + b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a]*(a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} + \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^2d}$$

$$= \frac{(a+3b)x}{2(a+b)^2} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\sqrt{a}(a+3b)(c+dx) + 4b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a}(a+b)\sinh(2(c+dx))}{4\sqrt{a}(a+b)^2d}$$

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
[Out] (2*Sqrt[a]*(a + 3*b)*(c + d*x) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/S
qrt[a]] + Sqrt[a]*(a + b)*Sinh[2*(c + d*x)])/(4*Sqrt[a]*(a + b)^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(65) = 130.

Time = 1.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

method	result
risch	$\frac{ax}{2(a+b)^2} + \frac{3xb}{2(a+b)^2} + \frac{e^{2dx+2c}}{8d(a+b)} - \frac{e^{-2dx-2c}}{8d(a+b)} + \frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c} + \frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a(a+b)^2d} - \frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c} - \frac{2\sqrt{-a}}{a}\right.}{2a(a+b)^2d}$
derivativedivides	$\frac{-\frac{1}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{2}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} + \frac{(a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2} + \frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{}{(4a+4}}{}$
default	$\frac{-\frac{1}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{2}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} + \frac{(a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2} + \frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{}{(4a+4}}{}$

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/(a+b)^2+3/2*x/(a+b)^2*b+1/8/d/(a+b)*exp(2*d*x+2*c)-1/8/d/(a+b)*exp(
-2*d*x-2*c)+1/2/a*(-a*b)^(1/2)*b/(a+b)^2/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2
)+a-b)/(a+b))-1/2/a*(-a*b)^(1/2)*b/(a+b)^2/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1
/2)-a+b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 948, normalized size of antiderivative = 12.31

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)
*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d
*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sq
rt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)
*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a + 3*b)*d*x*cosh(d*x + c)
+ (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a - b)/((a^2 + 2*a*b + b^2)*d*co
sh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ 2*a*b + b^2)*d*sinh(d*x + c)^2), 1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*
x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(2
*(a + 3*b)*d*x*cosh(d*x + c) + (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a -
 b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d
*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.10

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &\quad - \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &\quad - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &\quad + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &\quad - \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}(a+b)d} \\ &\quad + \frac{dx+c}{2\,(a+b)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((
a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d
) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x -
 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/2*(d*x + c)/((a + b)*d)
 + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Giac [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 880, normalized size of antiderivative = 11.43

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{2c+2dx}}{8d(a+b)} - \frac{e^{-2c-2dx}}{8d(a+b)} + \frac{x(a+3b)}{2(a+b)^2} + \frac{\operatorname{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{2\left(2b^3\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}+2ab^2\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}\right)}{d(a+b)^5\sqrt{b^3}(a^3+3a^2b+3ab^2+b^3)\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}} - \frac{}{b^2}\right.\right.\right.}{}$$

[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)

[Out] exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) + (x*(a + 3*b))/(2*(a + b)^2) + (atan((exp(2*c)*exp(2*d*x)*((2*(2*b^3*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^2*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(d*(a + b)^5*(b^3)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - ((a - b)*(2*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) - a^4*d*(b^3)^(1/2) - 2*a^3*b*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))) + ((a - b)*(4*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) + a^4*d*(b^3)^(1/2) + 4*a^3*b*d*(b^3)^(1/2) + 6*a^2*b^2*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^4*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + (b^4*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + 3*a^2*b^2*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^3*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a^3*b*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))*(b^3)^(1/2))/(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)

3.108 $\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

```
[Out] sinh(d*x+c)/(a+b)/d+b*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(3/2)/d
/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3757, 396, 211}

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^{3/2}} + \frac{\sinh(c+dx)}{d(a+b)}$$

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) +
 Sinh[c + d*x]/((a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\sinh(c+dx)}{(a+b)d} + \frac{b\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)d}\\
&= \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) +
 Sinh[c + d*x]/((a + b)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(45) = 90$.

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.81

method	result
risch	$\frac{e^{dx+c}}{2d(a+b)}-\frac{e^{-dx-c}}{2d(a+b)}-\frac{b\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}$
derivativedivides	$\frac{-\frac{2}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2ba\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)}{2a\sqrt{(}}\right.}{a+b}}{d}$
default	$\frac{-\frac{2}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2ba\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)}{2a\sqrt{(}}\right.}{a+b}}{d}$

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/(a+b)*exp(d*x+c)-1/2/d/(a+b)*exp(-d*x-c)-1/2/(-a^2-a*b)^(1/2)*b/(a+b)
/d*ln(exp(2*d*x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/2/(-a^2-a*b)^(1/2
)*b/(a+b)/d*ln(exp(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 766, normalized size of antiderivative = 14.45

$$\int\frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$=\left[\frac{(a^2+ab)\cosh(dx+c)^2+2(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2-\sqrt{-a^2-a}}{}\right.$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*si
nh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
```

```
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*
x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 +
 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
 b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
b)) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b
+ a*b^2)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)
*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + 2*sqrt(a^2 + a
*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c)^3
 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a
 - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/
sqrt(a^2 + a*b)) + 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*ar
ctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - a^2 - a*b)/((
a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x
 + c))]
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\cosh\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\cosh\left(dx+c\right)}{b\tanh\left(dx+c\right)^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b
*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b
*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)
), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.91

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{c+dx}}{2d(a+b)} - \frac{e^{-c-dx}}{2d(a+b)} - \frac{b\ln\left(\sqrt{-a}\sqrt{a+b}+2a\,e^{c+dx}-\sqrt{-a}\,e^{2c+2dx}\sqrt{a+b}\right)}{2\sqrt{-a}\,d\,(a+b)^{3/2}} + \frac{b\ln\left(2a\,e^{c+dx}-\sqrt{-a}\sqrt{a+b}+\sqrt{-a}\,e^{2c+2dx}\sqrt{a+b}\right)}{2\sqrt{-a}\,d\,(a+b)^{3/2}}$$

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
[Out] exp(c + d*x)/(2*d*(a + b)) - exp(- c - d*x)/(2*d*(a + b)) - (b*log((-a)^(1/
2)*(a + b)^(1/2) + 2*a*exp(c + d*x) - (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(
1/2)))/(2*(-a)^(1/2)*d*(a + b)^(3/2)) + (b*log(2*a*exp(c + d*x) - (-a)^(1/2
)*(a + b)^(1/2) + (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2)))/(2*(-a)^(1/2)
*d*(a + b)^(3/2))
```

3.109 $\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

```
[Out] arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3757, 211}

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(28) = 56.

Time = 0.90 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

method	result	size
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}$	102
derivativedivides	$\frac{2a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	152
default	$\frac{2a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	152

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1
)+1/2/(-a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-
1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 511, normalized size of antiderivative = 14.19

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2\,(3\,a+b)\cosh(dx+c)^2+2\,\left(3\,(a+b)\cosh(dx\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(}\right.}{}\right.$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(
d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
 c) + a + b))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*
x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^
3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*
x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh
(d*x + c) + sinh(d*x + c))/a))/((a^2 + a*b)*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*tanh(d*x + c)^2 + a), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.08

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{4\,a^2\,d^2\,e^{d\,x}\,e^c-e^{d\,x}\,e^c\,\sqrt{a^2\,d^2+b\,a\,d^2}\,\sqrt{a\,d^2\,(a+b)}+e^{3\,c}\,e^{3\,d\,x}\,\sqrt{a^2\,d^2+b\,a\,d^2}\,\sqrt{a\,d^2\,(a+b)}}{2\,a\,d\,\sqrt{a\,d^2\,(a+b)}}\right)+\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\,\sqrt{a\,d^2\,(a+b)}}{2\,a\,d}\right)}{\sqrt{a^2\,d^2+b\,a\,d^2}}$$

```
[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] (atan((4*a^2*d^2*exp(d*x)*exp(c) - exp(d*x)*exp(c)*(a^2*d^2 + a*b*d^2)^(1/2
)*(a*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(a^2*d^2 + a*b*d^2)^(1/2)*(a*
d^2*(a + b))^(1/2))/(2*a*d*(a*d^2*(a + b))^(1/2))) + atan((exp(d*x)*exp(c)*
(a*d^2*(a + b))^(1/2))/(2*a*d)))/(a^2*d^2 + a*b*d^2)^(1/2)
```

3.110 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 211}

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(24) = 48.

Time = 3.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

method	result	size
risch	$-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}$	114
derivativedivides	$\frac{2a\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{d}$	160
default	$\frac{2a\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{d}$	160

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)-2*a*b)
/(a+b)/(-a*b)^(1/2))+1/2/(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b
*(-a*b)^(1/2)+2*a*b)/(a+b)/(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(24) = 48$.

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 14.22

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)\sinh(dx+c)^4+2(a^2-b^2)\cosh(dx+c)^2+}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c}\right.}{}\right.$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh
(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)
^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(
3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/(a*b*d), sqrt(a*b)*ar
ctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
 (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b))/(a*b*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}d}$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^2+a}\,dx$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{a\,\sqrt{a\,b\,d^2}-b\,\sqrt{a\,b\,d^2}+a\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{a\,b\,d^2}+b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{a\,b\,d^2}}{2\,a\,b\,d}\right)}{\sqrt{a\,b\,d^2}}$$

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
[Out] atan((a*(a*b*d^2)^(1/2) - b*(a*b*d^2)^(1/2) + a*exp(2*c)*exp(2*d*x)*(a*b*d^
2)^(1/2) + b*exp(2*c)*exp(2*d*x)*(a*b*d^2)^(1/2))/(2*a*b*d))/(a*b*d^2)^(1/2
)
```

3.111 $\int \frac{\text{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\text{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\arctan(\sinh(c+dx))}{bd} + \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

```
[Out] -arctan(sinh(d*x+c))/b/d+arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2
)/b/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3757, 400, 209, 211}

$$\int \frac{\text{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd} - \frac{\arctan(\sinh(c+dx))}{bd}$$

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b*d)) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c +
d*x])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{bd} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{bd}\\
&= -\frac{\arctan(\sinh(c+dx))}{bd} + \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$
\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}
$$

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(((Sqrt[a + b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2*Ar
cTan[Tanh[(c + d*x)/2]])/(b*d))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.71 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.62

method	result
risch	$\frac{i\ln(\mathrm{e}^{dx+c}-i)}{db}-\frac{i\ln(\mathrm{e}^{dx+c}+i)}{db}+\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-a(a+b)}\,\mathrm{e}^{dx+c}}{a+b}-1\right)}{2adb}-\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-a}}{}\right.}{2adb}$
derivativedivides	$\frac{\frac{2a(a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{b}-\frac{2\arctan\left(\tanh\left(\frac{dx}{2}\right.\right.}{b}}{d}$
default	$\frac{\frac{2a(a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{b}-\frac{2\arctan\left(\tanh\left(\frac{dx}{2}\right.\right.}{b}}{d}$

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] I/d/b*ln(exp(d*x+c)-I)-I/d/b*ln(exp(d*x+c)+I)+1/2/a*(-a*(a+b))^(1/2)/d/b*ln
(exp(2*d*x+2*c)+2*(-a*(a+b))^(1/2)/(a+b)*exp(d*x+c)-1)-1/2/a*(-a*(a+b))^(1/
2)/d/b*ln(exp(2*d*x+2*c)-2*(-a*(a+b))^(1/2)/(a+b)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 9.82

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$=\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2\,(3\,a+b)\cosh(dx+c)^2+2\left(3\,(a+b)\cosh(dx+c)^2-\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b}\right.}{}\right.$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
```

```
(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3
 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c)
+ (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a) + a + b)/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), (
sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c)))
 + sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) +
 (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a +
b)) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -2*arctan(e^(d*x + c))/(b*d) + 8*integrate(1/4*((a*e^(3*c) + b*e^(3*c))*e^(
3*d*x) + (a*e^c + b*e^c)*e^(d*x))/(a*b + b^2 + (a*b*e^(4*c) + b^2*e^(4*c))*
e^(4*d*x) + 2*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 449, normalized size of antiderivative = 8.16

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b}\left(2\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\sqrt{a+b}\sqrt{a\,b^2\,d^2}}{2\,a\,b\,d}\right)-2\operatorname{atan}\left(\frac{\left(e^{dx}\,e^{c}\left(\frac{64\left(2\,a\,b^2\,d\sqrt{a+b}-6\,a^2\,b\,d\sqrt{a+b}\right)}{a^3\,b^3\,d^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}+\frac{32\left(3\,a^2\sqrt{a\,b^2\,d^2}-b^2\sqrt{a\,b^2\,d^2}+2\,a\,b\sqrt{a\,b^2\,d^2}\right)}{a^3\,b^2\,d\,(a+b)^{3/2}\,\left(a^2+2\,a\,b+b^2\right)\sqrt{a\,b^2\,d^2}}\right.\cdots}{\cdots}\right)\cdots\right)}{2\sqrt{a\,b^2\,d^2}} - \frac{2\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(9\,a^2\sqrt{b^2\,d^2}+b^2\sqrt{b^2\,d^2}-6\,a\,b\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b-6\,d\,a\,b^2+d\,b^3}\right)}{\sqrt{b^2\,d^2}}$$

```
[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] ((a + b)^(1/2)*(2*atan((exp(d*x)*exp(c)*(a + b)^(1/2)*(a*b^2*d^2)^(1/2))/(2
*a*b*d)) - 2*atan(((exp(d*x)*exp(c)*((64*(2*a*b^2*d*(a + b)^(1/2) - 6*a^2*b
*d*(a + b)^(1/2)))/(a^3*b^3*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) + (32*(3*a^2
*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*(a*b^2*d^2)^(1/2)))/(a^3
*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^2)^(1/2))) - (32*exp(3*c)
*exp(3*d*x)*(3*a^2*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*(a*b^2
*d^2)^(1/2)))/(a^3*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^2)^(1/2
)))*(a^4*b*(a + b)*(a*b^2*d^2)^(1/2) + a^2*b^3*(a + b)*(a*b^2*d^2)^(1/2) +
2*a^3*b^2*(a + b)*(a*b^2*d^2)^(1/2)))/(192*a - 64*b))))/(2*(a*b^2*d^2)^(1/2
)) - (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + b^2*(b^2*d^2)^(1/2)
- 6*a*b*(b^2*d^2)^(1/2)))/(b^3*d - 6*a*b^2*d + 9*a^2*b*d)))/(b^2*d^2)^(1/2)
```

3.112 $\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} - \frac{\tanh(c+dx)}{bd}$$

```
[Out] (a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/d/a^(1/2)-tanh(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 396, 211}

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} - \frac{\tanh(c+dx)}{bd}$$

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\tanh(c+dx)}{bd} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{bd} \\
&= \frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} - \frac{\tanh(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} - \frac{\tanh(c+dx)}{bd}$$

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(42) = 84$.

Time = 21.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.94

method	result
derivativedivides	$\frac{\frac{2a(a+b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{b}-\frac{2\tan\ldots}{b\left(\tanh\ldots\right.}}{d}$
default	$\frac{\frac{2a(a+b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{b}-\frac{2\tan\ldots}{b\left(\tanh\ldots\right.}}{d}$
risch	$\frac{2}{bd\left(e^{2dx+2c}+1\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a}{2\sqrt{-ab}\,db}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}\ldots}{(a\ldots}\right.}{2\sqrt{-ab}\ldots}$

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b*a*(a+b)*(-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a
-2*b)*a)^(1/2))+1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*
b)*a)^(1/2)))-2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 649, normalized size of antiderivative = 12.98

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[-\frac{\left((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a+b\right)\sqrt{-ab}\,\mathrm{l}\ldots}{\ldots}\right.$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a + b)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2
+ 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x +
c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4
+ 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a
- b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^
2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
 b)) - 4*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x +
 c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), (((a + b)*cosh(d*x + c)^2 + 2*(a
+ b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt(a*
b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x +
 c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 2*a*b)/(a*b^2*d*c
osh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x +
 c)^2 + a*b^2*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^4\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}bd}-\frac{2}{(be^{(-2\,dx-2\,c)}+b)d}$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -(a + b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b
)*b*d) - 2/((b*e^(-2*d*x - 2*c) + b)*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^2+a}\,dx$$

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.52

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}\right)\,\left(a+b\right)}{2\,\sqrt{-a}\,b^{3/2}\,d} - \frac{\ln\left(\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}\right)\,\left(a+b\right)}{2\,\sqrt{-a}\,b^{3/2}\,d}$$

[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)

[Out] 2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x))/b - (2*(a*d + b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d))*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d) - (log((2*(a*d + b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d) - (4*exp(2*c + 2*d*x))/b)*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d)

3.113 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(2a+3b)\arctan(\sinh(c+dx))}{2b^2d} + \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a+3*b)*arctan(sinh(d*x+c))/b^2/d+(a+b)^(3/2)*arctan(sinh(d*x+c)*(a+
b)^(1/2)/a^(1/2))/b^2/d/a^(1/2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3757, 425, 536, 209, 211}

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} - \frac{(2a+3b)\arctan(\sinh(c+dx))}{2b^2d} - \frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd}$$

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a + 3*b)*ArcTan[Sinh[c + d*x]])/(b^2*d) + ((a + b)^(3/2)*ArcTan[(S
qrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) - (Sech[c + d*x]*Tanh[c
 + d*x])/(2*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{2bd}\end{aligned}$$

$$= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2b^2d}$$

$$= -\frac{(2a+3b)\arctan(\sinh(c+dx))}{2b^2d} + \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2(2a+3b)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + b\operatorname{sech}(c+dx)\tanh(c+dx)}{2b^2d}$$

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((2*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a]
 + 2*(2*a + 3*b)*ArcTan[Tanh[(c + d*x)/2]] + b*Sech[c + d*x]*Tanh[c + d*x])
/(b^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(74) = 148$.

Time = 50.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.74

method	result
derivativedivides	$-\frac{2\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh \cdots}{\sqrt{\left(2\sqrt{(a \cdots}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b} \cdots}} \cdots\right)}{d}$
default	$-\frac{2\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh \cdots}{\sqrt{\left(2\sqrt{(a \cdots}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b} \cdots}} \cdots\right)}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{db\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}-i\right)a}{d\,b^2}+\frac{3i\ln\left(e^{dx+c}-i\right)}{2db}-\frac{i\ln\left(e^{dx+c}+i\right)a}{d\,b^2}-\frac{3i\ln\left(e^{dx+c}+i\right)}{2db}+\frac{\sqrt{-a(a+b)}\,\ln \cdots}{\cdots}$

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*((-1/2*tanh(1/2*d*x+1/2*c)^3*b+1/2*b*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^2+1)^2+1/2*(2*a+3*b)*arctan(tanh(1/2*d*x+1/2*c)))+2/b^2*(a^
2+2*a*b+b^2)*a*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*
a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1
/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 1584, normalized size of antiderivative = 18.42

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d
*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*co
sh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a +
```

```
b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(-(a + b)/a)*log(((a + b)*cosh
(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x +
c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b
)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*s
inh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a
*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c
))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*((2*a + 3*b)
*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a + 3*b
)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(3*(2*a + 3*b)*cosh(d
*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a + 3*b)*cosh(d*x + c)^3 + (
2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3*b)*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x +
 c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x
+ c))*sinh(d*x + c)), -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)
^2 + b*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*
(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
 c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt((a + b)/a)*arcta
n(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
+ 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c
) + a + b)*sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*
x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/
a)/(a + b)) + ((2*a + 3*b)*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (2*a + 3*b)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2
 + 2*(3*(2*a + 3*b)*cosh(d*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a
+ 3*b)*cosh(d*x + c)^3 + (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3
*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x
 + c)^2 - b)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d +
 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x +
c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2), x)

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{b\tanh(dx+c)^2+a}\,dx$$

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] -(e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d) - (2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*integrate(1/16*((a^2*e^(3*c) + 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e^c + 2*a*b*e^c + b^2*e^c)*e^(d*x))/(a*b^2 + b^3 + (a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)

Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{b\tanh(dx+c)^2+a}\,dx$$

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 1012, normalized size of antiderivative = 11.77

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \frac{2\,\mathrm{atan}\Bigg(\Big(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\Big(\frac{64\left(12\,a^2\,b^4\,d\,\sqrt{a^3+3\,a^2\,b+3\,a\,b^2+b^3}-2\,a\,b^5\,d\,\sqrt{a^3+3\,a^2\,b+3\,a\,b^2+b^3}+18\,a^3\,b^3\,d\,\sqrt{a^3+3\,a^2\,b+3\,a\,b^2+b^3}+6\,a^4\,b^2\,d\,\sqrt{a^3+3\,a^2\,b+3\,a\,b^2+\cdots}\right.}{a^3\,b^9\,d^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}}{\cdots}$$

$$-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(18\,a^7\,\sqrt{b^4\,d^2}+3\,b^7\,\sqrt{b^4\,d^2}+30\,a^2\,b^5\,\sqrt{b^4\,d^2}+342\,a^3\,b^4\,\sqrt{b^4\,d^2}+555\,a^4\,b^3\,\sqrt{b^4\,d^2}+396\,a^5\,b^2\,\sqrt{b^4\cdots}\right.}{b^8\,d\,\sqrt{4\,a^2+12\,a\,b+9\,b^2}-12\,a\,b^7\,d\,\sqrt{4\,a^2+12\,a\,b+9\,b^2}+18\,a^2\,b^6\,d\,\sqrt{4\,a^2+12\,a\,b+9\,b^2}+102\,a^3\,b^5\,d\,\sqrt{4\,a^2+12\,a\,b+9\,b^2}+117\,a^4\,b^4\,d\,\sqrt{4\,a^2+\cdots}}\cdots}{\sqrt{b^4\,d^2}}$$

$$-\frac{\mathrm{e}^{c+d\,x}}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{2\,\mathrm{e}^{c+d\,x}}{b\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

```
[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] ((2*atan(((exp(d*x)*exp(c)*((64*(12*a^2*b^4*d*(3*a*b^2 + 3*a^2*b + a^3 + b^
3)^(1/2) - 2*a*b^5*d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 18*a^3*b^3*d*(
3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 6*a^4*b^2*d*(3*a*b^2 + 3*a^2*b + a^3
 + b^3)^(1/2)))/(a^3*b^9*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*a^5*(a
*b^4*d^2)^(1/2) - b^5*(a*b^4*d^2)^(1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) + 15*a^
4*b*(a*b^4*d^2)^(1/2) + 20*a^2*b^3*(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*b^4*d^
2)^(1/2)))/(a^3*b^7*d*((a + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d^2)^(1/
2))) + (32*exp(3*c)*exp(3*d*x)*(3*a^5*(a*b^4*d^2)^(1/2) - b^5*(a*b^4*d^2)^(
1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) + 15*a^4*b*(a*b^4*d^2)^(1/2) + 20*a^2*b^3*
(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*b^4*d^2)^(1/2)))/(a^3*b^7*d*((a + b)^3)^(
1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d^2)^(1/2)))*(a^2*b^7*(a*b^4*d^2)^(1/2) + 2
*a^3*b^6*(a*b^4*d^2)^(1/2) + a^4*b^5*(a*b^4*d^2)^(1/2)))/(384*a*b^2 + 576*a
^2*b + 192*a^3 - 64*b^3)) + 2*atan((exp(d*x)*exp(c)*(a + b)^2*(a*b^4*d^2)^(
1/2))/(2*a*b^2*d*((a + b)^3)^(1/2))))*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2)
)/(2*(a*b^4*d^2)^(1/2)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^4*d^2)^(1/2) +
3*b^7*(b^4*d^2)^(1/2) + 30*a^2*b^5*(b^4*d^2)^(1/2) + 342*a^3*b^4*(b^4*d^2)^
(1/2) + 555*a^4*b^3*(b^4*d^2)^(1/2) + 396*a^5*b^2*(b^4*d^2)^(1/2) - 34*a*b^
6*(b^4*d^2)^(1/2) + 135*a^6*b*(b^4*d^2)^(1/2)))/(b^8*d*(12*a*b + 4*a^2 + 9*
b^2)^(1/2) - 12*a*b^7*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) + 18*a^2*b^6*d*(12*a
*b + 4*a^2 + 9*b^2)^(1/2) + 102*a^3*b^5*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) +
117*a^4*b^4*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) + 54*a^5*b^3*d*(12*a*b + 4*a^2
 + 9*b^2)^(1/2) + 9*a^6*b^2*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2)))*(12*a*b + 4*
a^2 + 9*b^2)^(1/2))/(b^4*d^2)^(1/2) - exp(c + d*x)/(b*d*(exp(2*c + 2*d*x) +
 1)) + (2*exp(c + d*x))/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.114 $\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}$$

```
[Out] (a+b)^2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(5/2)/d/a^(1/2)-(a+2*b)*tanh(
d*x+c)/b^2/d+1/3*tanh(d*x+c)^3/b/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3756, 398, 211}

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}$$

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a + 2*b)*Tanh[c + d*x])/(b^2*d) + Tanh[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}\\
&= \frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$
\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(3a+5b+b\,\text{sech}^2(c+dx))\tanh(c+dx)}{3b^2d}
$$

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(3*a + 5*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*b^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(65) = 130$.

Time = 115.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.36

method	result
derivativedivides	$\frac{\frac{2a(a^2+2ab+b^2)\left(-\frac{(a-\sqrt{(a+b)b}+b)\,\mathrm{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(2\sqrt{(a+b)b}-a-2b)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{(2\sqrt{(a+b)b}-a-2b)a}}+\frac{(-a-\sqrt{(a+b)b}-b)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(2\sqrt{(a+b)b}+a+2b)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{(2\sqrt{(a+b)b}+a+2b)a}}\right)}{b^2}}{d}$
default	$\frac{\frac{2a(a^2+2ab+b^2)\left(-\frac{(a-\sqrt{(a+b)b}+b)\,\mathrm{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(2\sqrt{(a+b)b}-a-2b)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{(2\sqrt{(a+b)b}-a-2b)a}}+\frac{(-a-\sqrt{(a+b)b}-b)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(2\sqrt{(a+b)b}+a+2b)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{(2\sqrt{(a+b)b}+a+2b)a}}\right)}{b^2}}{d}$
risch	$\frac{2a\,e^{4dx+4c}+2b\,e^{4dx+4c}+4\,e^{2dx+2c}a+8b\,e^{2dx+2c}+2a+\frac{10b}{3}}{b^2d(e^{2dx+2c}+1)^3}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a^2}{2\sqrt{-ab}\,d\,b^2}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{}}{}\right.}{\sqrt{}}$

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*a*(a^2+2*a*b+b^2)*(-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/
((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b
)*b)^(1/2)-a-2*b)*a)^(1/2))+1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((
2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b
)^(1/2)+a+2*b)*a)^(1/2)))+2/b^2*((-a-2*b)*tanh(1/2*d*x+1/2*c)^5+(-2*a-8/3*b
)*tanh(1/2*d*x+1/2*c)^3+(-a-2*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^
2+1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 2032, normalized size of antiderivative = 27.09

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 20*a*b^2
 + 24*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 24*(a^2*b + 2*a*b^2 + 3*(a^2*b +
```

a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)^6 + 6*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^3*d*sinh(d*x + c)^6 + 3*a*b^3*d*cosh(d*x + c)^4 + 3*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d + 3*(5*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^4 + 4*(5*a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^3*d*cosh(d*x + c)^4 + 6*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^2 + 6*(a*b^3*d*cosh(d*x + c)^5 + 2*a*b^3*d*cosh(d*x + c)^3 + a*b^3*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 24*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 6*a^2*b + 10*a*b^2 + 12*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 12*(a^2*b + 2*a*b^2 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 24*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)^6 + 6*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^3*d*sinh(d*x + c)^6 + 3*a*b^3*d*cosh(d*x + c)^4 + 3*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d + 3*(5*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^4 + 4*(5*a*b^3*d*co

```
sh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^3*d*cos
h(d*x + c)^4 + 6*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^2 + 6*(a*
b^3*d*cosh(d*x + c)^5 + 2*a*b^3*d*cosh(d*x + c)^3 + a*b^3*d*cosh(d*x + c))*
sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{2\left(6\,(a+2\,b)e^{(-2\,dx-2\,c)}+3\,(a+b)e^{(-4\,dx-4\,c)}+3\,a+5\,b\right)}{3\,(3\,b^2e^{(-2\,dx-2\,c)}+3\,b^2e^{(-4\,dx-4\,c)}+b^2e^{(-6\,dx-6\,c)}+b^2)d}-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}b^2d}$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -2/3*(6*(a + 2*b)*e^(-2*d*x - 2*c) + 3*(a + b)*e^(-4*d*x - 4*c) + 3*a + 5*b
)/((3*b^2*e^(-2*d*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c)
+ b^2)*d) - (a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/(sqrt(a*b)*b^2*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}(dx+c)^6}{b\tanh(dx+c)^2+a}\,dx$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.36

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{4}{b\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{8}{3\,b\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}+\frac{2\,\left(a+b\right)}{b^2\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{b^2}-\frac{2\,\left(a+b\right)\,\left(a+b+a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}\right)\,{\left(a+b\right)}^2}{2\,\sqrt{-a}\,b^{5/2}\,d}-\frac{\ln\left(\frac{2\,\left(a+b\right)\,\left(a+b+a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{b^2}\right)\,{\left(a+b\right)}^2}{2\,\sqrt{-a}\,b^{5/2}\,d}$$

```
[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] 4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 8/(3*b*d*(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (2*(a + b))/(b^2*d*
(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x)*(a + b))/b^2 - (2*(a +
 b)*(a + b + a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2))
)*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d) - (log((2*(a + b)*(a + b + a*exp(2*c
+ 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2)) - (4*exp(2*c + 2*d*x)*
(a + b))/b^2)*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d)
```

3.115 $\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d} + \frac{(a+3b)\sinh(c+dx)}{(a+b)^3 d} + \frac{\sinh^3(c+dx)}{3(a+b)^2 d} + \frac{b^3\sinh(c+dx)}{2a(a+b)^3 d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*b^2*(6*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(7/2)
/d+(a+3*b)*sinh(d*x+c)/(a+b)^3/d+1/3*sinh(d*x+c)^3/(a+b)^2/d+1/2*b^3*sinh(d
*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3757, 398, 393, 211}

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{7/2}} + \frac{b^3\sinh(c+dx)}{2ad(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)} + \frac{\sinh^3(c+dx)}{3d(a+b)^2} + \frac{(a+3b)\sinh(c+dx)}{d(a+b)^3}$$

```
[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a +
 b)^(7/2)*d) + ((a + 3*b)*Sinh[c + d*x])/((a + b)^3*d) + Sinh[c + d*x]^3/(3
*(a + b)^2*d) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*d*(a + (a + b)*Sinh[c +
d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3}+\frac{x^2}{(a+b)^2}+\frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3(a+(a+b)x^2)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^3d}\end{aligned}$$

$$= \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d} + \frac{\sinh^3(c+dx)}{3(a+b)^2d} + \frac{b^3\sinh(c+dx)}{2a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(b^2(6a+b))\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2a(a+b)^3d}$$

$$= \frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d} + \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d} + \frac{\sinh^3(c+dx)}{3(a+b)^2d} + \frac{b^3\sinh(c+dx)}{2a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{6b^2(6a+b)\arctan\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{7/2}} + \frac{3\left(3a+11b+\frac{4b^3}{a(a-b+(a+b)\cosh(2(c+dx)))}\right)\sinh(c+dx)}{(a+b)^3} + \frac{\sinh(3(c+dx))}{(a+b)^2}}{12d}$$

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((-6*b^2*(6*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(a
 + b)^(7/2)) + (3*(3*a + 11*b + (4*b^3)/(a*(a - b + (a + b)*Cosh[2*(c + d*x
)])))*Sinh[c + d*x])/(a + b)^3 + Sinh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(114) = 228$.

Time = 26.64 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.95

method	result
derivativedivides	$-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+3b}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{}{2(a+b)^2}$
default	$-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+3b}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{}{2(a+b)^2}$
risch	$\frac{e^{3dx+3c}}{24d(a^2+2ab+b^2)}+\frac{3\,e^{dx+c}a}{8(a^2+2ab+b^2)(a+b)d}+\frac{11\,e^{dx+c}b}{8(a^2+2ab+b^2)(a+b)d}-\frac{3\,e^{-dx-c}a}{8(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{11\,e^{-dx-c}b}{8(a^3+3a^2b+3a\,b^2}$

```
[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c
)-1)^2-(a+3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)-1/3/(a+b)^2/(1+tanh(1/2*d*x+
1/2*c))^3+1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^2-(a+3*b)/(a+b)^3/(1+tanh(1/2
*d*x+1/2*c))+2/(a+b)^3*b^2*((-1/2*b/a*tanh(1/2*d*x+1/2*c)^3+1/2*b/a*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)+1/2*(6*a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/(
(2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*
b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a
+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1
/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3642 vs. 2(114) = 228.

Time = 0.34 (sec) , antiderivative size = 6934, normalized size of antiderivative = 54.17

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^3}{(b\tanh(dx+c)^2+a)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/24*(a^3 + 2*a^2*b + a*b^2 - (a^3*e^(10*c) + 2*a^2*b*e^(10*c) + a*b^2*e^(
10*c))*e^(10*d*x) - (11*a^3*e^(8*c) + 42*a^2*b*e^(8*c) + 31*a*b^2*e^(8*c))*
e^(8*d*x) - 2*(5*a^3*e^(6*c) + 4*a^2*b*e^(6*c) - 49*a*b^2*e^(6*c) + 12*b^3*
e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 4*a^2*b*e^(4*c) - 49*a*b^2*e^(4*c)
+ 12*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) + 42*a^2*b*e^(2*c) + 31*a*b^2
*e^(2*c))*e^(2*d*x))/((a^5*d*e^(7*c) + 4*a^4*b*d*e^(7*c) + 6*a^3*b^2*d*e^(7
*c) + 4*a^2*b^3*d*e^(7*c) + a*b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) +
 2*a^4*b*d*e^(5*c) - 2*a^2*b^3*d*e^(5*c) - a*b^4*d*e^(5*c))*e^(5*d*x) + (a^
5*d*e^(3*c) + 4*a^4*b*d*e^(3*c) + 6*a^3*b^2*d*e^(3*c) + 4*a^2*b^3*d*e^(3*c)
 + a*b^4*d*e^(3*c))*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + b^3*e^
(3*c))*e^(3*d*x) + (6*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4 + (a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c
) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 2*a^4*b
*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.116 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a+5b)x}{2(a+b)^3}+\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*(a+5*b)*x/(a+b)^3+1/2*b^(3/2)*(5*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2
))/a^(3/2)/(a+b)^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2
)-1/2*(a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 212, 211}

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^3}-\frac{b(a-b)\tanh(c+dx)}{2ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x(a+5b)}{2(a+b)^3}$$

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + 5*b)*x)/(2*(a + b)^3) + (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(
a + b)*d*(a + b*Tanh[c + d*x]^2)) - ((a - b)*b*Tanh[c + d*x])/(2*a*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-2(a^2+4ab+b^2)-2(a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)^2d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{(b^2(5a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)^3d}\\
&\quad + \frac{(a+5b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^3d}\\
&= \frac{(a+5b)x}{2(a+b)^3} + \frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3d}\\
&\quad + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2(a+5b)(c+dx) + \frac{2b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + (a+b)\sinh(2(c+dx)) + \frac{2b^2(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{4(a+b)^3d}$$

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

[Out] (2*(a + 5*b)*(c + d*x) + (2*b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/a^(3/2) + (a + b)*Sinh[2*(c + d*x)] + (2*b^2*(a + b)*Sinh[2*(c
+ d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(4*(a + b)^3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(124) = 248$.

Time = 4.57 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.76

method	result
derivativedivides	$\frac{-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+5b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\cdots}{2(a+b)^2\cdots}}{\cdots}$
default	$\frac{-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+5b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\cdots}{2(a+b)^2\cdots}}{\cdots}$
risch	$\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{5xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2dx+2c}}{8d(a^2+2ab+b^2)}-\frac{e^{-2dx-2c}}{8d(a^2+2ab+b^2)}-\frac{b^2\left(e^{2dx+2c}a\cdots\right.}{d(a+b)^3a\left(a\,e^{4dx+4c}+b\,e^{4dx+\cdots}\right.}$

[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^2+1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2
c))+1/2(a+5*b)/(a+b)^3*ln(1+tanh(1/2*d*x+1/2*c))+1/2/(a+b)^2/(tanh(1/2*d*
x+1/2*c)-1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/(a+b)^3*(-a-5*b)*ln(t
anh(1/2*d*x+1/2*c)-1)-2/(a+b)^3*b^2*((-1/2*(a+b)/a*tanh(1/2*d*x+1/2*c)^3-1/
2*(a+b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(5*a+b)*(1/2*(a+((a+b)*b)^(1/2)+b)/
a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/(
(a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(124) = 248$.

Time = 0.34 (sec) , antiderivative size = 4324, normalized size of antiderivative = 30.89

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*c
osh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 + 2*
(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 -
a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3
+ 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 +
 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 +
 4*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
+ c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3
- 4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b
 + 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4
- a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x - 24*(a*b^2 - b^3
 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*((5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh(d*x + c)^6 + 2*(5*
a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b - 8*a*b^2 - 2*b^3 + 15*(
5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b +
 6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + (5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(5*a^2*
b + 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^2*b + 6*a*b^2 + b^3 + 12*(5*a^2*b
- 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 6*a*b^2
 + b^3)*cosh(d*x + c)^5 + 4*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^3 + (5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c
)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)
*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a
*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*
sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*co
```

sh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^6 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^2 + 2*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 + 2*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 + 4*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4 - a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x - 24*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh(d*x + c)^6 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b - 8*a*b^2 - 2*b^3 + 15*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^2*b + 6*a*b^2 + b^3 + 12*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^5 + 4*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^3 + (5*a^

```
2*b + 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a
 + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sin
h(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
+ c)^7 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^5
- 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (a^3 +
3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
+ c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 +
6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^6
+ 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*(a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^4*b -
 2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^3 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^2 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)
^2 + 2*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5
 + 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b
 + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh^2{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(124) = 248$.

Time = 0.38 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.00

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

$$-\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

$$-\frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d}$$

$$+\frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d}$$

$$-\frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d}$$

$$+\frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(2\,dx+2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)e^{(4\,dx+4\,c)}+2\,(a^5+2\,a^4b-2\,a^2}$$

$$-\frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(-2\,dx-2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+2\,(a^5+2\,a^4b-2\,a^2b^3-ab^4)e^{(-2\,dx-2\,c)}+(a^5+4\,a^4b+6\,a^3b^2+4}$$

$$+\frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)}}$$

$$+\frac{dx+c}{2\,(a^2+2\,ab+b^2)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a^2+2\,ab+b^2)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a^2+2\,ab+b^2)d}$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a
 + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*(
3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/4*(3*a*b + b^2)*arctan(1/2
*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqr
t(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a
^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 +
 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x
 - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b
 - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
```

```
*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x
 - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a
*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*
c))*d) + 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2*d*x + 2*c)/((a^2
+ 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [F]

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.117 $\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{(a+b)^2d} + \frac{b^2\sinh(c+dx)}{2a(a+b)^2d\,(a+(a+b)\sinh^2(c+dx))}$$

```
[Out] 1/2*b*(4*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/d
+sinh(d*x+c)/(a+b)^2/d+1/2*b^2*sinh(d*x+c)/a/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)
^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3757, 398, 393, 211}

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{5/2}} + \frac{b^2\sinh(c+dx)}{2ad(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)} + \frac{\sinh(c+dx)}{d(a+b)^2}$$

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b
)^(5/2)*d) + Sinh[c + d*x]/((a + b)^2*d) + (b^2*Sinh[c + d*x])/(2*a*(a + b)
^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2}+\frac{b(2a+b)+2b(a+b)x^2}{(a+b)^2(a+(a+b)x^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\sinh(c+dx)}{(a+b)^2d}+\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^2d}\\
&= \frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}\\
&\quad+\frac{(b(4a+b))\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2a(a+b)^2d}\\
&= \frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}}+\frac{\sinh(c+dx)\left(2+\frac{b^2}{a\left(a+(a+b)\sinh^2(c+dx)\right)}\right)}{(a+b)^2}}{2d}$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
[Out] ((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)
^(5/2)) + (Sinh[c + d*x]*(2 + b^2/(a*(a + (a + b)*Sinh[c + d*x]^2))))/(a +
b)^2)/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(89) = 178.

Time = 1.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.83

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2b\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(4a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a}{\sqrt{(2}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+}\right.}}\right.}{}\right.}{(a+b)^2}}{d}$
default	$\frac{-\frac{1}{(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2b\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(4a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a}{\sqrt{(2}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+}\right.}}\right.}{}\right.}{(a+b)^2}}{d}$
risch	$\frac{e^{dx+c}}{2d(a^2+2ab+b^2)}-\frac{e^{-dx-c}}{2d(a^2+2ab+b^2)}+\frac{b^2e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a+b)^2a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{b\ln\left(e^{2dx+2}\right.}{\sqrt{-a^2}}$

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))+2*b/(a+b)^2*((-1/2*b/a*tanh(1/2*d*x
+1/2*c)^3+1/2*b/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(4*a+b)*(1/2*(((a+b)*b)^(1/
2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/
a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/(a+b)^2/(tanh(1/2*d*x+1/2
*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1815 vs. $2(89) = 178$.

Time = 0.35 (sec) , antiderivative size = 3502, normalized size of antiderivative = 34.67

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x
+ c)^6 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + 2*(a^4 - 2
*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b + a^2*b
^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 - 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2 + 2*(
15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b^2 - 2*
a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d*x + c)
^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b
^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4
*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c) + (5*(4
*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^
2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh
(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
 c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^
```

2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(3*(a^4 +
2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*
cosh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d
*x + c))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)
^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*si
nh(d*x + c)^4 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*sinh(d*
x + c)^5 + 2*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5
*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6
 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a
^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^
3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3
- a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2
*b^4)*d)*sinh(d*x + c)), 1/2*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 6
*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b +
 a^2*b^2)*sinh(d*x + c)^6 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x +
c)^4 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^
3*b + a^2*b^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d
*x + c))*sinh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c
)^2 + (15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b
^2 - 2*a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*
b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d
x + c)^5 + 2(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*
b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
2*(5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)
+ (5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 +
6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)
arctan(1/2((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d
*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((4*a^2*b + 5*a*b^2
+ b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d*x + c)^5 + 2*(4*a^2*b - 3*a*b^2
- b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 5*a*b^2 + b^3)*co
sh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2
+ (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c) + (5*(4*a^2*b + 5*a*b^2 + b^3)*co
sh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(
d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh
(d*x + c) + sinh(d*x + c))/a) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c
)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (a^4 - 2*a^3*

```
b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 4*a^5*b + 6*a^
4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^6 + 4*a^5*b
+ 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^5 + 2*(a^6 + 2*a^5*b - 2
*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^
4)*d)*sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d
*cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*c
osh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*s
inh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cos
h(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 +
(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh{(dx+c)}}{\left(b\tanh{(dx+c)}^2+a\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) - 3*
a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) - 3*a*b*e^(2*c) + 2*b
^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + 3*a^3*b*d*e^(5*c) + 3*a^2*b^2*d*e^
(5*c) + a*b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^4*d*e^(3*c) + a^3*b*d*e^(3*c) - a
^2*b^2*d*e^(3*c) - a*b^3*d*e^(3*c))*e^(3*d*x) + (a^4*d*e^c + 3*a^3*b*d*e^c
+ 3*a^2*b^2*d*e^c + a*b^3*d*e^c)*e^(d*x)) + 1/2*integrate(2*((4*a*b*e^(3*c)
 + b^2*e^(3*c))*e^(3*d*x) + (4*a*b*e^c + b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a
*b^3*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c)
- a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.118 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+1
/2*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3757, 393, 211}

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}} + \frac{b\sinh(c+dx)}{2ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}$$

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^
(3/2)*d) + (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{(2a+b)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2a(a+b)d}\\
&= \frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d}+\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{b\sinh(c+dx)}{a\left(a+(a+b)\sinh^2(c+dx)\right)}}{2(a+b)d}$$

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a +
b]) + (b*Sinh[c + d*x])/(a*(a + (a + b)*Sinh[c + d*x]^2)))/(2*(a + b)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(71) = 142$.

Time = 4.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.04

method	result
derivativedivides	$\frac{\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}+\frac{(2a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-\cdots\right.}{2a\sqrt{\cdots}}\cdots\right.}{a+b}}{d}$
default	$\frac{\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}+\frac{(2a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-\cdots\right.}{2a\sqrt{\cdots}}\cdots\right.}{a+b}}{d}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{da(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{2\sqrt{-a^{2}-ab}\,(a+b)d}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{d\cdots}}{\sqrt{-a^{2}\cdots}}\cdots\right.}{4\sqrt{-a^{2}-ab}\,(a\cdots}$

[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2*(-1/2*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/a/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+(2*a+b)/(a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(71) = 142$.

Time = 0.29 (sec) , antiderivative size = 2041, normalized size of antiderivative = 24.59

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 - ((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +

(2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*cosh(d*x + c
)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*sinh(
d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^3
 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh
(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
 c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(a^2*b + a
*b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 +
4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^5 + a^4*b - a
^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c
)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d*cosh(
d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*(a^2*b
 + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x + c)^3
 + ((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 -
 a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 +
2*a^2 - a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*
b + b^2)*cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((2
*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2
- a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^
2)*cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) -
 2*(a^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(
d*x + c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^
5 + a^4*b - a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*
a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)
*sinh(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2

```
*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e
^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*
e^(3*d*x) + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c
) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a*b^2*e^(
2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

3.119 $\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/b^(1/2)+1/2*tanh(d*x+c)/a
/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3756, 205, 211}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tanh[c + d*
x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2ad} \\
&= \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\tanh(c+dx)}{a+b\tanh^2(c+dx)}}{2a^{3/2}d}$$

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tanh[c + d*x])/
(a + b*Tanh[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(54) = 108$.

Time = 14.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.18

method	result
risch	$-\frac{e^{2dx+2c}a-b\,e^{2dx+2c}+a+b}{(a+b)da\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(e^{2dx+2c}\cdots\right.}{\cdots}$
derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}-\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}+\frac{\left(-a+\sqrt{(a+b)b}\cdots\right.}{2a\sqrt{(\cdots}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}-\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}+\frac{\left(-a+\sqrt{(a+b)b}\cdots\right.}{2a\sqrt{(\cdots}}}{d}$

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(exp(2*d*x+2*c)*a-b*exp(2*d*x+2*c)+a+b)/(a+b)/d/a/(a*exp(4*d*x+4*c)+b*exp(
4*d*x+4*c)+2*exp(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)-1/4/(-a*b)^(1/2)/d/a*
ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)-2*a*b)/(a+b)/(-a*b)^(1/2))
+1/4/(-a*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)+2*a*
b)/(a+b)/(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(54) = 108$.

Time = 0.30 (sec) , antiderivative size = 1515, normalized size of antiderivative = 22.95

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a
^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 -
b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
```

```
2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b
^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x
 + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)
^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*
(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x +
c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) +
a + b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^
3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2
*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4
*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)*d)*sinh(d*x
 + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3
)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1
/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^
2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x +
c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^
2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)
^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
b)*sqrt(a*b)/(a*b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(
a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a
^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2
 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)
*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d
*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}^2\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(54) = 108.

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{\left(a^3+2\,a^2b+ab^2+2\,(a^3-ab^2)e^{(-2\,dx-2\,c)}+(a^3+2\,a^2b+ab^2)e^{(-4\,dx-4\,c)}\right)d} - \frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}ad}$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2
)*e^(-2*d*x - 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) - 1/2*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

3.120 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}d}+\frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*(a+b)^(1
/2)/a^(1/2))/a^(3/2)/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 205, 211}

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}}+\frac{\sinh(c+dx)}{2ad\left((a+b)\sinh^2(c+dx)+a\right)}$$

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]*d) + Sin
h[c + d*x]/(2*a*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2ad} \\
&= \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + \frac{\sqrt{a}\sinh(c+dx)}{a+(a+b)\sinh^2(c+dx)}}{2a^{3/2}d}$$

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/Sqrt[a + b] + (Sqrt[a]*Sinh[c
+ d*x])/(a + (a + b)*Sinh[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(60) = 120$.

Time = 35.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{da\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{4\sqrt{-a^2-ab}\,da}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right.}{4\sqrt{-a^2-ab}\,da}$
derivativedivides	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(\sqrt{(a+b)b}+b\right)\operatorname{arctan}}{2a\sqrt{(a+b)b}\,\sqrt{}}}{d}$
default	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(\sqrt{(a+b)b}+b\right)\operatorname{arctan}}{2a\sqrt{(a+b)b}\,\sqrt{}}}{d}$

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/a/(a*exp(4*d*x+4*c)+b*exp(4*d*x+4*c)+2*exp(
2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)-1/4/(-a^2-a*b)^(1/2)/d/a*ln(exp(2*d*x+
2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/4/(-a^2-a*b)^(1/2)/d/a*ln(exp(2*d
*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 1555, normalized size of antiderivative = 21.60

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cos
h(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((
a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt
(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*
x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x
```

```
 + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 +
 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
 b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
b)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2
- a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^
2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4
+ 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2
 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x +
c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*
cosh(d*x + c)^3 + 6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*
b)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(
a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(a^2 + a*b)*arctan(1/
2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
 b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2
+ 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((a + b)*cosh(d*x + c)^4 + 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4
*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*s
qrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a
) - 2*(a^2 + a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 -
a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 +
 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 +
2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 +
 (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)
^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^3*d + 2*a^
2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(
4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/8*(
e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))*e^(
4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

3.121 $\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{2abd\,(a+b\tanh^2(c+dx))}$$

```
[Out] -1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d+1/2*(a+b)*
tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3756, 393, 211}

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{(a+b)\tanh(c+dx)}{2abd\,(a+b\tanh^2(c+dx))} - \frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}$$

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*b^(3/2)*d)
+ ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2abd}\\
&= -\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$
\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(-a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\sqrt{b}(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{3/2}b^{3/2}d}
$$

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(a + b
)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2
)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(65) = 130.

Time = 73.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.36

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}-\frac{(a-b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{(-a\cdots}{\cdots}\right)}{b}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}-\frac{(a-b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{(-a\cdots}{\cdots}\right)}{b}}{d}$
risch	$-\frac{e^{2dx+2c}a-b\,e^{2dx+2c}+a+b}{dab\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,db}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{\cdots}}{\cdots}\right)}{4\sqrt{\cdots}}$

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a+b)/a/b*tanh(1/2*d*x+1/2*c)^3-1/2*(a+b)/a/b*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)-(a-b)/b*(-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)
*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)
-a-2*b)*a)^(1/2))+1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+
2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(65) = 130.

Time = 0.28 (sec) , antiderivative size = 1443, normalized size of antiderivative = 18.74

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
```

```
a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(
a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2
 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))/((a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^3)*d*cosh
(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^2 - a^2*b
^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2*b^3)*d*c
osh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^2)*
cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a^2 - b
^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -
b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*(
(a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
 c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c
)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)))/((a^3*
b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^
3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^
2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c
))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^4\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{(a^2b+ab^2+2\,(a^2b-ab^2)e^{(-2\,dx-2\,c)}+(a^2b+ab^2)e^{(-4\,dx-4\,c)})d} + \frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}abd}$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(-
2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/2*(a - b)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^4}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{1}{\cosh(c+d\,x)^4\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

3.122 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{\arctan(\sinh(c+dx))}{b^2d} - \frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] arctan(sinh(d*x+c))/b^2/d+1/2*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c)^
2)-1/2*(2*a-b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/
b^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3757, 425, 536, 209, 211}

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = -\frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b)\sinh(c+dx)}{2abd\left((a+b)\sinh^2(c+dx)+a\right)} + \frac{\arctan(\sinh(c+dx))}{b^2d}$$

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*
Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) + ((a + b)*Sinh[c + d*x])/(2*a*b
*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a+b)\sinh(c+dx)}{2abd\,(a+(a+b)\sinh^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{a-b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{2abd}
\end{aligned}$$

$$= \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{((2a-b)(a+b))\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2ab^2d}$$

$$= \frac{\arctan(\sinh(c+dx))}{b^2d} - \frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{(a-b)\left((2a^2+ab-b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+4a^{3/2}\sqrt{a+b}\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a+b)\left((2a^2\right.}{2a^{3/2}b^2\sqrt{a+b}\,d(}$$

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*((2*a^2 + a*b - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] +
 4*a^(3/2)*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]) + (a + b)*((2*a^2 + a*b -
 b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 4*a^(3/2)*Sqrt[a + b]*A
rcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)] + 2*Sqrt[a]*b*(a + b)^(3/2)*Sin
h[c + d*x])/(2*a^(3/2)*b^2*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]
))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(90) = 180.

Time = 136.48 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.68

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}+\frac{\left(2a^{2}+ab-b^{2}\right)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{2}}{b^{2}}}{d}$
default	$-\frac{\frac{2\left(\frac{\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a}+\frac{\left(2a^{2}+ab-b^{2}\right)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{2}}{b^{2}}}{d}$
risch	$\frac{e^{dx+c}(a+b)\left(e^{2dx+2c}-1\right)}{dab\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{i\ln\left(e^{dx+c}+i\right)}{d\,b^{2}}-\frac{i\ln\left(e^{dx+c}-i\right)}{d\,b^{2}}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2dx}\right.}{}$

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*((1/2*(a+b)*b/a*tanh(1/2*d*x+1/2*c)^3-1/2*(a+b)*b/a*tanh(1/2*d*
x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)+1/2*(2*a^2+a*b-b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/
2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a
+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2
*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b
)^(1/2)-a-2*b)*a)^(1/2))))+2/b^2*arctan(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(90) = 180$.

Time = 0.35 (sec) , antiderivative size = 2140, normalized size of antiderivative = 20.98

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b - b^2)*cosh(d*x + c
)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b -
b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^
2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + 2*a
^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x +
 c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 -
 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*
x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(
d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqr
t(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3
*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*((a^2 + a*b)*cosh(
d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh
(d*x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c
)^2 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)
^3 + (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(
d*x + c)) - 4*(a*b + b^2)*cosh(d*x + c) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2
- a*b - b^2)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b
^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x
+ c)^4 + 2*(a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*c
osh(d*x + c)^2 + (a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d
 + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 - a*b^3)*d*cosh(d*x +
c))*sinh(d*x + c)), 1/2*(2*(a*b + b^2)*cosh(d*x + c)^3 + 6*(a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b -
 b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3
 + (2*a^2 + a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x +
 c)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sin
h(d*x + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 +
 (2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan
(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((2*a^2 + a*b - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
2*a^2 + a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^
2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*
x + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*
a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan(1/2
*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 +
 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b)) + 4*((a^2 + a*b)*cosh(d*x
 + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*
x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2
 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3
```

```
+ (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - 2*(a*b + b^2)*cosh(d*x + c) + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 - a
*b - b^2)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2
+ a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c
)^4 + 2*(a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh
(d*x + c)^2 + (a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d +
4*((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 - a*b^3)*d*cosh(d*x + c))
*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^5\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^2*b*d + a*
b^2*d + (a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrat
e(1/32*((2*a^2*e^(3*c) + a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (2*a^2*e^c
+ a*b*e^c - b^2*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e
^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^5\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2), x)
```

3.123 $\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^2} dx = -\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{(a+b)^2\tanh(c+dx)}{2ab^2d\,(a+b\tanh^2(c+dx))}$$

```
[Out] -1/2*(3*a-b)*(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/d+ta
nh(d*x+c)/b^2/d+1/2*(a+b)^2*tanh(d*x+c)/a/b^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 398, 393, 211}

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^2} dx = -\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{(a+b)^2\tanh(c+dx)}{2ab^2d\,(a+b\tanh^2(c+dx))} + \frac{\tanh(c+dx)}{b^2d}$$

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*b
^(5/2)*d) + Tanh[c + d*x]/(b^2*d) + ((a + b)^2*Tanh[c + d*x])/(2*a*b^2*d*(a
 + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2b(a+b)x^2}{b^2(a+bx^2)^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{b^2d}-\frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}\\
&= \frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\,(a+b\tanh^2(c+dx))}\\
&\quad -\frac{((3a-b)(a+b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2ab^2d}\\
&= -\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\,(a+b\tanh^2(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}(a+b)^2\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}+2\sqrt{b}\tanh(c+dx)}{2b^{5/2}d}$$

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) + (
Sqrt[b]*(a + b)^2*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])
) + 2*Sqrt[b]*Tanh[c + d*x])/(2*b^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(85) = 170$.

Time = 258.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.15

method	result
derivativedivides	$\frac{\frac{2\left(\frac{(a^2+2ab+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{(a^2+2ab+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+(3a^2+2ab-b^2)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{b^2}}{d}$
default	$\frac{\frac{2\left(\frac{(a^2+2ab+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{(a^2+2ab+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+(3a^2+2ab-b^2)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{b^2}}{d}$
risch	$-\frac{3a^2e^{4dx+4c}+2ab\,e^{4dx+4c}-e^{4dx+4c}b^2+6a^2e^{2dx+2c}-2ab\,e^{2dx+2c}+3a^2+4ab+b^2}{a\,b^2d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}+1\right)}-\frac{3a\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2}{(a+b)\sqrt{-ab}}\right.}{4\sqrt{-ab}\,d\,b^2}$

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*((1/2*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a^2+2*a*b+b^2
)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(3*a^2+2*a*b-b^2)*(1/2*(a+((a+b)*b)^(1/2)
+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)
```

```
/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+2/b^2*tanh(1/2*d*x+1/2*c)/
(tanh(1/2*d*x+1/2*c)^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(85) = 170$.

Time = 0.31 (sec) , antiderivative size = 2869, normalized size of antiderivative = 29.58

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 2*a^
2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 2*a^2*b^2 - a*b
^3)*sinh(d*x + c)^4 + 12*a^3*b + 16*a^2*b^2 + 4*a*b^3 + 8*(3*a^3*b - a^2*b^
2)*cosh(d*x + c)^2 + 8*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d
*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5
 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*b - 5*a
*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15*(3*a^3
+ 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 5
*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 + (9*a^3 + 3*
a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b + a*b^2 - b^3
)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3 + 3*a^2*b -
5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 5*a^2*b + a
*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x
+ c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2
 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 -
b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c
)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
 b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b
)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*((3*a^3*b + 2*a^2*b^2 -
a*b^3)*cosh(d*x + c)^3 + (3*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/
((a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^5 + (a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 -
a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3
```

```
*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x +
 c)^2 + 4*(5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^
4 + 6*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*si
nh(d*x + c)^2 + (a^3*b^3 + a^2*b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x
 + c)^5 + 2*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)
*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cos
h(d*x + c)^4 + 8*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^
3 + 2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*sinh(d*x + c)^4 + 6*a^3*b + 8*a^2*b^2 +
 2*a*b^3 + 4*(3*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 4*(3*a^3*b - a^2*b^2 + 3
*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 +
 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)
*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x +
 c)^6 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*
b - 5*a*b^2 + b^3 + 15*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^
3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2
*b + a*b^2 - b^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*
(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2
 + b^3 + 6*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 2*(3*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^
2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 +
 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*s
qrt(a*b)/(a*b)) + 8*((3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^3 + (3*a^3
*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + a^2*b^4)*d*cosh(d*x
 + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3
+ a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + (1
5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x
 + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + a^2*b^4
)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 - a^2*b^4)*d*c
osh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^3 + a^2*
b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 - a^2*b^
4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^6\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{3a^2+4ab+b^2+2(3a^2-ab)e^{(-2dx-2c)}+(3a^2+2ab-b^2)e^{(-4dx-4c)}}{\left(a^2b^2+ab^3+(3a^2b^2-ab^3)e^{(-2dx-2c)}+(3a^2b^2-ab^3)e^{(-4dx-4c)}+(a^2b^2+ab^3)e^{(-6dx-6c)}\right)d}$$
$$+ \frac{(3a^2+2ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}ab^2d}$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] (3*a^2 + 4*a*b + b^2 + 2*(3*a^2 - a*b)*e^(-2*d*x - 2*c) + (3*a^2 + 2*a*b -
b^2)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 - a*b^3)*e^(-2*d*x -
2*c) + (3*a^2*b^2 - a*b^3)*e^(-4*d*x - 4*c) + (a^2*b^2 + a*b^3)*e^(-6*d*x -
 6*c))*d) + 1/2*(3*a^2 + 2*a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c)
+ a - b)/sqrt(a*b))/(sqrt(a*b)*a*b^2*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^6}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^6\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2), x)
```

3.124 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(4a+5b)\arctan(\sinh(c+dx))}{2b^3d} - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(4*a+5*b)*arctan(sinh(d*x+c))/b^3/d-1/2*(4*a-b)*(a+b)^(3/2)*arctan(sinh
(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a+b)*(2*a+b)*sinh(d*x+c)/a/
b^2/d/(a+(a+b)*sinh(d*x+c)^2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+(a+b)*sinh
(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

= {3757, 425, 541, 536, 209, 211}

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(4a+5b)\arctan(\sinh(c+dx))}{2b^3d} + \frac{(2a+b)(a+b)\sinh(c+dx)}{2ab^2d\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd\left((a+b)\sinh^2(c+dx)+a\right)}$$

```
[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) - ((4*a - b)*(a + b)^(3/2)*Ar
cTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a + b)*(2*
a + b)*Sinh[c + d*x])/(2*a*b^2*d*(a + (a + b)*Sinh[c + d*x]^2)) - (Sech[c +
 d*x]*Tanh[c + d*x])/(2*b*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{2bd} \\
&= \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{2(2a^2+2ab-b^2)-2(a+b)(2a+b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{4ab^2d} \\
&= \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} \\
&\quad - \frac{((4a-b)(a+b)^2)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2ab^3d} \\
&\quad + \frac{(4a+5b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2b^3d} \\
&= \frac{(4a+5b)\arctan(\sinh(c+dx))}{2b^3d} - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} \\
&\quad + \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2\sqrt{a}b(a+b)^{5/2}\sinh(c+dx)+(a-b)\left((4a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+2a^{3/2}\sqrt{a+b}(4a+5b)\operatorname{ar}\right.}{}$$

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*Sqrt[a]*b*(a + b)^(5/2)*Sinh[c + d*x] + (a - b)*((4*a - b)*(a + b)^2*Arc
Tan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b
)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c +
d*x]) + (a + b)*Cosh[2*(c + d*x)]*((4*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Csch
[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b)*ArcTan[Tanh[(c
+ d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]))/(2*a^(3/2)
*b^3*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(139) = 278$.

Time = 0.30 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.26

$$\frac{\frac{2\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+(5b+4a)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^3}-\frac{2\left(\frac{\frac{b\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{b\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{\left(4a^3+7a\right.}{}\right.}{}}{d}$$

```
[In] int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(2/b^3*((-1/2*tanh(1/2*d*x+1/2*c)^3*b+1/2*b*tanh(1/2*d*x+1/2*c))/(tanh(
1/2*d*x+1/2*c)^2+1)^2+1/2*(5*b+4*a)*arctan(tanh(1/2*d*x+1/2*c)))-2/b^3*((1/
2*b*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3-1/2*b*(a^2+2*a*b+b^2)/a*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)+1/2*(4*a^3+7*a^2*b+2*a*b^2-b^3)*(1/2*(((a+b)*b)^(1/2)+b)
/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a
+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3490 vs. $2(139) = 278$.

Time = 0.37 (sec) , antiderivative size = 6396, normalized size of antiderivative = 41.26

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

```
[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**7/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^7}{(b\tanh(dx+c)^2+a)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] ((2*a^2*e^(7*c) + 3*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) -
 a*b*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) - a*b*e^(3*c) + b^2*
e^(3*c))*e^(3*d*x) - (2*a^2*e^c + 3*a*b*e^c + b^2*e^c)*e^(d*x))/(4*a^2*b^2*
d*e^(6*d*x + 6*c) + 4*a^2*b^2*d*e^(2*d*x + 2*c) + a^2*b^2*d + a*b^3*d + (a^
2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 2*(3*a^2*b^2*d*e^(4*c) - a*b
^3*d*e^(4*c))*e^(4*d*x)) + (4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(
b^3*d) - 128*integrate(1/128*((4*a^3*e^(3*c) + 7*a^2*b*e^(3*c) + 2*a*b^2*e^
(3*c) - b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 7*a^2*b*e^c + 2*a*b^2*e^c - b
^3*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*
d*x) + 2*(a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^7\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2), x)
```

3.125 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx = {} & \frac{(a+7b)x}{2(a+b)^4} + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4d} \\ & + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\ & - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\ & - \frac{(a-3b)b(4a+b)\tanh(c+dx)}{8a^2(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

```
[Out] 1/2*(a+7*b)*x/(a+b)^4+1/8*b^(3/2)*(35*a^2+14*a*b+3*b^2)*arctan(b^(1/2)*tanh
(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b
*tanh(d*x+c)^2)^2-1/4*(2*a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
^2-1/8*(a-3*b)*b*(4*a+b)*tanh(d*x+c)/a^2/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3756, 425, 541, 536, 212, 211}

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{b(a-3b)(4a+b)\tanh(c+dx)}{8a^2d(a+b)^3\left(a+b\tanh^2(c+dx)\right)} + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^4} - \frac{b(2a-b)\tanh(c+dx)}{4ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{x(a+7b)}{2(a+b)^4}$$

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 7*b)*x)/(2*(a + b)^4) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((2*a - b)*b*Tanh[c +
 d*x])/(4*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*b*(4*a + b)
*Tanh[c + d*x])/(8*a^2*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+2b+5bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-2(2a^2+8ab+3b^2)-6(2a-b)bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{8a(a+b)^2d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad - \frac{(a-3b)b(4a+b)\tanh(c+dx)}{8a^2(a+b)^3d\left(a+b\tanh^2(c+dx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{2(4a^3+24a^2b+11ab^2+3b^3)+2(a-3b)b(4a+b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{16a^2(a+b)^3d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad - \frac{(a-3b)b(4a+b)\tanh(c+dx)}{8a^2(a+b)^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{(a+7b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^4d} \\
&\quad + \frac{\left(b^2(35a^2+14ab+3b^2)\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+7b)x}{2(a+b)^4} + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4d} \\
&\quad + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\
&\quad - \frac{(a-3b)b(4a+b)\tanh(c+dx)}{8a^2(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

$$
\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx
$$

$$
= \frac{4(a+7b)(c+dx) + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + 2(a+b)\sinh(2(c+dx)) + \frac{4b^3(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx}}{8(a+b)^4d}
$$

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*(a + 7*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) + 2*(a + b)*Sinh[2*(c + d*x)] + (4*b^3*(a
 + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (b^2*(
a + b)*(13*a + 3*b)*Sinh[2*(c + d*x)])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*
x)])))/(8*(a + b)^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(180) = 360$.

Time = 27.89 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.53

method	result
derivativedivides	$\frac{-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+7b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^4}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2(a+b)^3}}{}$
default	$\frac{-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+7b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^4}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2(a+b)^3}}{}$
risch	$\frac{xa}{2(a+b)(a^3+3a^2b+3a\,b^2+b^3)}+\frac{7xb}{2(a+b)(a^3+3a^2b+3a\,b^2+b^3)}+\frac{e^{2dx+2c}}{8(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{e^{-2dx-2c}}{8(a^3+3a^2b+3a\,b^2+b^3)d}$

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^2+1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2
*c))+1/2*(a+7*b)/(a+b)^4*ln(1+tanh(1/2*d*x+1/2*c))+1/2/(a+b)^3/(tanh(1/2*d*
x+1/2*c)-1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/(a+b)^4*(-a-7*b)*ln(t
anh(1/2*d*x+1/2*c)-1)-2*b^2/(a+b)^4*((-1/8*(13*a^2+18*a*b+5*b^2)/a*tanh(1/2
*d*x+1/2*c)^7-1/8*(39*a^3+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)
^5-1/8*(39*a^3+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3-1/8*(13*
a^2+18*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8/a*(35*a^2+14*a*b+3*b^2)
*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/
2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6780 vs. $2(180) = 360$.

Time = 0.46 (sec) , antiderivative size = 13887, normalized size of antiderivative = 70.14

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. $2(180) = 360$.

Time = 0.57 (sec) , antiderivative size = 1806, normalized size of antiderivative = 9.12

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*(
(a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b
^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*
b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/16*(15*a^2*b + 10*a*b
^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4*a^3*b^2 -
 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a
```

```
*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*
b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b
^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15
*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a
^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a
^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*
d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 -
 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x
- 2*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*
x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x
- 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5
+ a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)
*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*
b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 1
5*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c)
)*d) + 1/8*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^
2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 +
 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x
 - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 +
4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x -
 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)
*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^
3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + 1/2*(d*x + c)/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d) + 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) -
1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [F]

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\cosh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\cosh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.126 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d}+\frac{\sinh(c+dx)}{(a+b)^3d}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+
b)^(7/2)/d+sinh(d*x+c)/(a+b)^3/d+1/4*b^3*sinh(d*x+c)/a/(a+b)^3/d/(a+(a+b)*s
inh(d*x+c)^2)^2+3/8*b^2*(4*a+b)*sinh(d*x+c)/a^2/(a+b)^3/d/(a+(a+b)*sinh(d*x
+c)^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3757, 398, 1171, 393, 211}

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3b^2(4a+b)\sinh(c+dx)}{8a^2d(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{4ad(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)^2}+\frac{\sinh(c+dx)}{d(a+b)^3}$$

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*
a^(5/2)*(a + b)^(7/2)*d) + Sinh[c + d*x]/((a + b)^3*d) + (b^3*Sinh[c + d*x]
)/(4*a*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh
[c + d*x])/(8*a^2*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3} + \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+b)^3(a+(a+b)x^2)^3}\right)\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{(a+b)^3d} \\
&= \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12ab(a+b)x^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4a(a+b)^3d} \\
&= \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2} \\
&\quad + \frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)} \\
&\quad + \frac{(3b(8a^2+4ab+b^2))\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{8a^2(a+b)^3d} \\
&= \frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d} + \frac{\sinh(c+dx)}{(a+b)^3d} \\
&\quad + \frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2} \\
&\quad + \frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$
\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx
= \frac{\frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^{7/2}} + \frac{\sinh(c+dx)\left(8+\frac{3b^3}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{2b^2\left(6a+b+6(a+b)\sinh^2(c+dx)\right)}{a\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{8d}
$$

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a
^(5/2)*(a + b)^(7/2)) + (Sinh[c + d*x]*(8 + (3*b^3)/(a^2*(a + (a + b)*Sinh[
c + d*x]^2)) + (2*b^2*(6*a + b + 6*(a + b)*Sinh[c + d*x]^2))/(a*(a + (a + b
)*Sinh[c + d*x]^2)^2)))/(a + b)^3)/(8*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(140) = 280$.

Time = 7.20 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.44

method	result
derivativedivides	$\frac{2b\left(\frac{-\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}-\frac{3\left(4a^{2}+15ab+4b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}}+\frac{3\left(4a^{2}+15ab+4b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}}+\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\right.}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a\right)^{2}}\right.}{(a+b}$
default	$\frac{2b\left(\frac{-\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}-\frac{3\left(4a^{2}+15ab+4b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}}+\frac{3\left(4a^{2}+15ab+4b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}}+\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\right.}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a\right)^{2}}\right.}{(a+b}$
risch	$\frac{e^{dx+c}}{2(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})d}-\frac{e^{-dx-c}}{2(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})d}+\frac{b^{2}e^{dx+c}\left(12a^{2}e^{6dx+6c}+15ab\,e^{6dx+6c}+3b^{2}e^{6dx+6c}+12a^{2}e^{4dx+}\right.}{4(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})(a}$

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a+b)^3*((-1/8*b*(12*a+5*b)/a*tanh(1/2*d*x+1/2*c)^7-3/8*(4*a^2+15*
a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5+3/8*(4*a^2+15*a*b+4*b^2)/a^2*b*tanh(
1/2*d*x+1/2*c)^3+1/8*b*(12*a+5*b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+3/8/a*(8*a^
2+4*a*b+b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)
+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)
^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2
))))-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)-1/(a+b)^3/(1+tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6260 vs. 2(140) = 280.

Time = 0.43 (sec) , antiderivative size = 11392, normalized size of antiderivative = 73.97

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a^
2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) - 4*a^3*b*e^(8*c) + 2*a^2*b^2*e
^(8*c) + 15*a*b^3*e^(8*c) + 3*b^4*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) - 8*a
^3*b*e^(6*c) + 32*a^2*b^2*e^(6*c) - 25*a*b^3*e^(6*c) - 9*b^4*e^(6*c))*e^(6*
d*x) + (4*a^4*e^(4*c) - 8*a^3*b*e^(4*c) + 32*a^2*b^2*e^(4*c) - 25*a*b^3*e^(
4*c) - 9*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) - 4*a^3*b*e^(2*c) + 2*a^2*
b^2*e^(2*c) + 15*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c)
+ 5*a^6*b*d*e^(9*c) + 10*a^5*b^2*d*e^(9*c) + 10*a^4*b^3*d*e^(9*c) + 5*a^3*b
^4*d*e^(9*c) + a^2*b^5*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 3*a^6*b*d*
e^(7*c) + 2*a^5*b^2*d*e^(7*c) - 2*a^4*b^3*d*e^(7*c) - 3*a^3*b^4*d*e^(7*c) -
 a^2*b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 7*a^6*b*d*e^(5*c) + 6*
a^5*b^2*d*e^(5*c) + 6*a^4*b^3*d*e^(5*c) + 7*a^3*b^4*d*e^(5*c) + 3*a^2*b^5*d
*e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 3*a^6*b*d*e^(3*c) + 2*a^5*b^2*d*e^
(3*c) - 2*a^4*b^3*d*e^(3*c) - 3*a^3*b^4*d*e^(3*c) - a^2*b^5*d*e^(3*c))*e^(3
```

```
*d*x) + (a^7*d*e^c + 5*a^6*b*d*e^c + 10*a^5*b^2*d*e^c + 10*a^4*b^3*d*e^c +
5*a^3*b^4*d*e^c + a^2*b^5*d*e^c)*e^(d*x)) + 1/2*integrate(3/2*((8*a^2*b*e^(
3*c) + 4*a*b^2*e^(3*c) + b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 4*a*b^2*e^
c + b^3*e^c)*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a
^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*
b^4*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 2*a^5*b*e^(2*c) - 2*a^3*b^3*e^(2*
c) - a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx=\int \frac{\cosh(dx+c)}{\left(b\tanh(dx+c)^2+a\right)^3}\,dx$$

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)}{\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.127 $\int \frac{\text{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\text{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+
b)^(5/2)/d+1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2
)^2+3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3757, 424, 393, 211}

$$\int \frac{\text{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)} + \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}} + \frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^
(5/2)*(a + b)^(5/2)*d) + (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{\text{Subst}\left(\int \frac{4a+3b+(4a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4a(a+b)d}
\end{aligned}
$$

$$= \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(8a^2+8ab+3b^2)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{8a^2(a+b)^2d}$$

$$= \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{2\sqrt{a}b(8a^2-ab-3b^2+(8a^2+11ab+3b^2)\cosh(2(c+dx)))\sinh(c+dx)}{(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a
 + b)^(5/2)) + (2*Sqrt[a]*b*(8*a^2 - a*b - 3*b^2 + (8*a^2 + 11*a*b + 3*b^2)
*Cosh[2*(c + d*x)])*Sinh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c +
d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(130) = 260$.

Time = 19.44 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.74

method	result
derivativedivides	$\frac{\frac{-\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(8a^{2}+29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(8a^{2}+29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a\left(a^{2}+2ab+b^{2}\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a\right)^{2}}+\cdots}{d}$
default	$\frac{\frac{-\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(8a^{2}+29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(8a^{2}+29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a\left(a^{2}+2ab+b^{2}\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a\right)^{2}}+\cdots}{d}$
risch	$\frac{(8a^{2}e^{6dx+6c}+11ab\,e^{6dx+6c}+3b^{2}e^{6dx+6c}+8a^{2}e^{4dx+4c}-13ab\,e^{4dx+4c}-9\,e^{4dx+4c}b^{2}-8a^{2}e^{2dx+2c}+13ab\,e^{2dx+2c}+9\,e^{2dx+\cdots}}{4(a^{2}+2ab+b^{2})\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^{2}d\,a^{2}}$

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(8*a^2
+29*a*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/8*(8*a^2+29*a
*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/8*b*(8*a+5*b)/a/(a
^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+
1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/4*(8*a^2+8*a*b+3*b^2)/a/(a^2+2*
a*b+b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/
2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4377 vs. $2(130) = 260$.

Time = 0.37 (sec) , antiderivative size = 7909, normalized size of antiderivative = 54.92

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/4*((8*a^2*b*e^(7*c) + 11*a*b^2*e^(7*c) + 3*b^3*e^(7*c))*e^(7*d*x) + (8*a^
2*b*e^(5*c) - 13*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c
) - 13*a*b^2*e^(3*c) - 9*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 11*a*b^2*e
^c + 3*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d + a
^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a^3
*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*b*
d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d
*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2*a
^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*
e^(3*c) + 8*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c
 + 3*b^2*e^c)*e^(d*x))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5*e^(4*c)
+ 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c))*e^(4*d*x) + 2*(a^5
*e^(2*c) + a^4*b*e^(2*c) - a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x
)
```

Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

3.128 $\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad\,(a+b\tanh^2(c+dx))^2} + \frac{3\tanh(c+dx)}{8a^2d\,(a+b\tanh^2(c+dx))}$$

```
[Out] 3/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d/b^(1/2)+1/4*tanh(d*x+c)/a
/d/(a+b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3756, 205, 211}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{3\tanh(c+dx)}{8a^2d\,(a+b\tanh^2(c+dx))} + \frac{\tanh(c+dx)}{4ad\,(a+b\tanh^2(c+dx))^2}$$

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Tanh[c
+ d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*a^2*d*(a +
b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}\\
&= \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^2d}\\
&= \frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{\frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}+\frac{\tanh(c+dx)\left(5a+3b\tanh^2(c+dx)\right)}{a^2\left(a+b\tanh^2(c+dx)\right)^2}}{8d}$$

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[b]) + (Tanh[c +
d*x]*(5*a + 3*b*Tanh[c + d*x]^2))/(a^2*(a + b*Tanh[c + d*x]^2)^2))/(8*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(82) = 164.

Time = 63.99 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.96

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-\frac{3\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{\cdots}{\sqrt{(\cdots}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a\cdots}}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-\frac{3\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{\cdots}{\sqrt{(\cdots}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a\cdots}}\cdots\right.}{\cdots}}{d}$
risch	$-\frac{5a^3e^{6dx+6c}-a^2b\,e^{6dx+6c}-9a\,b^2e^{6dx+6c}-3\,e^{6dx+6c}b^3+15a^3e^{4dx+4c}-a^2b\,e^{4dx+4c}+9a\,b^2e^{4dx+4c}+9\,e^{4dx+4c}b^3+15a^3e^{2\cdots}}{4(a^2+2ab+b^2)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2d\cdots}\right.}$

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c)
^5-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/a*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2-3/4/a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1
/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2768 vs. $2(82) = 164$.

Time = 0.34 (sec) , antiderivative size = 5840, normalized size of antiderivative = 60.83

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^2\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(82) = 164$.

Time = 0.40 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.81

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{5a^3+13a^2b+11ab^2+3b^3+(15a^3+13a^2b-11ab^2-9b^3)e^{(-2\,da}}{4\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4+4\left(a^6+2a^5b-2a^3b^3-a^2b^4\right)e^{(-2\,dx-2\,c)}+2\left(3a^6+4a^5b+2a^4b^2\right.\right.}$$
$$- \frac{3\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2d}$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/4*(5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3 + (15*a^3 + 13*a^2*b - 11*a*b^2 -
9*b^3)*e^(-2*d*x - 2*c) + (15*a^3 - a^2*b + 9*a*b^2 + 9*b^3)*e^(-4*d*x - 4*
c) + (5*a^3 - a^2*b - 9*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^6 + 4*a^5*b +
6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e
^(-2*d*x - 2*c) + 2*(3*a^6 + 4*a^5*b + 2*a^4*b^2 + 4*a^3*b^3 + 3*a^2*b^4)*e
^(-4*d*x - 4*c) + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c)
+ (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) -
3/8*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2
*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{\left(b\tanh(dx+c)^2+a\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{1}{\cosh(c+d\,x)^2\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

3.129 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{(4a+3b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d}+\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/8*(4*a+3*b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(3/2)/d
+1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)^2+1/8*(4*a+3*b)*sinh(d
*x+c)/a^2/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3757, 393, 205, 211}

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{(4a+3b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{3/2}}+\frac{(4a+3b)\sinh(c+dx)}{8a^2d(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{b\sinh(c+dx)}{4ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + 3*b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b
)^(3/2)*d) + (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2)
^2) + ((4*a + 3*b)*Sinh[c + d*x])/(8*a^2*(a + b)*d*(a + (a + b)*Sinh[c + d*
x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{\left(\frac{3}{a}+\frac{1}{a+b}\right)\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4d}\end{aligned}$$

$$= \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(4a+3b)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{8a^2(a+b)d}$$

$$= \frac{(4a+3b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{8\sinh(c+dx)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2} + (4a+3b)\left(\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}} + \frac{5a\sinh(c+dx)+3(a+b)\sinh^3(c+dx)}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{24(a+b)d}$$

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-8*Sinh[c + d*x])/(a + (a + b)*Sinh[c + d*x]^2)^2 + (4*a + 3*b)*((3*ArcTa
n[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) + (5*a*Sinh[c
 + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a^2*(a + (a + b)*Sinh[c + d*x]^2)^2))
)/(24*(a + b)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(115) = 230.

Time = 103.41 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.65

method	result
derivativedivides	$\frac{\frac{-\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a(a+b)}-\frac{\left(4a^{2}+13ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a^{2}(a+b)}+\frac{\left(4a^{2}+13ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a^{2}(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a\right)^{2}}+\frac{(4a+3b}{}}{d}$
default	$\frac{\frac{-\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a(a+b)}-\frac{\left(4a^{2}+13ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a^{2}(a+b)}+\frac{\left(4a^{2}+13ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a^{2}(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a\right)^{2}}+\frac{(4a+3b}{}}{d}$
risch	$\frac{(4a^{2}e^{6dx+6c}+7ab\,e^{6dx+6c}+3b^{2}e^{6dx+6c}+4a^{2}e^{4dx+4c}-ab\,e^{4dx+4c}-9\,e^{4dx+4c}b^{2}-4a^{2}e^{2dx+2c}+ab\,e^{2dx+2c}+9\,e^{2dx+2c}b^{2}-4}{4(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^{2}d\,a^{2}}$

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*(4*a^2+13*a*b+12*b
^2)/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2+13*a*b+12*b^2)/a^2/(a+b)*tan
h(1/2*d*x+1/2*c)^3+1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x
+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/4/a*
(4*a+3*b)/(a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1
/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)
*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3627 vs. 2(115) = 230.

Time = 0.36 (sec) , antiderivative size = 6614, normalized size of antiderivative = 51.27

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(b\tanh(dx+c)^2+a)^3}\,dx$$

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/4*((4*a^2*e^(7*c) + 7*a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - a*b*e^(5*c) - 9*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - a*b*e^(3*c) - 9*b^2*e^(3*c))*e^(3*d*x) - (4*a^2*e^c + 7*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^5*d + 3*a^4*b*d + 3*a^3*b^2*d + a^2*b^3*d + (a^5*d*e^(8*c) + 3*a^4*b*d*e^(8*c) + 3*a^3*b^2*d*e^(8*c) + a^2*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^5*d*e^(6*c) + a^4*b*d*e^(6*c) - a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^5*d*e^(4*c) + a^4*b*d*e^(4*c) + a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(4*c))*e^(4*d*x) + 4*(a^5*d*e^(2*c) + a^4*b*d*e^(2*c) - a^3*b^2*d*e^(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/32*((4*a*e^(3*c) + 3*b*e^(3*c))*e^(3*d*x) + (4*a*e^c + 3*b*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)

Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(b\tanh(dx+c)^2+a)^3}\,dx$$

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

3.130 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{(a-3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -1/8*(a-3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(3/2)/d+1/4*(a+b
)*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a-3*b)*tanh(d*x+c)/a^2/b/d/(
a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 393, 205, 211}

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{(a-3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((a - 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*b^(3/2)*d
) + ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b
)*Tanh[c + d*x])/(8*a^2*b*d*(a + b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}\\
&= \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}\\
&\quad - \frac{(a-3b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^2bd}\\
&= -\frac{(a-3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{(-a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}}+\frac{\sqrt{a}\left(a^2+6ab-3b^2+\left(a^2+4ab+3b^2\right)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((-a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(3/2) + (Sqrt[a]*(a
^2 + 6*a*b - 3*b^2 + (a^2 + 4*a*b + 3*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(101) = 202.

Time = 185.48 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.90

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8ab}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ab}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-}{d}$
default	$\frac{-\frac{2\left(-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8ab}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ab}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-}{d}$
risch	$-\frac{a^3e^{6dx+6c}-a^2b\,e^{6dx+6c}-5a\,b^2e^{6dx+6c}-3\,e^{6dx+6c}b^3+3a^3e^{4dx+4c}+7a^2b\,e^{4dx+4c}-3a\,b^2e^{4dx+4c}+9\,e^{4dx+4c}b^3+3a^3e^{2c}}{4(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+\right.}$

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2+11*a*b+12*b^2)/a
^2/b*tanh(1/2*d*x+1/2*c)^5-1/8*(3*a^2+11*a*b+12*b^2)/a^2/b*tanh(1/2*d*x+1/2
*c)^3-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/4/a*(a-3*b)/b*(-1/2*(a-
((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*ar
```

```
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))+1/2*(-a-((
a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2677 vs. $2(101) = 202$.

Time = 0.34 (sec) , antiderivative size = 5659, normalized size of antiderivative = 49.21

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(101) = 202$.

Time = 0.46 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.13

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$
$$= \frac{a^3+5a^2b+7ab^2+3b^3+(3a^3+13a^2b+ab^2-9b^3)e^{(-2dx-2c)}+(3a^3+7a}{4(a^5b+3a^4b^2+3a^3b^3+a^2b^4+4(a^5b+a^4b^2-a^3b^3-a^2b^4)e^{(-2dx-2c)}+2(3a^5b+a^4b^2+a^3b^3+3a^2b^4)e^{(}}$$
$$+ \frac{(a-3b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2bd}$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/4*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b^2 - 9*b^3)*e
^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) + (a
```

```
^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^5*b + 3*a^4*b^2 + 3*a^3
*b^3 + a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-2*d*x - 2*c) +
 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^5*b +
a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 3*a^4*b^2 + 3*a^3*
b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/8*(a - 3*b)*arctan(1/2*((a + b)*e^(
-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

3.131 $\int \frac{\text{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}d}+\frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/4*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+(a+b
)*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/d/(a+b
)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 205, 211}

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a+b}}+\frac{3\sinh(c+dx)}{8a^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{4ad\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a + b]*d) +
 Sinh[c + d*x]/(4*a*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/
(8*a^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4ad}\\
&= \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{8a^2d}\\
&= \frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2}\\
&\quad + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+\frac{\sqrt{a}\sinh(c+dx)\left(5a+3(a+b)\sinh^2(c+dx)\right)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2}}{8a^{5/2}d}$$

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[a + b] + (Sqrt[a]*Sin
h[c + d*x]*(5*a + 3*(a + b)*Sinh[c + d*x]^2))/(a + (a + b)*Sinh[c + d*x]^2)
^2)/(8*a^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(90) = 180.

Time = 0.22 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.65

$$\frac{\frac{-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a}+\frac{3(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2}-\frac{3(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}+\frac{\frac{3\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{8a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{3\left(\sqrt{(a+b)b}-b\right)\text{arc}\cdots}{8a\sqrt{(a+b)\cdots}}}{a}}{d}$$

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7+3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^5-
3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^3+5/8/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+3/4
/a*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/
2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. 2(90) = 180.

Time = 0.32 (sec) , antiderivative size = 5077, normalized size of antiderivative = 48.82

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^5\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**3, x)

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/4*(3*(a*e^(7*c) + b*e^(7*c))*e^(7*d*x) + (11*a*e^(5*c) - 9*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) - 9*b*e^(3*c))*e^(3*d*x) - 3*(a*e^c + b*e^c)*e^(d*x))/(a^4*d + 2*a^3*b*d + a^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*d*e^(4*c) - 2*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 32*integrate(3/128*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)

Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^5\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3), x)
```

3.132 $\int \frac{\text{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{\text{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\text{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(5/2)
/d+1/4*(a+b)*sech(d*x+c)^2*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2+3/8*(1/a
^2-1/b^2)*tanh(d*x+c)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 424, 393, 211}

$$\int \frac{\text{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)} + \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\tanh(c+dx)\text{sech}^2(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) + ((a + b)*Sech[c + d*x]^2*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Tanh[c + d*x])/(8*d*(a + b*Tanh[c +
d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a+b)\text{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-a+3b+(3a-b)x^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}\end{aligned}$$

$$= \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3(a^2-b^2)\tanh(c+dx)}{8a^2b^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{(3a^2-2ab+3b^2)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^2b^2d}$$

$$= \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3(a^2-b^2)\tanh(c+dx)}{8a^2b^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a}\sqrt{b}(a+b)\left(3a^2-10ab+3b^2+3(a^2-b^2)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}b^{5/2}d}$$

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - (Sqrt[a]
*Sqrt[b]*(a + b)*(3*a^2 - 10*a*b + 3*b^2 + 3*(a^2 - b^2)*Cosh[2*(c + d*x)])
*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*b^(5/
2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(117) = 234.

Time = 0.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.87

$$-\frac{\frac{2\left(\frac{(3a^2-2ab-5b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a\,b^2}+\frac{(9a^3+14a^2b-7a\,b^2-12b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b^2}+\frac{(9a^3+14a^2b-7a\,b^2-12b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b^2}+\frac{(3a^2-2ab-5b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a\,b^2}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}}{d}$$

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-2*(1/8*(3*a^2-2*a*b-5*b^2)/a/b^2*tanh(1/2*d*x+1/2*c)^7+1/8*(9*a^3+14*
a^2*b-7*a*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3+14*a^2*b-7*a
*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*(3*a^2-2*a*b-5*b^2)/a/b^2*ta
```

```
nh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2-1/4/a*(3*a^2-2*a*b+3*b^2)/b^2*(1/2*(a+((a+b)*b)^(
1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2
)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(117) = 234$.

Time = 0.34 (sec) , antiderivative size = 5233, normalized size of antiderivative = 39.95

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(117) = 234$.

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{3a^3+3a^2b-3ab^2-3b^3+(9a^3-13a^2b-13ab^2+9b^3)e^{(-2dx-2c)}+3(3a^3-5a^2b+5ab^2-3b^3)e}{4(a^4b^2+2a^3b^3+a^2b^4+4(a^4b^2-a^2b^4)e^{(-2dx-2c)}+2(3a^4b^2-2a^3b^3+3a^2b^4)e^{(-4dx-4c)}+4(a^4b^2-a^2} - \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2b^2d}$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 - 13*a^2*b - 13*a*b^2 + 9*
b^3)*e^(-2*d*x - 2*c) + 3*(3*a^3 - 5*a^2*b + 5*a*b^2 - 3*b^3)*e^(-4*d*x - 4
*c) + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^(-6*d*x - 6*c))/((a^4*b^2 + 2*a^3*b
^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^4*b^2 - 2*a^
3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 - a^2*b^4)*e^(-6*d*x - 6*c
) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) - 1/8*(3*a^2 - 2*a
*b + 3*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(
a*b)*a^2*b^2*d)
```

Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^6}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^6\,\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3), x)
```

3.133 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\arctan(\sinh(c+dx))}{b^3d} + \frac{\sqrt{a+b}(8a^2-4ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} + \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] -arctan(sinh(d*x+c))/b^3/d+1/4*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c)
^2)^2-1/8*(4*a-3*b)*(a+b)*sinh(d*x+c)/a^2/b^2/d/(a+(a+b)*sinh(d*x+c)^2)+1/8
*(8*a^2-4*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/a^
(5/2)/b^3/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3757, 425, 541, 536, 209, 211\}$

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left((a+b)\sinh^2(c+dx)+a\right)} + \frac{\sqrt{a+b}(8a^2-4ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} + \frac{(a+b)\sinh(c+dx)}{4abd\left((a+b)\sinh^2(c+dx)+a\right)^2} - \frac{\arctan(\sinh(c+dx))}{b^3d}$$

```
[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b^3*d)) + (Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Arc
Tan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) + ((a + b)*Sinh
[c + d*x])/(4*a*b*d*(a + (a + b)*Sinh[c + d*x]^2)^2) - ((4*a - 3*b)*(a + b)
*Sinh[c + d*x])/(8*a^2*b^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a-3b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4abd} \\
&= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{4a^2-ab+3b^2-(4a-3b)(a+b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{8a^2b^2d} \\
&= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{b^3d} \\
&\quad + \frac{\left((a+b)\left(8a^2-4ab+3b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{8a^2b^3d} \\
&= -\frac{\arctan(\sinh(c+dx))}{b^3d} + \frac{\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} \\
&\quad + \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{2\sqrt{a+b}(8a^2-4ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}}+\frac{2(8a^3+4a^2b-ab^2+3b^3)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+64\arctan\left(\tanh\left(\frac{1}{2}(c+\right.\right.}{}$$

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/32*((2*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x]
)/Sqrt[a + b]])/a^(5/2) + (2*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*ArcTan[(Sqrt
[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + 64*ArcTan[Tanh[(c
+ d*x)/2]] + (I*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Log[a - b + (a + b)*Cos
h[2*(c + d*x)]])/a^(5/2) - (I*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*Log[a - b +
 (a + b)*Cosh[2*(c + d*x)]])/(a^(5/2)*Sqrt[a + b]) - (32*b^2*(a + b)*Sinh[c
 + d*x])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (8*b*(4*a^2 + a*b - 3*
b^2)*Sinh[c + d*x])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*x)]))))/(b^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(142) = 284$.

Time = 0.21 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.49

$$\frac{\frac{\frac{2\left(\frac{b(4a^2-ab-5b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}+\frac{(4a^3+23a^2b+7ab^2-12b^3)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{(4a^3+23a^2b+7ab^2-12b^3)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{b(4a^2-ab-5b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}}{b^3}}{d}$$

```
[In] int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(2/b^3*((1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^3+23*
a^2*b+7*a*b^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^5-1/8*(4*a^3+23*a^2*b+7*a*b
^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^3-1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/2*d
*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2+1/8/a*(8*a^3+4*a^2*b-a*b^2+3*b^3)*(1/2*(((a+b)*b)^(1/2)+b
)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((
a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
```

```
2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-2/b^3*arctan(tanh(1/2*d*x+1/2*c
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4457 vs. $2(142) = 284$.

Time = 0.38 (sec) , antiderivative size = 8070, normalized size of antiderivative = 51.73

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^7}{(b\tanh(dx+c)^2+a)^3}\,dx$$

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/4*((4*a^3*e^(7*c) + 5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e
^(7*d*x) + (4*a^3*e^(5*c) - 19*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e^(
5*c))*e^(5*d*x) - (4*a^3*e^(3*c) - 19*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) + 9*
b^3*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c
)*e^(d*x))/(a^4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) + 2*a^
3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c) - a^2
*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d*e^(4*c) - 2*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) - a^2*b^4*d*e^(2*c))*
e^(2*d*x)) - 2*arctan(e^(d*x + c))/(b^3*d) + 128*integrate(1/512*((8*a^3*e^
(3*c) + 4*a^2*b*e^(3*c) - a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (8*a^3
*e^c + 4*a^2*b*e^c - a*b^2*e^c + 3*b^3*e^c)*e^(d*x))/(a^3*b^3 + a^2*b^4 + (
a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*c) - a^2*b^4
*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^7\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3), x)
```

3.134 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=(a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*(a+b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3712, 3554, 8}

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^5(c+dx)}{5d}$$

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - ((a + b)*Tanh[c + d*x]^3)/(3*d) - (
b*Tanh[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int\tanh^4(c+dx)\,dx\\ &= -\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int\tanh^2(c+dx)\,dx\\ &= -\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int 1\,dx\\ &= (a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int\tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\text{arctanh}(\tanh(c+dx))}{d}+\frac{b\text{arctanh}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}-\frac{b\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d
*x])/d - (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]
^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

method	result
parallelrisch	$\frac{-3\tanh(dx+c)^5 b-5\tanh(dx+c)^3 a-5b\tanh(dx+c)^3+15adx+15dxb-15a\tanh(dx+c)-15b\tanh(dx+c)}{15d}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^5 b}{5}-\frac{\tanh(dx+c)^3 a}{3}-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^5 b}{5}-\frac{\tanh(dx+c)^3 a}{3}-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)}{2}}{d}$
parts	$\frac{b\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{a\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\right.}{}$
risch	$ax+bx+\frac{4a\,e^{8dx+8c}+6b\,e^{8dx+8c}+12a\,e^{6dx+6c}+12b\,e^{6dx+6c}+\frac{44a\,e^{4dx+4c}}{3}+\frac{56b\,e^{4dx+4c}}{3}+\frac{28\,e^{2dx+2c}a}{3}+\frac{28b\,e^{2dx+2c}}{3}+}{d(e^{2dx+2c}+1)^5}$

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-3*tanh(d*x+c)^5*b-5*tanh(d*x+c)^3*a-5*b*tanh(d*x+c)^3+15*a*d*x+15*d*
x*b-15*a*tanh(d*x+c)-15*b*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 6.28

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(15\,(a+b)dx+20\,a+23\,b)\cosh\left(dx+c\right)^5+5\,(15\,(a+b)dx+20\,a+23\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-(}{}$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*((15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^5 + 5*(15*(a + b)*d*x +
20*a + 23*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a + 23*b)*sinh(d*x + c)^5
+ 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 - 5*(2*(20*a + 23*b)*cos
h(d*x + c)^2 + 8*a + 5*b)*sinh(d*x + c)^3 + 5*(2*(15*(a + b)*d*x + 20*a + 2
3*b)*cosh(d*x + c)^3 + 3*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + 10*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c) - 5*((20*a +
23*b)*cosh(d*x + c)^4 + 3*(8*a + 5*b)*cosh(d*x + c)^2 + 4*a + 10*b)*sinh(d*
x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d
*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 1
0*d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\begin{cases} ax-\frac{a\tanh^3(c+dx)}{3d}-\frac{a\tanh(c+dx)}{d}+bx-\frac{b\tanh^5(c+dx)}{5d}-\frac{b\tanh^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d} & \text{for } d\neq 0\\ x\left(a+b\tanh^2(c)\right)\tanh^4(c) & \text{otherwise}\end{cases}$$

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*tanh(c + d*x)**3/(3*d) - a*tanh(c + d*x)/d + b*x - b*tan
h(c + d*x)**5/(5*d) - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)*tanh(c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(50) = 100.

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.69

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{1}{15}b\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*
e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c
) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.48

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{15\,(dx+c)(a+b) + \frac{2\,\left(30\,ae^{(8\,dx+8\,c)}+45\,be^{(8\,dx+8\,c)}+90\,ae^{(6\,dx+6\,c)}+90\,be^{(6\,dx+6\,c)}+110\,ae^{(4\,dx+4\,c)}+140\,be^{(4\,dx+4\,c)}+70\,ae^{(2\,dx+2\,c)}+\cdots\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*(a + b) + 2*(30*a*e^(8*d*x + 8*c) + 45*b*e^(8*d*x + 8*c)
 + 90*a*e^(6*d*x + 6*c) + 90*b*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) + 14
0*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 70*b*e^(2*d*x + 2*c) + 20*a +
23*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b) - \frac{\tanh(c+d\,x)^3\,(a+b)}{3\,d} - \frac{b\tanh(c+d\,x)^5}{5\,d} - \frac{\tanh(c+d\,x)\,(a+b)}{d}$$

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (tanh(c + d*x)^3*(a + b))/(3*d) - (b*tanh(c + d*x)^5)/(5*d) - (
tanh(c + d*x)*(a + b))/d
```

3.135 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{(a+b)\tanh^2(c+dx)}{2d}-\frac{b\tanh^4(c+dx)}{4d}$$

```
[Out] (a+b)*ln(cosh(d*x+c))/d-1/2*(a+b)*tanh(d*x+c)^2/d-1/4*b*tanh(d*x+c)^4/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3712, 3554, 3556}

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^4(c+dx)}{4d}$$

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - ((a + b)*Tanh[c + d*x]^2)/(2*d) - (b*Tanh[
c + d*x]^4)/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\tanh^4(c+dx)}{4d} + (a+b)\int \tanh^3(c+dx)\,dx \\
&= -\frac{(a+b)\tanh^2(c+dx)}{2d} - \frac{b\tanh^4(c+dx)}{4d} + (a+b)\int \tanh(c+dx)\,dx \\
&= \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{(a+b)\tanh^2(c+dx)}{2d} - \frac{b\tanh^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{-4(a+b)\log(\cosh(c+dx)) + 2(a+b)\tanh^2(c+dx) + b\tanh^4(c+dx)}{4d}$$

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] -1/4*(-4*(a + b)*Log[Cosh[c + d*x]] + 2*(a + b)*Tanh[c + d*x]^2 + b*Tanh[c
+ d*x]^4)/d
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 b}{4}-\frac{\tanh(dx+c)^2 a}{2}-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 b}{4}-\frac{\tanh(dx+c)^2 a}{2}-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$
parallelrisch	$-\frac{\tanh(dx+c)^4 b+4adx+4dxb+2\tanh(dx+c)^2 a+2b\tanh(dx+c)^2+4\ln(1-\tanh(dx+c))a+4\ln(1-\tanh(dx+c))b}{4d}$
parts	$\frac{a\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh}{}\right.}{d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2\,e^{2dx+2c}\left(a\,e^{4dx+4c}+2b\,e^{4dx+4c}+2\,e^{2dx+2c}a+2b\,e^{2dx+2c}+a+2b\right)}{d\left(e^{2dx+2c}+1\right)^4}+\frac{\ln\left(e^{2dx+2c}+1\right)}{d}$

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*tanh(d*x+c)^4*b-1/2*tanh(d*x+c)^2*a-1/2*b*tanh(d*x+c)^2-1/2*(a+b)
*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 1205, normalized size of antiderivative = 24.59

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + (a + b)*d*x*sinh(d*x + c)^8 + 2*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^
6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x - a - 2*b)*sinh(d*x +
 c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 + 3*(2*(a + b)*d*x - a - 2*b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4
 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x + 15*(2*(a + b)*d*x -
a - 2*b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*co
sh(d*x + c)^5 + 5*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^3 + (3*(a + b)*d*
x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x + 2*(2*(a + b)*
d*x - a - 2*b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 + 15*(2*
(a + b)*d*x - a - 2*b)*cosh(d*x + c)^4 + 2*(a + b)*d*x + 6*(3*(a + b)*d*x -
 2*a - 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x
+ c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8
+ 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 + 30*(a
 + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x
+ c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c
```

```
)^3 + 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 + 15*(a + b)
*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 8*(
(a + b)*cosh(d*x + c)^7 + 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x +
c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 + 3*(2*(a
 + b)*d*x - a - 2*b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d
*x + c)^3 + (2*(a + b)*d*x - a - 2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^4(c+dx)}{4d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) + b*
x - b*log(tanh(c + d*x) + 1)/d - b*tanh(c + d*x)**4/(4*d) - b*tanh(c + d*x)
**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.43

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right) + a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a*(x + c/d + log(e^(-2*d*x - 2
*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\left((a+2\,b)e^{(6\,dx+6\,c)}+2\,(a+b)e^{(4\,dx+4\,c)}+(a+2\,b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*((a + 2*b)*e^(6*
d*x + 6*c) + 2*(a + b)*e^(4*d*x + 4*c) + (a + 2*b)*e^(2*d*x + 2*c))/(e^(2*d
*x + 2*c) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{\tanh(c+d\,x)^2\,(a+b)}{2\,d}-\frac{b\tanh(c+d\,x)^4}{4\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

```
[In] int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (tanh(c + d*x)^2*(a + b))/(2*d) - (b*tanh(c + d*x)^4)/(4*d) - (
log(tanh(c + d*x) + 1)*(a + b))/d
```

3.136 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=(a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3712, 3554, 8}

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^3(c+dx)}{3d}$$

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\tanh^3(c+dx)}{3d} - (-a-b)\int \tanh^2(c+dx)\,dx \\
&= -\frac{(a+b)\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d} - (-a-b)\int 1\,dx \\
&= (a+b)x - \frac{(a+b)\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\text{arctanh}(\tanh(c+dx))}{d} + \frac{b\text{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{b\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d}$$

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d
*x])/d - (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

method	result
parallelrisch	$-\frac{b\tanh(dx+c)^{3}-3adx-3dxb+3a\tanh(dx+c)+3b\tanh(dx+c)}{3d}$
derivativedivides	$\frac{-\frac{b\tanh(dx+c)^{3}}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b\tanh(dx+c)^{3}}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)+1)}{2}}{d}$
risch	$ax+bx+\frac{2a\,e^{4dx+4c}+4b\,e^{4dx+4c}+4\,e^{2dx+2c}a+4b\,e^{2dx+2c}+2a+\frac{8b}{3}}{d\left(e^{2dx+2c}+1\right)^{3}}$
parts	$\frac{a\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^{3}}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx\right.}{2}}{d}$

[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] -1/3*(b*tanh(d*x+c)^3-3*a*d*x-3*d*x*b+3*a*tanh(d*x+c)+3*b*tanh(d*x+c))/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)^3+3\,(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-(3\,a+4}{3\,\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+\right.\right.}$$

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] 1/3*((3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c)^3 + 3*(3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a + 4*b)*sinh(d*x + c)^3 + 3*(3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c) - 3*((3*a + 4*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax-\frac{a\tanh\left(c+dx\right)}{d}+bx-\frac{b\tanh^{3}\left(c+dx\right)}{3d}-\frac{b\tanh\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(a+b\tanh^{2}\left(c\right)\right)\tanh^{2}\left(c\right) & \text{otherwise} \end{cases}$$

[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)

```
[Out] Piecewise((a*x - a*tanh(c + d*x)/d + b*x - b*tanh(c + d*x)**3/(3*d) - b*tan
h(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(34) = 68$.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{3}b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right) + a\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*
e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a*(x + c/
d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(a+b)+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+4\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*(a + b) + 2*(3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) + 6
*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a + 4*b)/(e^(2*d*x + 2*c) + 1)
^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{b\tanh(c+d\,x)^3}{3\,d}-\frac{\tanh(c+d\,x)\,\left(a+b\right)}{d}$$

```
[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
[Out] x*(a + b) - (b*tanh(c + d*x)^3)/(3*d) - (tanh(c + d*x)*(a + b))/d
```

3.137 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] (a+b)*ln(cosh(d*x+c))/d-1/2*b*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3712, 3556}

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
```

```
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{b\tanh^2(c+dx)}{2d} - (-a-b)\int \tanh(c+dx)\,dx$$
$$= \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{b\tanh^2(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\log(\cosh(c+dx))}{d} + \frac{b\log(\cosh(c+dx))}{d} - \frac{b\tanh^2(c+dx)}{2d}$$

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*Log[Cosh[c + d*x]])/d + (b*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(
2*d)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	49
default	$\frac{-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	49
parts	$\frac{a\ln(\cosh(dx+c))}{d}+\frac{b\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	52
parallelrisch	$-\frac{2adx+2dxb+b\tanh(dx+c)^2+2\ln(1-\tanh(dx+c))a+2\ln(1-\tanh(dx+c))b}{2d}$	55
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2b\,\mathrm{e}^{2dx+2c}}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+1\right)a}{d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+1\right)b}{d}$	86

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/2*b*tanh(d*x+c)^2-1/2*(a+b)*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(tanh(d*
x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 399, normalized size of antiderivative = 12.87

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+b)}{}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x + 2*((a + b)*d*x - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a + b)*d*x - b)*sinh(d*x + c)
^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh
(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 + ((a + b)*d*x - b)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
+ d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sin
h(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d + b*x - b*log(tanh(c + d*x) + 1
)/d - b*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + \frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d
*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\,be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*b*e^(2*d*x + 2*c
)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{b\tanh(c+d\,x)^2}{2\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

```
[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
[Out] x*(a + b) - (b*tanh(c + d*x)^2)/(2*d) - (log(tanh(c + d*x) + 1)*(a + b))/d
```

3.138 $\int \left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=ax+bx-\frac{b\tanh(c+dx)}{d}$$

```
[Out] a*x+b*x-b*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3554, 8}

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=ax-\frac{b\tanh(c+dx)}{d}+bx$$

```
[In] Int[a + b*Tanh[c + d*x]^2,x]
```

```
[Out] a*x + b*x - (b*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= ax + b\int \tanh^2(c+dx)\,dx \\ &= ax - \frac{b\tanh(c+dx)}{d} + b\int 1\,dx \\ &= ax + bx - \frac{b\tanh(c+dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \left(a + b\tanh^2(c+dx)\right)\,dx = ax + \frac{b\text{arctanh}(\tanh(c+dx))}{d} - \frac{b\tanh(c+dx)}{d}$$

```
[In] Integrate[a + b*Tanh[c + d*x]^2,x]
[Out] a*x + (b*ArcTanh[Tanh[c + d*x]])/d - (b*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
parallelrisch	$-\frac{b(-dx+\tanh(dx+c))}{d}+ax$	22
risch	$ax+bx+\frac{2b}{d\left(e^{2dx+2c}+1\right)}$	27
default	$ax+\frac{b\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	41
parts	$ax+\frac{b\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	41
derivativedivides	$\frac{-b\tanh(dx+c)+\frac{(-a-b)\ln(\tanh(dx+c)-1)}{2}-\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	51

```
[In] int(a+b*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -b*(-d*x+tanh(d*x+c))/d+a*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=\frac{((a+b)dx+b)\cosh\left(dx+c\right)-b\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)}$$

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="fricas")
[Out] (((a + b)*d*x + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=ax+b\left(\begin{cases} x-\frac{\tanh\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\tanh^2\left(c\right) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*tanh(d*x+c)**2,x)
[Out] a*x + b*Piecewise((x - tanh(c + d*x)/d, Ne(d, 0)), (x*tanh(c)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+ax$$

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="maxima")
[Out] b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=ax+\frac{\left(dx+c+\frac{2}{e^{(2\,dx+2\,c)}+1}\right)b}{d}$$

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="giac")
[Out] a*x + (d*x + c + 2/(e^(2*d*x + 2*c) + 1))*b/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=x\,(a+b)-\frac{b\tanh(c+d\,x)}{d}$$

```
[In] int(a + b*tanh(c + d*x)^2,x)
[Out] x*(a + b) - (b*tanh(c + d*x))/d
```

3.139 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}$$

```
[Out] b*ln(cosh(d*x+c))/d+a*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3706, 3556}

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\log(\sinh(c+dx))}{d}+\frac{b\log(\cosh(c+dx))}{d}$$

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*Log[Cosh[c + d*x]])/d + (a*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3706

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x],
x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Rubi steps

$$\text{integral} = a\int \coth(c+dx)\,dx + b\int \tanh(c+dx)\,dx$$
$$= \frac{b\log(\cosh(c+dx))}{d} + \frac{a\log(\sinh(c+dx))}{d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\log(\cosh(c+dx))}{d} + \frac{a(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{d}$$

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] (b*Log[Cosh[c + d*x]])/d + (a*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
default	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
parallelrisch	$\frac{(-a-b)\ln(1-\tanh(dx+c))+a\ln(\tanh(dx+c))-(a+b)xd}{d}$	41
risch	$-ax-bx-\frac{2bc}{d}-\frac{2ac}{d}+\frac{\ln(e^{2dx+2c}+1)b}{d}+\frac{a\ln(e^{2dx+2c}-1)}{d}$	58

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(sinh(d*x+c))+b*ln(cosh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.80

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+b)dx - b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) - a\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -((a + b)*d*x - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - a*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\coth\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d} + \frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] b*log(e^(d*x + c) + e^(-d*x - c))/d + a*log(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-b\log\left(e^{(2\,dx+2\,c)}+1\right)-a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -((d*x + c)*(a + b) - b*log(e^(2*d*x + 2*c) + 1) - a*log(abs(e^(2*d*x + 2*c
) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 228, normalized size of antiderivative = 9.12

$$\begin{aligned}\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{a\,\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,e^{4\,c}\,e^{4\,d\,x}+4\,b^2\,e^{4\,c}\,e^{4\,d\,x}-8\,a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d}\\&\quad-b\,x-\frac{\mathrm{atan}\left(\frac{a\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}-\frac{b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}\right)\,\sqrt{a^2-2\,a\,b+b^2}}{\sqrt{-d^2}}-a\,x\\&\quad+\frac{b\,\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,e^{4\,c}\,e^{4\,d\,x}+4\,b^2\,e^{4\,c}\,e^{4\,d\,x}-8\,a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d}\end{aligned}$$

```
[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2),x)
[Out] (a*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(4*d*x) + 4*b^2*exp(4*c)*e
xp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - b*x - (atan((a*exp(2*c)*exp
(2*d*x)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)) - (b*exp(2*c)*exp(2*d*x
)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2))/(
-d^2)^(1/2) - a*x + (b*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(4*d*x
) + 4*b^2*exp(4*c)*exp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d)
```

3.140 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=(a+b)x-\frac{a\coth(c+dx)}{d}$$

```
[Out] (a+b)*x-a*coth(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3710, 8}

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=x(a+b)-\frac{a\coth(c+dx)}{d}$$

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - (a*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
```

```
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a\coth(c+dx)}{d} - \int(-a-b)\,dx\\ &= (a+b)x - \frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = bx - \frac{a\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}$$

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] b*x - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

method	result	size
risch	$ax+bx-\frac{2a}{d\left(e^{2dx+2c}-1\right)}$	27
parallelrisch	$\frac{-a+\tanh(dx+c)dx(a+b)}{d\tanh(dx+c)}$	29
derivativedivides	$\frac{-\frac{a}{\tanh(dx+c)}+\left(-\frac{b}{2}-\frac{a}{2}\right)\ln(\tanh(dx+c)-1)+\left(\frac{b}{2}+\frac{a}{2}\right)\ln(\tanh(dx+c)+1)}{d}$	51
default	$\frac{-\frac{a}{\tanh(dx+c)}+\left(-\frac{b}{2}-\frac{a}{2}\right)\ln(\tanh(dx+c)-1)+\left(\frac{b}{2}+\frac{a}{2}\right)\ln(\tanh(dx+c)+1)}{d}$	51

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] a*x+b*x-2*a/d/(exp(2*d*x+2*c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\cosh\left(dx+c\right)-\left((a+b)dx+a\right)\sinh\left(dx+c\right)}{d\sinh\left(dx+c\right)}$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -(a*cosh(d*x + c) - ((a + b)*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(14) = 28$.

Time = 4.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.00

$$\begin{aligned}\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= a\left(\begin{cases} x\coth^2\left(c\right) & \text{for}\ d=0\\ -\frac{\log\left(-e^{-dx}\right)\coth^2\left(dx+\log\left(-e^{-dx}\right)\right)}{d} & \text{for}\ c=\log\left(-e^{-dx}\right)\\ x\coth^2\left(dx+\log\left(e^{-dx}\right)\right) & \text{for}\ c=\log\left(e^{-dx}\right)\\ x-\frac{1}{d\tanh\left(c+dx\right)} & \text{otherwise}\end{cases}\right)\\ &\quad + b\left(\begin{cases} x & \text{for}\ |x|<1\\ G_{2,2}^{1,1}\left(\begin{matrix}1 & 2\\ 1 & 0\end{matrix}\middle|\,x\right)+G_{2,2}^{0,2}\left(\begin{matrix}2,1 & \\ & 1,0\end{matrix}\middle|\,x\right) & \text{otherwise}\end{cases}\right)\end{aligned}$$

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
[Out] a*Piecewise((x*coth(c)**2, Eq(d, 0)), (-log(-exp(-d*x))*coth(d*x + log(-exp
(-d*x)))**2/d, Eq(c, log(-exp(-d*x)))), (x*coth(d*x + log(exp(-d*x)))**2, E
q(c, log(exp(-d*x)))), (x - 1/(d*tanh(c + d*x)), True)) + b*Piecewise((x, A
bs(x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()),
 ((), (1, 0)), x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = a\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+bx$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b*x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(dx+c)(a+b)-\frac{2\,a}{e^{(2\,dx+2\,c)}-1}}{d}$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] ((d*x + c)*(a + b) - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{2\,a}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$

```
[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
[Out] x*(a + b) - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

3.141 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*a*coth(d*x+c)^2/d+(a+b)*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3710, 12, 3556}

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^2(c+dx)}{2d}$$

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*(a*Coth[c + d*x]^2)/d + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a\coth^2(c+dx)}{2d} + \int (a+b)\coth(c+dx)\,dx \\
&= -\frac{a\coth^2(c+dx)}{2d} + (a+b)\int \coth(c+dx)\,dx \\
&= -\frac{a\coth^2(c+dx)}{2d} + \frac{(a+b)\log(\sinh(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$
\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-a\coth^2(c+dx)+2(a+b)(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{2d}
$$

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] (-(a*Coth[c + d*x]^2) + 2*(a + b)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])
)/(2*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
parallelrisch	$\frac{(-2a-2b)\ln(1-\tanh(dx+c))+(2a+2b)\ln(\tanh(dx+c))-\coth(dx+c)^2a-2(a+b)xd}{2d}$	59
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2a\,e^{2dx+2c}}{d\left(e^{2dx+2c}-1\right)^2}+\frac{a\ln\left(e^{2dx+2c}-1\right)}{d}+\frac{\ln\left(e^{2dx+2c}-1\right)b}{d}$	86

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+b*ln(sinh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 13.13

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+b)}{}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x - 2*((a + b)*d*x - a)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 - (a + b)*d*x + a)*sinh(d*x + c)
^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a + b)*sinh(d*x + c)^4 - 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 - a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (a + b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh
(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 - ((a + b)*d*x - a)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
+ d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sin
h(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\coth^3\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.42

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$+\frac{b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b*log(e^(d*x +
c) - e^(-d*x - c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=-\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,ae^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*a*e^(2*d*x
+ 2*c)/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,(a+b)}{d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$
$$-\frac{2\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-x\,(a+b)$$

```
[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
 - (2*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - x*(a + b)
```

3.142 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=(a+b)x-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}$$

```
[Out] (a+b)*x-(a+b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3710, 12, 3554, 8}

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\coth(c+dx)}{d}+x(a+b)-\frac{a\coth^3(c+dx)}{3d}$$

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\coth^3(c+dx)}{3d} + \int (a+b)\coth^2(c+dx)\,dx \\
&= -\frac{a\coth^3(c+dx)}{3d} + (a+b)\int \coth^2(c+dx)\,dx \\
&= -\frac{(a+b)\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d} + (a+b)\int 1\,dx \\
&= (a+b)x - \frac{(a+b)\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\begin{aligned}
&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\
&= -\frac{a\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d} \\
&\quad - \frac{b\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}
\end{aligned}$$

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/3*(a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])/
d - (b*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-\coth(dx+c)^{3}a+(-3a-3b)\coth(dx+c)+3(a+b)xd}{3d}$	39
derivativedivides	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^{3}}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
default	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^{3}}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
risch	$ax+bx-\frac{2\left(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}-6\,e^{2dx+2c}a-6b\,e^{2dx+2c}+4a+3b\right)}{3d\left(e^{2dx+2c}-1\right)^{3}}$	81

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-coth(d*x+c)^3*a+(-3*a-3*b)*coth(d*x+c)+3*(a+b)*x*d)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(34) = 68.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(4\,a+3\,b)\cosh\left(dx+c\right)^{3}+3\,(4\,a+3\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-(3\,(a+b)dx+4\,a+3\,b)\sinh\left(dx\right.}{3\,\left(d\sinh\left(dx+c\right)^{3}+3\,\left(d\,c\right.\right.}$$

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -1/3*((4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x +
c)^2 - (3*(a + b)*d*x + 4*a + 3*b)*sinh(d*x + c)^3 - 3*b*cosh(d*x + c) + 3*
(3*(a + b)*d*x - (3*(a + b)*d*x + 4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*s
inh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^{2}\left(c+dx\right)\right)\coth^{4}\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(34) = 68$.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{3}\,a\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right) + b\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)$$

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(a+b)-\frac{2\left(6\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+4\,a+3\,b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*(a + b) - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 4*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.50

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\frac{2\,b}{3\,d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a+b)}{3\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1} - \frac{\frac{2\,(2\,a+b)}{3\,d}-\frac{4\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(2\,a+b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1} + x\,(a+b) - \frac{2\,(2\,a+b)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

```
[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] ((2*b)/(3*d) - (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*x) - 2*
exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) - (4*b*exp(2*c + 2*d*x))/(3*d)
 + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1) + x*(a + b) - (2*(2*a + b))/(3*d*(exp(2*c +
2*d*x) - 1))
```

3.143 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\coth^2(c+dx)}{2d}-\frac{a\coth^4(c+dx)}{4d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(a+b)*coth(d*x+c)^2/d-1/4*a*coth(d*x+c)^4/d+(a+b)*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3710, 12, 3554, 3556}

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^4(c+dx)}{4d}$$

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((a + b)*Coth[c + d*x]^2)/d - (a*Coth[c + d*x]^4)/(4*d) + ((a + b)*Log
[Sinh[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\coth^4(c+dx)}{4d} + \int (a+b)\coth^3(c+dx)\,dx \\
&= -\frac{a\coth^4(c+dx)}{4d} + (a+b)\int \coth^3(c+dx)\,dx \\
&= -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + (a+b)\int \coth(c+dx)\,dx \\
&= -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + \frac{(a+b)\log(\sinh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2(a+b)\coth^2(c+dx) + a\coth^4(c+dx) - 4(a+b)(\log(\cosh(c+dx)) + \log(\tanh(c+dx)))}{4d}$$

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
[Out] -1/4*(2*(a + b)*Coth[c + d*x]^2 + a*Coth[c + d*x]^4 - 4*(a + b)*(Log[Cosh[c
 + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)}{d}$
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)}{d}$
parallelrisch	$\frac{(-4a-4b)\ln(1-\tanh(dx+c))+(4a+4b)\ln(\tanh(dx+c))-\coth(dx+c)^4a+(-2a-2b)\coth(dx+c)^2-4(a+b)xd}{4d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2\,e^{2dx+2c}\left(2a\,e^{4dx+4c}+b\,e^{4dx+4c}-2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+2a+b\right)}{d\left(e^{2dx+2c}-1\right)^4}+\frac{a\ln\left(e^{2dx+2c}-1\right)}{d}$

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b*(ln(sinh(d*x
+c))-1/2*coth(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 1216, normalized size of antiderivative = 24.82

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + (a + b)*d*x*sinh(d*x + c)^8 - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^
6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 - 2*(a + b)*d*x + 2*a + b)*sinh(d*x +
 c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 - 3*(2*(a + b)*d*x - 2*a - b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4
 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x - 15*(2*(a + b)*d*x -
2*a - b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*co
sh(d*x + c)^5 - 5*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*(a + b)*d*
x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x - 2*(2*(a + b)*
d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 - 15*(2*
(a + b)*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*(a + b)*d*x + 6*(3*(a + b)*d*x -
 2*a - 2*b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x
+ c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8
- 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*
x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 - 30*(a
```

```
 + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x
+ c)^5 - 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c
)^3 - 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 - 15*(a + b)
*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 8*(
(a + b)*cosh(d*x + c)^7 - 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x +
c)^3 - (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 - 3*(2*(a
 + b)*d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d
*x + c)^3 - (2*(a + b)*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)\coth^5\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(45) = 90.

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.20

$$\begin{aligned}&\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&=a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,d}}\right.\\&\quad\left.+b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\right.\end{aligned}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6
```

```
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b*(x + c
/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)
/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(45) = 90$.

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\left((2\,a+b)e^{(6\,dx+6\,c)}-2\,(a+b)e^{(4\,dx+4\,c)}+(2\,a+b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*((2*a + b)*
e^(6*d*x + 6*c) - 2*(a + b)*e^(4*d*x + 4*c) + (2*a + b)*e^(2*d*x + 2*c))/(e
^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.61

$$\begin{aligned}\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,(a+b)}{d}-\frac{2\,(2\,a+b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-x\,(a+b)\\ &\quad-\frac{2\,(4\,a+b)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}\\ &\quad-\frac{4\,a}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}\end{aligned}$$

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*(2*a + b))/(d*(exp(2*c + 2*d*
x) - 1)) - x*(a + b) - (2*(4*a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d
*x) + 1)) - (8*a)/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6
*d*x) - 1)) - (4*a)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c
 + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.144 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*(a+b)^2*tanh(d*x+c)^3/d-1/5*b*(2*a+b)*t
anh(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 212}

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - ((a + b)^2*Tanh[c + d*x]^3)/(3*
d) - (b*(2*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-(a+b)^2-(a+b)^2x^2-b(2a+b)x^4-b^2x^6+\frac{a^2+2ab+b^2}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\tanh^5(c+dx)}{5d}\\
&\quad -\frac{b^2\tanh^7(c+dx)}{7d}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= (a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}\\
&\quad -\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(83) = 166$.

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.29

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\operatorname{arctanh}(\tanh(c+dx))}{d} + \frac{2ab\operatorname{arctanh}(\tanh(c+dx))}{d} + \frac{b^2\operatorname{arctanh}(\tanh(c+dx))}{d} - \frac{a^2\tanh(c+dx)}{d} - \frac{2ab\tanh(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d} - \frac{a^2\tanh^3(c+dx)}{3d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{2ab\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^7(c+dx)}{7d}$$

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*Ar
cTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d -
 (b^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]
^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^5)/(5*d) - (
b^2*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

method	result
parallelrisch	$-\frac{15\tanh(dx+c)^7 b^2+42\tanh(dx+c)^5 ab+21\tanh(dx+c)^5 b^2+35\tanh(dx+c)^3 a^2+70\tanh(dx+c)^3 ab+35b^2\tanh(dx+c)}{105d}$
derivativedivides	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^5 ab}{5}-\frac{2\tanh(dx+c)^3 ab}{3}-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^5 b^2}{5}-\frac{\tanh(dx+c)^3 a^2}{3}-}{d}$
default	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^5 ab}{5}-\frac{2\tanh(dx+c)^3 ab}{3}-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^5 b^2}{5}-\frac{\tanh(dx+c)^3 a^2}{3}-}{d}$
parts	$\frac{b^2\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{a^2\left(-\frac{\tanh(dx+c}{3}\right.}{}$
risch	$a^2x+2abx+b^2x+\frac{4a^2e^{12dx+12c}+12ab\,e^{12dx+12c}+8b^2e^{12dx+12c}+20a^2e^{10dx+10c}+48ab\,e^{10dx+10c}+24b^2e^{10dx}}{}$

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(15*tanh(d*x+c)^7*b^2+42*tanh(d*x+c)^5*a*b+21*tanh(d*x+c)^5*b^2+35*t
anh(d*x+c)^3*a^2+70*tanh(d*x+c)^3*a*b+35*b^2*tanh(d*x+c)^3-105*a^2*d*x-210*
a*b*d*x-105*b^2*d*x+105*a^2*tanh(d*x+c)+210*a*b*tanh(d*x+c)+105*b^2*tanh(d*
x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(77) = 154$.

Time = 0.26 (sec) , antiderivative size = 796, normalized size of antiderivative = 9.59

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(105\,(a^2+2\,ab+b^2)dx+140\,a^2+322\,ab+176\,b^2)\cosh\left(dx+c\right)^7+7\,(105\,(a^2+2\,ab+b^2)dx+140\,a^2+32}{}$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x
 + c)^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cos
h(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2 + 161*a*b + 88*b^2)*sinh(d*x + c)^7
+ 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x +
c)^5 - 14*(3*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^2 + 40*a^2 + 71*a*b
+ 28*b^2)*sinh(d*x + c)^5 + 35*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 32
2*a*b + 176*b^2)*cosh(d*x + c)^3 + (105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 +
 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*(a^2 + 2*a*b +
 b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 - 14*(5*(70*a^2 +
161*a*b + 88*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x
+ c)^2 + 60*a^2 + 123*a*b + 84*b^2)*sinh(d*x + c)^3 + 7*(3*(105*(a^2 + 2*a*
b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 10*(105*(a^2
+ 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + 9*(105*
(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(
d*x + c)^2 + 35*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)
*cosh(d*x + c) - 14*((70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^6 + 5*(40*a^
2 + 71*a*b + 28*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 + 41*a*b + 28*b^2)*cosh(d*
x + c)^2 + 30*a^2 + 75*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*
x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^
5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d
*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(70) = 140$.

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.99

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{a^2\tanh(c+dx)}{d}+2abx-\frac{2ab\tanh^5(c+dx)}{5d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{2ab\tanh(c+dx)}{d}+b^2x-\frac{b^2\tanh^7}{7} \\ x\left(a+b\tanh^2(c)\right)^2\tanh^4(c) \end{cases}$$

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)**3/(3*d) - a**2*tanh(c + d*x)/d + 2*
a*b*x - 2*a*b*tanh(c + d*x)**5/(5*d) - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a*b
*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**7/(7*d) - b**2*tanh(c + d*x
)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)),
 (x*(a + b*tanh(c)**2)**2*tanh(c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(77) = 154$.

Time = 0.21 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.45

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{105}b^2\left(105x+\frac{105c}{d}-\frac{8\left(203e^{(-2dx-2c)}+609e^{(-4dx-4c)}+770e^{(-6dx-6c)}+770e^{(-8dx-8c)}+315e^{(-10}\right.}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+}\right.$$
$$+\frac{2}{15}ab\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$
$$+\frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/15*a*b*(15*x + 15*c/d - 2*(70
*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*d
*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^2*(3*x + 3
*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(77) = 154.

Time = 0.38 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.61

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{105\left(a^2+2\,ab+b^2\right)(dx+c)+\frac{4\left(105\,a^2e^{(12\,dx+12\,c)}+315\,abe^{(12\,dx+12\,c)}+210\,b^2e^{(12\,dx+12\,c)}+525\,a^2e^{(10\,dx+10\,c)}+1260\,abe^{(10\,dx+10\,c)}\right.}{}}{}$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(105*a^2*e^(12*d*x + 12*c) + 3
15*a*b*e^(12*d*x + 12*c) + 210*b^2*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x +
10*c) + 1260*a*b*e^(10*d*x + 10*c) + 630*b^2*e^(10*d*x + 10*c) + 1120*a^2*e
^(8*d*x + 8*c) + 2555*a*b*e^(8*d*x + 8*c) + 1540*b^2*e^(8*d*x + 8*c) + 1330
*a^2*e^(6*d*x + 6*c) + 3080*a*b*e^(6*d*x + 6*c) + 1540*b^2*e^(6*d*x + 6*c)
+ 945*a^2*e^(4*d*x + 4*c) + 2121*a*b*e^(4*d*x + 4*c) + 1218*b^2*e^(4*d*x +
4*c) + 385*a^2*e^(2*d*x + 2*c) + 812*a*b*e^(2*d*x + 2*c) + 406*b^2*e^(2*d*x
 + 2*c) + 70*a^2 + 161*a*b + 88*b^2)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right)-\frac{\tanh(c+d\,x)\,(a+b)^2}{d}-\frac{\tanh(c+d\,x)^5\,\left(b^2+2\,a\,b\right)}{5\,d}-\frac{b^2\tanh(c+d\,x)^7}{7\,d}-\frac{\tanh(c+d\,x)^3\,\left(a^2+2\,a\,b+b^2\right)}{3\,d}$$

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^5*(2*a
*b + b^2))/(5*d) - (b^2*tanh(c + d*x)^7)/(7*d) - (tanh(c + d*x)^3*(2*a*b +
a^2 + b^2))/(3*d)
```

3.145 $\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{(a+b)^2\tanh^2(c+dx)}{2d} - \frac{b(2a+b)\tanh^4(c+dx)}{4d} - \frac{b^2\tanh^6(c+dx)}{6d}$$

```
[Out] (a+b)^2*ln(cosh(d*x+c))/d-1/2*(a+b)^2*tanh(d*x+c)^2/d-1/4*b*(2*a+b)*tanh(d*
x+c)^4/d-1/6*b^2*tanh(d*x+c)^6/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = -\frac{b(2a+b)\tanh^4(c+dx)}{4d} - \frac{(a+b)^2\tanh^2(c+dx)}{2d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{b^2\tanh^6(c+dx)}{6d}$$

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - ((a + b)^2*Tanh[c + d*x]^2)/(2*d) - (b*(
2*a + b)*Tanh[c + d*x]^4)/(4*d) - (b^2*Tanh[c + d*x]^6)/(6*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-(a+b)^2-\frac{(a+b)^2}{-1+x}-b(2a+b)x-b^2x^2\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}\\
&\quad -\frac{b(2a+b)\tanh^4(c+dx)}{4d}-\frac{b^2\tanh^6(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{-12(a+b)^2\log(\cosh(c+dx))+6(a+b)^2\tanh^2(c+dx)+3b(2a+b)\tanh^4(c+dx)+2b^2\tanh^6(c+d}{12d}$$

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/12*(-12*(a + b)^2*Log[Cosh[c + d*x]] + 6*(a + b)^2*Tanh[c + d*x]^2 + 3*b
*(2*a + b)*Tanh[c + d*x]^4 + 2*b^2*Tanh[c + d*x]^6)/d
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4ab}{2}-\tanh(dx+c)^2ab-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^4b^2}{4}-\frac{a^2\tanh(dx+c)^2}{2}-\frac{b^2\tanh(dx+c)^2}{2}-\tan}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4ab}{2}-\tanh(dx+c)^2ab-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^4b^2}{4}-\frac{a^2\tanh(dx+c)^2}{2}-\frac{b^2\tanh(dx+c)^2}{2}-\tan}{d}$
parallelrisch	$-\frac{2\tanh(dx+c)^6b^2+6\tanh(dx+c)^4ab+3\tanh(dx+c)^4b^2+12a^2dx+24abdx+12b^2dx+6a^2\tanh(dx+c)^2+12\tanh(dx+c)}{12d}$
parts	$\frac{a^2\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(d}{2}\right.}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}+\frac{2\,e^{2dx+2c}(3a^2e^{8dx+8c}+12ab\,e^{8dx+8c}+9b^2e^{8dx+8c}+12a^2e^{6dx}}{}$

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*tanh(d*x+c)^4*a*b-tanh(d*x+c)^2*a*b-1/2*(a^2+2*a*b+b^2)*ln(tanh(d
*x+c)-1)-1/4*tanh(d*x+c)^4*b^2-1/2*a^2*tanh(d*x+c)^2-1/2*b^2*tanh(d*x+c)^2-
1/6*tanh(d*x+c)^6*b^2+1/2*(-a^2-2*a*b-b^2)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3441 vs. $2(70) = 140$.

Time = 0.30 (sec) , antiderivative size = 3441, normalized size of antiderivative = 45.28

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^12 + 6*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^10
+ 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 3*(a^2 + 2*a*b + b^2)*d*x
 - a^2 - 4*a*b - 3*b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*c
osh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 1
2*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 1
5*(a^2 + 2*a*b + b^2)*d*x + 90*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3
*b^2)*cosh(d*x + c)^2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^8 + 24*(99*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 30*(3*(a^2 + 2*a*b + b^2)*d*x - a^
2 - 4*a*b - 3*b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 -
24*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*(a^2 + 2*a*b + b^2)
*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^6 + 315*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^
2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2
)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - 9*a^2 - 24*a*b - 17*b^2)
*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 63*(3*(
a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^5 + 7*(15*(a^2
+ 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^3 + (15*(a^2 +
2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4
+ 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 420*(3*(a^2 + 2*a*b + b^
2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*
d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x
 + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^
2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^4 + 4*(165*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^9 + 180*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*
cosh(d*x + c)^7 + 42*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)
*cosh(d*x + c)^5 + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2
)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 +
2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^2 + 6*(33*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^10 + 45*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b
 - 3*b^2)*cosh(d*x + c)^8 + 14*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b
```

- 12*b^2)*cosh(d*x + c)^6 + 10*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - a^2 - 4*a*b - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 315*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 36*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 50*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 12*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^11 + 5*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^9 + 2*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^7 + 2*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^5 + (15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c

```
)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh
(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d
*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x +
 c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3
+ 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d
*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x +
c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 +
 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(65) = 130$.

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.24

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^4(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x \\ x\left(a+b\tanh^2(c)\right)^2\tanh^3(c) \end{cases}$$

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(
2*d) + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**4/(2*d
) - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b**2*
tanh(c + d*x)**6/(6*d) - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**
2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.38

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{3}b^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-}\right.}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+}\right.$$
$$+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c
) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*
x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a^2*(x + c/d + lo
g(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(70) = 140$.

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.51

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{3\left(a^2+2\,ab+b^2\right)(dx+c)-3\left(a^2+2\,ab+b^2\right)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\left(3\left(a^2+4\,ab+3\,b^2\right)e^{(10\,dx+10\,c)}+6\left(2\,a^2+6\,ab+\cdots\right.\right.}{\cdots}}{3\,d}$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(e^(2*d*x
+ 2*c) + 1) - 2*(3*(a^2 + 4*a*b + 3*b^2)*e^(10*d*x + 10*c) + 6*(2*a^2 + 6*a
*b + 3*b^2)*e^(8*d*x + 8*c) + 2*(9*a^2 + 24*a*b + 17*b^2)*e^(6*d*x + 6*c) +
 6*(2*a^2 + 6*a*b + 3*b^2)*e^(4*d*x + 4*c) + 3*(a^2 + 4*a*b + 3*b^2)*e^(2*d
*x + 2*c))/(e^(2*d*x + 2*c) + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right)-\frac{{\tanh(c+d\,x)}^4\,\left(b^2+2\,a\,b\right)}{4\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-\frac{b^2\,{\tanh(c+d\,x)}^6}{6\,d}-\frac{{\tanh(c+d\,x)}^2\,\left(a^2+2\,a\,b+b^2\right)}{2\,d}$$

```
[In] int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^4*(2*a*b + b^2))/(4*d) - (log(tanh(c
 + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^6)/(6*d) - (tanh(c
 + d*x)^2*(2*a*b + a^2 + b^2))/(2*d)
```

3.146 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=(a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh(
d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 212}

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - (b*(2*a + b)*Tanh[c + d*x]^3)/(
3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-(a+b)^2 - b(2a+b)x^2 - b^2x^4 + \frac{a^2+2ab+b^2}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d}\\
&\quad - \frac{b^2\tanh^5(c+dx)}{5d} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= (a+b)^2x - \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.17

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a^2\text{arctanh}(\tanh(c+dx))}{d}+\frac{2ab\text{arctanh}(\tanh(c+dx))}{d}+\frac{b^2\text{arctanh}(\tanh(c+dx))}{d}-\frac{a^2\tanh(c+dx)}{d}-\frac{2ab\tanh(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

method	result
parallelrisch	$-\frac{3\tanh(dx+c)^5 b^2+10\tanh(dx+c)^3 ab+5b^2\tanh(dx+c)^3-15a^2dx-30abdx-15b^2dx+15a^2\tanh(dx+c)+30ab\tanh(dx}{15d}$
derivativedivides	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^3ab}{3}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^2\tanh(dx+c)^3}{3}-b^2\tanh(dx+c)-a^2\tanh(dx+c)-}{d}$
default	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^3ab}{3}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^2\tanh(dx+c)^3}{3}-b^2\tanh(dx+c)-a^2\tanh(dx+c)-}{d}$
parts	$\frac{a^2\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx}{2}\right.}{d}$
risch	$a^2x+2abx+b^2x+\frac{2a^2e^{8dx+8c}+8ab\,e^{8dx+8c}+6b^2e^{8dx+8c}+8a^2e^{6dx+6c}+24ab\,e^{6dx+6c}+12b^2e^{6dx+6c}+12a^2e^{4dx}}{d\left(e^{2d}\right.}$

[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] -1/15*(3*tanh(d*x+c)^5*b^2+10*tanh(d*x+c)^3*a*b+5*b^2*tanh(d*x+c)^3-15*a^2*d*x-30*a*b*d*x-15*b^2*d*x+15*a^2*tanh(d*x+c)+30*a*b*tanh(d*x+c)+15*b^2*tanh(d*x+c))/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 483, normalized size of antiderivative = 7.67

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{(15\,(a^2+2\,ab+b^2)dx+15\,a^2+40\,ab+23\,b^2)\cosh\left(dx+c\right)^5+5\,(15\,(a^2+2\,ab+b^2)dx+15\,a^2+40\,ab+2}{}$$

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)
^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c
)*sinh(d*x + c)^4 - (15*a^2 + 40*a*b + 23*b^2)*sinh(d*x + c)^5 + 5*(15*(a^2
 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 - 5*(2*(15*
a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^2 + 9*a^2 + 16*a*b + 5*b^2)*sinh(d*x +
 c)^3 + 5*(2*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d
*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(
d*x + c))*sinh(d*x + c)^2 + 10*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*
b + 23*b^2)*cosh(d*x + c) - 5*((15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^4 +
 3*(9*a^2 + 16*a*b + 5*b^2)*cosh(d*x + c)^2 + 6*a^2 + 8*a*b + 10*b^2)*sinh(
d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh
(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
 10*d*cosh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} a^2x - \frac{a^2\tanh\left(c+dx\right)}{d} + 2abx - \frac{2ab\tanh^3\left(c+dx\right)}{3d} - \frac{2ab\tanh\left(c+dx\right)}{d} + b^2x - \frac{b^2\tanh^5\left(c+dx\right)}{5d} - \frac{b^2\tanh^3\left(c+dx\right)}{3d} - \frac{b^2\tanh\left(c+\right.}{d} \\ x\left(a+b\tanh^2\left(c\right)\right)^2\tanh^2\left(c\right) \end{cases}$$

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)/d + 2*a*b*x - 2*a*b*tanh(c + d*x)**3
/(3*d) - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**
2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(
c)**2)**2*tanh(c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.67

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{15}b^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$
$$+\frac{2}{3}ab\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+a^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

```
[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
 1))) + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(59) = 118$.

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.46

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{15\left(a^2+2ab+b^2\right)(dx+c)+\frac{2\left(15a^2e^{(8dx+8c)}+60abe^{(8dx+8c)}+45b^2e^{(8dx+8c)}+60a^2e^{(6dx+6c)}+180abe^{(6dx+6c)}+90b^2e^{(6dx+6c}\right.}{}}{15d}$$

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

```
[Out] 1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) + 2*(15*a^2*e^(8*d*x + 8*c) + 60*a*b
*e^(8*d*x + 8*c) + 45*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 180*a*
b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 220*a
*b*e^(4*d*x + 4*c) + 140*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 140
*a*b*e^(2*d*x + 2*c) + 70*b^2*e^(2*d*x + 2*c) + 15*a^2 + 40*a*b + 23*b^2)/(
e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2ab+b^2\right)-\frac{\tanh(c+dx)\,(a+b)^2}{d}-\frac{\tanh(c+dx)^3\,(b^2+2ab)}{3d}-\frac{b^2\tanh(c+dx)^5}{5d}$$

```
[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^3*(2*a
*b + b^2))/(3*d) - (b^2*tanh(c + d*x)^5)/(5*d)
```

3.147 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}$$

```
[Out] (a+b)^2*ln(cosh(d*x+c))/d-1/2*b*(a+b)*tanh(d*x+c)^2/d-1/4*(a+b*tanh(d*x+c)^
2)^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 455, 45}

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - (b*(a + b)*Tanh[c + d*x]^2)/(2*d) - (a +
 b*Tanh[c + d*x]^2)^2/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-b(a+b)+\frac{(a+b)^2}{1-x}-b(a+bx)\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{-4(a+b)^2\log(\cosh(c+dx))+2b(2a+b)\tanh^2(c+dx)+b^2\tanh^4(c+dx)}{4d}$$

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] -1/4*(-4*(a + b)^2*Log[Cosh[c + d*x]] + 2*b*(2*a + b)*Tanh[c + d*x]^2 + b^2
*Tanh[c + d*x]^4)/d
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 b^2}{4}-\tanh(dx+c)^2 ab-\frac{b^2\tanh(dx+c)^2}{2}-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a^2-2ab-b^2)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 b^2}{4}-\tanh(dx+c)^2 ab-\frac{b^2\tanh(dx+c)^2}{2}-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a^2-2ab-b^2)\ln(\tanh(dx+c)+1)}{2}}{d}$
parts	$\frac{a^2\ln(\cosh(dx+c))}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+2ab\left(-\frac{\tanh(dx+c)^2}{2}\right.$
parallelrisch	$-\frac{\tanh(dx+c)^4b^2+4a^2dx+8abdx+4b^2dx+4\tanh(dx+c)^2ab+2b^2\tanh(dx+c)^2+4\ln(1-\tanh(dx+c))a^2+8\ln(1-\tanh(}{4d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}+\frac{4b\,e^{2dx+2c}(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a+b\,e^{2dx+2c}+a+}{d(e^{2dx+2c}+1)^4}$

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*tanh(d*x+c)^4*b^2-tanh(d*x+c)^2*a*b-1/2*b^2*tanh(d*x+c)^2-1/2*(a^
2+2*a*b+b^2)*ln(tanh(d*x+c)-1)+1/2*(-a^2-2*a*b-b^2)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 1638, normalized size of antiderivative = 28.74

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*((a^
2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*sinh(d*x + c)^
6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b + b^2)*d
*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d
*x - 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 30*((a^2 + 2*a*b + b^2)*d*x - a*b -
b^2)*cosh(d*x + c)^2 - 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d
*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x + 4*((a^2 + 2*a*b + b^2)*d*x - a*b -
b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((
a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^4 + (a^2 + 2*a*b + b^2)*d
```

```
*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^2 - a*b -
b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
8 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 + 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(
d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^6 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8
*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b + b^2)*d*x - a*
b - b^2)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh
(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6
+ 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*
x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^
7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
 c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x - \frac{a^2\log\left(\tanh\left(c+dx\right)+1\right)}{d} + 2abx - \frac{2ab\log\left(\tanh\left(c+dx\right)+1\right)}{d} - \frac{ab\tanh^2\left(c+dx\right)}{d} + b^2x - \frac{b^2\log\left(\tanh\left(c+dx\right)+1\right)}{d} - \frac{b^2\tanh^4\left(\right.}{4d} \\ x\left(a+b\tanh^2\left(c\right)\right)^2\tanh\left(c\right) \end{cases}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d + 2*a*b*x - 2*a*b*log(tan
h(c + d*x) + 1)/d - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x
```

```
) + 1)/d - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne(d,
 0)), (x*(a + b*tanh(c)**2)**2*tanh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.26

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{a^2\log\left(\cosh\left(dx+c\right)\right)}{d}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d
*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c) + 1))) + a^2*log(cosh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{4\left((ab+b^2)e^{(6\,dx+6\,c)}+(2\,ab+b^2)e^{(4\,dx+4\,c)}+(ab+b^2\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c) +
 1) - 4*((a*b + b^2)*e^(6*d*x + 6*c) + (2*a*b + b^2)*e^(4*d*x + 4*c) + (a*b
 + b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right)-\frac{\tanh(c+d\,x)^2\left(b^2+2\,a\,b\right)}{2\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-\frac{b^2\tanh(c+d\,x)^4}{4\,d}$$

```
[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^2*(2*a*b + b^2))/(2*d) - (log(tanh(c
 + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^4)/(4*d)
```

3.148 $\int \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \left(a+b\tanh^2(c+dx)\right)^2 dx=(a+b)^2x-\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)^2*x-b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3742, 398, 212}

$$\int \left(a+b\tanh^2(c+dx)\right)^2 dx=-\frac{b(2a+b)\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-b(2a+b) - b^2x^2 + \frac{(a+b)^2}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= (a+b)^2x - \frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\tanh(c+dx)\left(\frac{3(a+b)^2\text{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(6a+b\left(3+\tanh^2(c+dx)\right)\right)\right)}{3d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (Tanh[c + d*x]*((3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
d*x]^2] - b*(6*a + b*(3 + Tanh[c + d*x]^2))))/(3*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{b^2\tanh(dx+c)^3-3a^2dx-6abdx-3b^2dx+6ab\tanh(dx+c)+3b^2\tanh(dx+c)}{3d}$
derivativedivides	$\frac{-\frac{b^2\tanh(dx+c)^3}{3}-2ab\tanh(dx+c)-b^2\tanh(dx+c)-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}+\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b^2\tanh(dx+c)^3}{3}-2ab\tanh(dx+c)-b^2\tanh(dx+c)-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}+\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)+1)}{2}}{d}$
risch	$a^2x+2abx+b^2x+\frac{4b(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+6\,e^{2dx+2c}a+3b\,e^{2dx+2c}+3a+2b)}{3d(e^{2dx+2c}+1)^3}$
parts	$a^2x+\frac{b^2\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2ab\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)\cdot}{2}\right.}{d}$

```
[In] int((a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b^2*tanh(d*x+c)^3-3*a^2*d*x-6*a*b*d*x-3*b^2*d*x+6*a*b*tanh(d*x+c)+3*b
^2*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.67

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{(3\,(a^2+2\,ab+b^2)dx+6\,ab+4\,b^2)\cosh{(dx+c)}^3+3\,(3\,(a^2+2\,ab+b^2)dx+6\,ab+4\,b^2)\cosh{(dx+c)}\sin}{3\,(d\cosh{(dx+}}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*(a^
2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - 2*(3*
a*b + 2*b^2)*sinh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2
)*cosh(d*x + c) - 6*((3*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/
(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = \begin{cases} a^2x+2abx-\frac{2ab\tanh{(c+dx)}}{d}+b^2x-\frac{b^2\tanh^3{(c+dx)}}{3d}-\frac{b^2\tanh{(c+dx)}}{d} & \text{for } d\neq 0 \\ x\left(a+b\tanh^2{(c)}\right)^2 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c
+ d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**2
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.65

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right) + 2ab\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+a^2x$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a*b*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\left(a^2+2ab+b^2\right)(dx+c)+\frac{4\left(3abe^{(4dx+4c)}+3b^2e^{(4dx+4c)}+6abe^{(2dx+2c)}+3b^2e^{(2dx+2c)}+3ab+2b^2\right)}{\left(e^{(2dx+2c)}+1\right)^3}}{3d}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(
4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + 2*b^
2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx=x\,\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\tanh(c+d\,x)^3}{3\,d}-\frac{b\tanh(c+d\,x)\,\left(2\,a+b\right)}{d}$$

```
[In] int((a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x)^3)/(3*d) - (b*tanh(c + d*x)*(2*a
 + b))/d
```

3.149 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a^2\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

```
[Out] (a+b)^2*ln(cosh(d*x+c))/d+a^2*ln(tanh(d*x+c))/d-1/2*b^2*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 457, 84}

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d + (a^2*Log[Tanh[c + d*x]])/d - (b^2*Tanh[c
 + d*x]^2)/(2*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-b^2-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a^2\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2((a+b)^2\log(\cosh(c+dx))+a^2\log(\tanh(c+dx)))-b^2\tanh^2(c+dx)}{2d}$$

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (2*((a + b)^2*Log[Cosh[c + d*x]] + a^2*Log[Tanh[c + d*x]]) - b^2*Tanh[c + d
*x]^2)/(2*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{-2(a+b)^2\ln(1-\tanh(dx+c))+2a^2\ln(\tanh(dx+c))-b^2\tanh(dx+c)^2-2dx(a+b)^2}{2d}$
derivativedivides	$-\frac{\frac{b^2\tanh(dx+c)^2}{2}+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)-a^2\ln(\tanh(dx+c))}{d}$
default	$-\frac{\frac{b^2\tanh(dx+c)^2}{2}+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)-a^2\ln(\tanh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{4abc}{d}-\frac{2cb^2}{d}-\frac{2a^2c}{d}+\frac{2b^2e^{2dx+2c}}{d(e^{2dx+2c}+1)^2}+\frac{2ab\ln(e^{2dx+2c}+1)}{d}+\frac{\ln(e^{2dx+2c}+1)b^2}{d}$

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*(a+b)^2*ln(1-tanh(d*x+c))+2*a^2*ln(tanh(d*x+c))-b^2*tanh(d*x+c)^2-2
*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 668, normalized size of antiderivative = 13.63

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{(a^2+2\,ab+b^2)dx\cosh\left(dx+c\right)^4+4\left(a^2+2\,ab+b^2\right)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(a^2+2\,ab+b^2\right)dx}{}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 +
 2*a*b + b^2)*d*x + 2*((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - b^2)*
sinh(d*x + c)^2 - ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x +
 c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh
(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh
(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(
d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c
))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c))*
```

```
sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*s
inh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^2\coth\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&\quad+\frac{2\,ab\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^2\log\left(\sinh\left(dx+c\right)\right)}{d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 2*a*b*log(e^(d*x + c) + e^(-d*x - c
))/d + a^2*log(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.88

$$\begin{aligned}&\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{a^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+(2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{2\,ab\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+}{e^{(2\,dx+2\,c}}}{2\,d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (2*a*b + b^2)*log(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*a*b - 2*b^2)/(e^(2*
d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 210, normalized size of antiderivative = 4.29

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-x\,(a+b)^2-\frac{2\,b^2}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} + \frac{\ln\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)\,\left(d\,(b^2+2\,a\,b)+a^2\,d\right)}{2\,d^2} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(b^2\,\sqrt{-d^2}-a^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4-4\,a^3\,b+2\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{a^4-4\,a^3\,b+2\,a^2\,b^2+4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

```
[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a + b)^2 - (2*b^2)/(d*(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1)) + (log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b + b^
2) + a^2*d))/(2*d^2) + (atan((exp(2*c)*exp(2*d*x)*(b^2*(-d^2)^(1/2) - a^2*(
-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^
2*b^2)^(1/2)))*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^2)^(1
/2)
```

3.150 $\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = (a+b)^2x - \frac{a^2\coth(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a+b)^2*x-a^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = -\frac{a^2\coth(c+dx)}{d} + x(a+b)^2 - \frac{b^2\tanh(c+dx)}{d}$$

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (a^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b^2+\frac{a^2}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= (a+b)^2x-\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = 2abx+\frac{b^2\text{arctanh}(\tanh(c+dx))}{d} - \frac{a^2\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] 2*a*b*x + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Coth[c + d*x]*Hypergeometri
c2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d - (b^2*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-\coth(dx+c)a^2-b^2\tanh(dx+c)+dx(a+b)^2}{d}$	36
derivativedivides	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
default	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
risch	$a^2x+2abx+b^2x-\frac{2(a^2e^{2dx+2c}-e^{2dx+2c}b^2+a^2+b^2)}{d(e^{2dx+2c}+1)(e^{2dx+2c}-1)}$	82

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] (-coth(d*x+c)*a^2-b^2*tanh(d*x+c)+d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.69

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{(a^2+b^2)\cosh(dx+c)^2-2\left((a^2+2\,ab+b^2)dx+a^2+b^2\right)\cosh(dx+c)\sinh(dx+c)+(a^2+b^2)\sinh(}{2\,d\cosh(dx+c)\sinh(dx+c)}$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] -1/2*((a^2 + b^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*a*b + b^2)*d*x + a^2 + b^2)
*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2)/(d*
cosh(d*x + c)*sinh(d*x + c))
```

Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\coth^2(c+dx)\,dx$$

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = b^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + a^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+2\,abx$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 2*a*b*x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)(dx+c)-\frac{2\,\left(a^2e^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+a^2+b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] ((a^2 + 2*a*b + b^2)*(d*x + c) - 2*(a^2*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*
c) + a^2 + b^2)/(e^(4*d*x + 4*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\,(a+b)^2-\frac{\frac{2\,(a^2+b^2)}{d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2-b^2)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$

```
[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
[Out] x*(a + b)^2 - ((2*(a^2 + b^2))/d + (2*exp(2*c + 2*d*x)*(a^2 - b^2))/d)/(exp
(4*c + 4*d*x) - 1)
```

3.151 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*a^2*coth(d*x+c)^2/d+(a+b)^2*ln(cosh(d*x+c))/d+a*(a+2*b)*ln(tanh(d*x+c)
)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth^2(c+dx)}{2d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(a^2*Coth[c + d*x]^2)/d + ((a + b)^2*Log[Cosh[c + d*x]])/d + (a*(a + 2
*b)*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^3(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^2}+\frac{a(a+2b)}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= -\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$
\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-a^2\coth^2(c+dx)+2(a+b)^2\log(\cosh(c+dx))+2a(a+2b)\log(\tanh(c+dx))}{2d}
$$

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (-(a^2*Coth[c + d*x]^2) + 2*(a + b)^2*Log[Cosh[c + d*x]] + 2*a*(a + 2*b)*Lo
g[Tanh[c + d*x]])/(2*d)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{-2(a+b)^2\ln(1-\tanh(dx+c))+2a(a+2b)\ln(\tanh(dx+c))-\coth(dx+c)^2a^2-2dx(a+b)^2}{2d}$
derivativedivides	$-\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)+\frac{a^2}{2\tanh(dx+c)^2}-a(a+2b)\ln(\tanh(dx+c))}{d}$
default	$-\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)+\frac{a^2}{2\tanh(dx+c)^2}-a(a+2b)\ln(\tanh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}-\frac{2a^2e^{2dx+2c}}{d(e^{2dx+2c}-1)^2}+\frac{a^2\ln(e^{2dx+2c}-1)}{d}+\frac{2a\ln(e^{2dx+2c}-1)}{d}$

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*(a+b)^2*ln(1-tanh(d*x+c))+2*a*(a+2*b)*ln(tanh(d*x+c))-coth(d*x+c)^2
*a^2-2*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 677, normalized size of antiderivative = 13.02

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{(a^2+2\,ab+b^2)dx\cosh\left(dx+c\right)^4+4\,(a^2+2\,ab+b^2)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a^2+2\,ab+b^2)d}{}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 +
 2*a*b + b^2)*d*x - 2*((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2)*
sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
- b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*
sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2
 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4
*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c
))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c))*
```

```
sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*s
inh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x
 + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\coth^3\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.58

$$\begin{aligned}\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&\quad+\frac{b^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{2\,ab\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*
x + c) + e^(-d*x - c))/d + 2*a*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.71

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{b^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+(a^2+2\,ab)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a}{e^{(2\,dx+2\,c)}}}{2\,d}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + (a^2 + 2*a*b)*log(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^2*(e^(2*d*x + 2*c) + e^(-2*d*x -
 2*c)) + 2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 - 4*a*b)/(e^(2*
d*x + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.06

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\ln\left(e^{4c+4dx}-1\right)\,\left(d\,(a^2+2\,b\,a)+b^2\,d\right)}{2\,d^2}-\frac{2\,a^2}{d\,\left(e^{2c+2dx}-1\right)} - \frac{2\,a^2}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-x\,(a+b)^2 - \frac{\operatorname{atan}\left(\frac{e^{2c}\,e^{2dx}\,\left(a^2\,\sqrt{-d^2}-b^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b+2\,a^2\,b^2-4\,a\,b^3+b^4}}\right)\,\sqrt{a^4+4\,a^3\,b+2\,a^2\,b^2-4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

```
[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b + a^2) + b^2*d))/(2*d^2) - (2*a^2)/(d*
(exp(2*c + 2*d*x) - 1)) - (2*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1)) - x*(a + b)^2 - (atan((exp(2*c)*exp(2*d*x)*(a^2*(-d^2)^(1/2) - b^2*(
-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^
2*b^2)^(1/2)))*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^2)^(1
/2)
```

3.152 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=(a+b)^2x-\frac{a(a+2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}$$

```
[Out] (a+b)^2*x-a*(a+2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+x(a+b)^2$$

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (a*(a + 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}+\frac{a(a+2b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{a(a+2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= (a+b)^2x-\frac{a(a+2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$
\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{\coth(c+dx)\left(a\left(3a+6b+a\coth^2(c+dx)\right)-3(a+b)^2\text{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)\sqrt{\tanh^2(c+dx)}\right)}{3d}
$$

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/3*(Coth[c + d*x]*(a*(3*a + 6*b + a*Coth[c + d*x]^2) - 3*(a + b)^2*ArcTan
h[Sqrt[Tanh[c + d*x]^2]]*Sqrt[Tanh[c + d*x]^2]))/d
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-\coth(dx+c)^3 a^2-3a(a+2b)\coth(dx+c)+3dx(a+b)^2}{3d}$	43
derivativedivides	$-\frac{\frac{a^2}{3\tanh(dx+c)^3}+\frac{a(a+2b)}{\tanh(dx+c)}+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)}{d}$	84
default	$-\frac{\frac{a^2}{3\tanh(dx+c)^3}+\frac{a(a+2b)}{\tanh(dx+c)}+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)}{d}$	84
risch	$a^2x+2abx+b^2x-\frac{4a\left(3a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}-3\,\mathrm{e}^{2dx+2c}a-6b\,\mathrm{e}^{2dx+2c}+2a+3b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3}$	91

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(-coth(d*x+c)^3*a^2-3*a*(a+2*b)*coth(d*x+c)+3*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(41) = 82.

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.58

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{2\left(2a^2+3ab\right)\cosh\left(dx+c\right)^3+6\left(2a^2+3ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-\left(3\left(a^2+2ab+b^2\right)dx+4a^2\right.}{3\left(d\sinh\right.}$$

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] -1/3*(2*(2*a^2 + 3*a*b)*cosh(d*x + c)^3 + 6*(2*a^2 + 3*a*b)*cosh(d*x + c)*s
inh(d*x + c)^2 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*sinh(d*x + c)^
3 - 6*a*b*cosh(d*x + c) + 3*(3*(a^2 + 2*a*b + b^2)*d*x - (3*(a^2 + 2*a*b +
b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 6*a*b)*sinh(d*x + c))/(
d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\coth^4\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.65

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right) + 2ab\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+b^2x$$

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2*a*b*(
x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\left(a^2+2ab+b^2\right)(dx+c)-\frac{4\left(3a^2e^{(4dx+4c)}+3abe^{(4dx+4c)}-3a^2e^{(2dx+2c)}-6abe^{(2dx+2c)}+2a^2+3ab\right)}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(
4*d*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) + 2*a^2 + 3*a*
b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.07

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\,(a+b)^2-\frac{\frac{4\,e^{2\,c+2\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}-\frac{4\,a\,b}{3\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1} - \frac{\frac{4\,\left(a^2+b\,a\right)}{3\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}-\frac{8\,a\,b\,e^{2\,c+2\,d\,x}}{3\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1} - \frac{4\,\left(a^2+b\,a\right)}{3\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$

```
[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(a + b)^2 - ((4*exp(2*c + 2*d*x)*(a*b + a^2))/(3*d) - (4*a*b)/(3*d))/(exp
(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((4*(a*b + a^2))/(3*d) + (4*exp(4
*c + 4*d*x)*(a*b + a^2))/(3*d) - (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(2*c
 + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (4*(a*b + a^2))/(3
*d*(exp(2*c + 2*d*x) - 1))
```

3.153 $\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{a(a+2b)\coth^2(c+dx)}{2d} - \frac{a^2\coth^4(c+dx)}{4d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*a*(a+2*b)*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d+(a+b)^2*ln(cosh(d*x+
c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{a^2\coth^4(c+dx)}{4d} - \frac{a(a+2b)\coth^2(c+dx)}{2d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(a*(a + 2*b)*Coth[c + d*x]^2)/d - (a^2*Coth[c + d*x]^4)/(4*d) + ((a +
b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^5(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^3}+\frac{a(a+2b)}{x^2}+\frac{(a+b)^2}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= -\frac{a(a+2b)\coth^2(c+dx)}{2d}-\frac{a^2\coth^4(c+dx)}{4d}\\
&\quad+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{2a(a+2b)\coth^2(c+dx)+a^2\coth^4(c+dx)-4(a+b)^2(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{4d}$$

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/4*(2*a*(a + 2*b)*Coth[c + d*x]^2 + a^2*Coth[c + d*x]^4 - 4*(a + b)^2*(Lo
g[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-4(a+b)^2\ln(1-\tanh(dx+c))+4(a+b)^2\ln(\tanh(dx+c))-\coth(dx+c)^4a^2-2a\coth(dx+c)^2(a+2b)-4dx(a+b)^2}{4d}$
derivativedivides	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{4\tanh(dx+c)^4}+\frac{a(a+2b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2}{d}$
default	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{4\tanh(dx+c)^4}+\frac{a(a+2b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2c\,b^2}{d}-\frac{4a\,e^{2dx+2c}(a\,e^{4dx+4c}+b\,e^{4dx+4c}-e^{2dx+2c}a-2b\,e^{2dx+2c}+a+}{d(e^{2dx+2c}-1)^4}$

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-4*(a+b)^2*ln(1-tanh(d*x+c))+4*(a+b)^2*ln(tanh(d*x+c))-coth(d*x+c)^4*a
^2-2*a*coth(d*x+c)^2*(a+2*b)-4*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 1649, normalized size of antiderivative = 22.90

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 - 4*((a^
2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2
```

```
)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^
6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - 3*((a^2 + 2*a*b + b^2)*d
*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d
*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 30*((a^2 + 2*a*b + b^2)*d*x - a^2 -
a*b)*cosh(d*x + c)^2 - 2*a^2 - 4*a*b)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^5 - 10*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d
*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c))*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 -
a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 15*((
a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 - (a^2 + 2*a*b + b^2)*d
*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 +
a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
8 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 - 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(
d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 10*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^6 - 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8
*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 3*((a^2 + 2*a*b + b^2)*d*x - a^
2 - a*b)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh
(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6
+ 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*
x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*
sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^
7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
 c) + d)
```

Sympy [F]

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2\coth^5{(c+dx)}\,dx$$

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.28

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right) + 2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) + \frac{b^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*a*b*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*x +
c) - e^(-d*x - c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{4\left((a^2+ab)e^{(6\,dx+6\,c)}-(a^2+2\,ab)e^{(4\,dx+4\,c)}+(a^2+\cdots\right.}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2*
c) - 1)) + 4*((a^2 + a*b)*e^(6*d*x + 6*c) - (a^2 + 2*a*b)*e^(4*d*x + 4*c) +
 (a^2 + a*b)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.74

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-x\,(a+b)^2$$
$$-\frac{4\,(2\,a^2+b\,a)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{8\,a^2}{d\,\left(3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1\right)}$$
$$-\frac{4\,a^2}{d\,\left(6\,e^{4c+4dx}-4\,e^{2c+2dx}-4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}-\frac{4\,(a^2+b\,a)}{d\,\left(e^{2c+2dx}-1\right)}$$

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - x*(a + b)^2 - (4*(a*
b + 2*a^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(d*(3
*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a^2)/(
d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c +
 8*d*x) + 1)) - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1))
```

3.154 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=(a+b)^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*x-(a+b)^2*coth(d*x+c)/d-1/3*a*(a+2*b)*coth(d*x+c)^3/d-1/5*a^2*coth(
d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\coth^5(c+dx)}{5d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth(c+dx)}{d}+x(a+b)^2$$

```
[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Coth[c + d*x])/d - (a*(a + 2*b)*Coth[c + d*x]^3)/(
3*d) - (a^2*Coth[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{a(a+2b)}{x^4}+\frac{(a+b)^2}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d} \\
&\quad -\frac{a^2\coth^5(c+dx)}{5d}-\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= (a+b)^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= -\frac{a^2\coth^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(c+dx)\right)}{5d} \\
&\quad -\frac{2ab\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d} \\
&\quad -\frac{b^2\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}
\end{aligned}$$

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/5*(a^2*Coth[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2]
)/d - (2*a*b*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]
^2])/(3*d) - (b^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*
x]^2])/d
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{-3\coth(dx+c)^5a^2-5a\coth(dx+c)^3(a+2b)-15(a+b)^2\coth(dx+c)+15dx(a+b)^2}{15d}$
derivativedivides	$-\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{\tanh(dx+c)}+\frac{a^2}{5\tanh(dx+c)^5}+\frac{a(a+2b)}{3\tanh(dx+}}{d}$
default	$-\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{\tanh(dx+c)}+\frac{a^2}{5\tanh(dx+c)^5}+\frac{a(a+2b)}{3\tanh(dx+}}{d}$
risch	$a^2x+2abx+b^2x-\frac{2(45a^2e^{8dx+8c}+60ab\,e^{8dx+8c}+15b^2e^{8dx+8c}-90a^2e^{6dx+6c}-180ab\,e^{6dx+6c}-60b^2e^{6dx+6c}+}{}$

```
[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-3*coth(d*x+c)^5*a^2-5*a*coth(d*x+c)^3*(a+2*b)-15*(a+b)^2*coth(d*x+c)
+15*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 473, normalized size of antiderivative = 7.51

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(23\,a^2+40\,ab+15\,b^2)\cosh(dx+c)^5+5\,(23\,a^2+40\,ab+15\,b^2)\cosh(dx+c)\sinh(dx+c)^4-(15\,(a^2}{}$$

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 40*a*b + 15
*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2
+ 40*a*b + 15*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x +
c)^3 + 5*(15*(a^2 + 2*a*b + b^2)*d*x - 2*(15*(a^2 + 2*a*b + b^2)*d*x + 23*a
^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 23*a^2 + 40*a*b + 15*b^2)*sinh(d*x
+ c)^3 + 5*(2*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*
b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 4*a*b + 3*b^2)*cosh
```

```
(d*x + c) - 5*((15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh
(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*d*x - 3*(15*(a^2 + 2*a*b + b^2)*d*x +
23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 46*a^2 + 80*a*b + 30*b^2)*sinh(
d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3
+ 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int\left(a+b\tanh^2{(c+dx)}\right)^2\coth^6{(c+dx)}\,dx$$

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.67

$$\begin{aligned}&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{1}{15}a^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1)}\right)\\&\quad+\frac{2}{3}ab\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\\&\quad+b^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)\end{aligned}$$

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) -
 1))) + b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(59) = 118$.

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.46

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{15\,(a^2+2\,ab+b^2)(dx+c) - \frac{2\,(45\,a^2e^{(8\,dx+8\,c)}+60\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}-180\,abe^{(6\,dx+6\,c)}-60\,b^2e^{(6\,dx+6\,c)}}{}}{15\,d}$$

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) - 2*(45*a^2*e^(8*d*x + 8*c) + 60*a*b
*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) - 180*a*
b*e^(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 220*
a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) - 140
*a*b*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 23*a^2 + 40*a*b + 15*b^2)/(
e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 529, normalized size of antiderivative = 8.40

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}$$
$$+\frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,e^{6\,c+6\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(5\,a^2+4\,a\,b+3\,b^2)}{5\,d}}{6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}+x\,(a+b)^2$$
$$-\frac{\frac{2\,(3\,a^2+4\,a\,b+b^2)}{5\,d}-\frac{8\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{8\,e^{6\,c+6\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(5\,a^2+4\,a\,b+3\,b^2)}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}$$
$$-\frac{\frac{2\,(5\,a^2+4\,a\,b+3\,b^2)}{15\,d}-\frac{4\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1}-\frac{2\,(3\,a^2+4\,a\,b+b^2)}{5\,d\,(e^{2\,c+2\,d\,x}-1)}$$

```
[In] int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((2*(2*a*b + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d)
)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) + ((2*(2*a*b + b^2))/(5*d) +
(6*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) - (2*exp(6*c + 6*d*x)*(4*a*b + 3*a
^2 + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 5*a^2 + 3*b^2))/(5*d))/(6*e
xp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1) + x*(a + b)^2 - ((2*(4*a*b + 3*a^2 + b^2))/(5*d) - (8*exp(2*c + 2*d*
x)*(2*a*b + b^2))/(5*d) - (8*exp(6*c + 6*d*x)*(2*a*b + b^2))/(5*d) + (2*exp
(8*c + 8*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d) + (4*exp(4*c + 4*d*x)*(4*a*b + 5
*a^2 + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*
```

```
c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(4*a*b + 5*
a^2 + 3*b^2))/(15*d) - (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) + (2*exp(4*
c + 4*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(4*a*b + 3*a^2 + b^2))/(5*d*(exp(2*c +
2*d*x) - 1))
```

3.155 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a+b)^2\coth^2(c+dx)}{2d} - \frac{a(a+2b)\coth^4(c+dx)}{4d} - \frac{a^2\coth^6(c+dx)}{6d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*(a+b)^2*coth(d*x+c)^2/d-1/4*a*(a+2*b)*coth(d*x+c)^4/d-1/6*a^2*coth(d*x
+c)^6/d+(a+b)^2*ln(cosh(d*x+c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth^6(c+dx)}{6d} - \frac{a(a+2b)\coth^4(c+dx)}{4d} - \frac{(a+b)^2\coth^2(c+dx)}{2d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

```
[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)^2*Coth[c + d*x]^2)/d - (a*(a + 2*b)*Coth[c + d*x]^4)/(4*d) -
(a^2*Coth[c + d*x]^6)/(6*d) + ((a + b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2
*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^7(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^4}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^4}+\frac{a(a+2b)}{x^3}+\frac{(a+b)^2}{x^2}+\frac{(a+b)^2}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= -\frac{(a+b)^2\coth^2(c+dx)}{2d}-\frac{a(a+2b)\coth^4(c+dx)}{4d}-\frac{a^2\coth^6(c+dx)}{6d} \\
&\quad +\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{6(a+b)^2\coth^2(c+dx)+3a(a+2b)\coth^4(c+dx)+2a^2\coth^6(c+dx)-12(a+b)^2(\log(\cosh(c+dx)}{12d}$$

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] -1/12*(6*(a + b)^2*Coth[c + d*x]^2 + 3*a*(a + 2*b)*Coth[c + d*x]^4 + 2*a^2*
Coth[c + d*x]^6 - 12*(a + b)^2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-12(a+b)^2\ln(1-\tanh(dx+c))+12(a+b)^2\ln(\tanh(dx+c))-2\coth(dx+c)^6a^2-3a\coth(dx+c)^4(a+2b)-6\coth(dx+c)^2(a}{12d}$
derivativedivides	$-\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{2\tanh(dx+c)^2}+\left(-a^2-2ab-b^2\right)\ln(\tanh(dx}{d}$
default	$-\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)-1)+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{2\tanh(dx+c)^2}+\left(-a^2-2ab-b^2\right)\ln(\tanh(dx}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2c\,b^2}{d}-\frac{2\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+12ab\,e^{8dx+8c}+3b^2e^{8dx+8c}-18a^2e^{6dx}\right.}{}$

```
[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/12*(-12*(a+b)^2*ln(1-tanh(d*x+c))+12*(a+b)^2*ln(tanh(d*x+c))-2*coth(d*x+c
)^6*a^2-3*a*coth(d*x+c)^4*(a+2*b)-6*coth(d*x+c)^2*(a+b)^2-12*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3454 vs. 2(86) = 172.

Time = 0.31 (sec) , antiderivative size = 3454, normalized size of antiderivative = 37.54

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^12 - 6*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^10
```

+ 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x + 3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x - 90*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^8 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 30*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 315*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^4 - 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 17*a^2 + 24*a*b + 9*b^2)*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 63*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^5 + 7*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^3 - (15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 - 420*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^4 + 4*(165*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^9 - 180*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^7 + 42*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^5 - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^2 + 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^10 - 45*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^8 + 14*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^6 - 10*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^4 - 3*(a^2 + 2*a*b + b^2)*d*x + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 - 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^10 + 20*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(a^2 + 2*a*b +

```
b^2)*cosh(d*x + c)^6 - 315*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 - 5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^6 + 2
4*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 63*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^5 + 35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 5*(a^2 + 2*a*b + b^2)*co
sh(d*x + c))*sinh(d*x + c)^5 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*
(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 84*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 20*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^9 - 36*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^5 - 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 45*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^8 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 50*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a
^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 12*((a^2 + 2*a*b +
b^2)*cosh(d*x + c)^11 - 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 10*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^7 - 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^
2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^11 - 5*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^
2 - 4*a*b - b^2)*cosh(d*x + c)^9 + 2*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 -
 24*a*b - 8*b^2)*cosh(d*x + c)^7 - 2*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 -
 24*a*b - 9*b^2)*cosh(d*x + c)^5 + (15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 2
4*a*b - 8*b^2)*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x +
 c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^
4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 -
30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x +
 c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x +
 c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c
)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh
(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d
*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x +
 c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3
+ 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d
*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x +
c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 -
 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*co
sh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(86) = 172$.

Time = 0.20 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.24

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{3}a^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+3\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-}}\right.$$
$$+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6}\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(}}\right.$$
$$+b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 2*a*b*(x + c/d + log(e^(-d*x - c) + 1)/d +
 log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d
*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c)
 - e^(-8*d*x - 8*c) - 1))) + b^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e
^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x
- 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(86) = 172.

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.09

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{3\,(a^2+2\,ab+b^2)(dx+c)-3\,(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,(3\,(3\,a^2+4\,ab+b^2)e^{(10\,dx+10\,c)}-6\,(3\,a^2+6\,a}{}}{3\,d}$$

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(abs(e^(2*
d*x + 2*c) - 1)) + 2*(3*(3*a^2 + 4*a*b + b^2)*e^(10*d*x + 10*c) - 6*(3*a^2
+ 6*a*b + 2*b^2)*e^(8*d*x + 8*c) + 2*(17*a^2 + 24*a*b + 9*b^2)*e^(6*d*x + 6
*c) - 6*(3*a^2 + 6*a*b + 2*b^2)*e^(4*d*x + 4*c) + 3*(3*a^2 + 4*a*b + b^2)*e
^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.93

$$\begin{aligned}\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-\frac{2\,\left(3\,a^2+4\,a\,b+b^2\right)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}\\ &\quad-\frac{32\,a^2}{3\,d\,\left(15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}\\ &\quad-x\,(a+b)^2-\frac{2\,\left(9\,a^2+8\,a\,b+b^2\right)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,\left(13\,a^2+6\,b\,a\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}\\ &\quad-\frac{4\,\left(11\,a^2+2\,b\,a\right)}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}\\ &\quad-\frac{32\,a^2}{d\,\left(5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1\right)}\end{aligned}$$

```
[In] int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - (2*(4*a*b + 3*a^2 +
b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (32*a^2)/(3*d*(15*exp(4*c + 4*d*x) - 6*e
xp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c +
10*d*x) + exp(12*c + 12*d*x) + 1)) - x*(a + b)^2 - (2*(8*a*b + 9*a^2 + b^2)
)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(6*a*b + 13*a^2))/(3
*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(
```

```
2*a*b + 11*a^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^2)/(d*(5*exp(2*c + 2*d*x) - 10*exp(
4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x
) - 1))
```

3.156 $\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^3\tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+b)\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d}$$

```
[Out] (a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^3*tanh(d*x+c)^3/d-1/5*b*(3*a^2+3*
a*b+b^2)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+b)*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x+c
)^9/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 212}

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+b)\tanh^7(c+dx)}{7d} - \frac{(a+b)^3\tanh^3(c+dx)}{3d} - \frac{(a+b)^3\tanh(c+dx)}{d} + x(a+b)^3 - \frac{b^3\tanh^9(c+dx)}{9d}$$

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^3*Tanh[c + d*x]^3)/(3*
d) - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + b)*Tanh[
c + d*x]^7)/(7*d) - (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3 - (a+b)^3x^2 - b(3a^2+3ab+b^2)\,x^4 - b^2(3a+b)x^6 - b^3x^8 + \frac{a^3+3a^2b+3ab^2+b^3}{1-x^2}\right.\right.}{d}$$

$$= -\frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^3\tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+b)\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d} + \frac{(a+b)^3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^3\tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+b)\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d}$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\tanh(c+dx)\left(-315(a+b)^3 - 105(a+b)^3\tanh^2(c+dx) - 63b(3a^2+3ab+b^2)\tanh^4(c+dx) - 45b^2(3a\right.}{315d}$$

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-315*(a + b)^3 - 105*(a + b)^3*Tanh[c + d*x]^2 - 63*b*(3*a^
2 + 3*a*b + b^2)*Tanh[c + d*x]^4 - 45*b^2*(3*a + b)*Tanh[c + d*x]^6 - 35*b^
3*Tanh[c + d*x]^8 + (315*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tan
h[c + d*x]^2]))/(315*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(106) = 212$.

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.91

method	result
parallelrisch	$-\frac{35\tanh(dx+c)^9 b^3+135a\,b^2\tanh(dx+c)^7+45\tanh(dx+c)^7 b^3+189a^2 b\tanh(dx+c)^5+189\tanh(dx+c)^5 a\,b^2+63b^3\tanh(d\ldots}{}$
derivativedivides	$\frac{-3a^2 b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-\frac{3a\,b^2\tanh(dx+c)^7}{7}-\frac{3a^2 b\tanh(dx+c)^5}{5}-\frac{3\tanh(dx+c)^5 a\,b^2}{5}-\tanh(dx+c)^3 a^2 b-a\,b^2\tan\ldots}{}$
default	$\frac{-3a^2 b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-\frac{3a\,b^2\tanh(dx+c)^7}{7}-\frac{3a^2 b\tanh(dx+c)^5}{5}-\frac{3\tanh(dx+c)^5 a\,b^2}{5}-\tanh(dx+c)^3 a^2 b-a\,b^2\tan\ldots}{}$
parts	$\frac{b^3\left(-\frac{\tanh(dx+c)^9}{9}-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{a^3(\ldots}{}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x+\frac{\frac{352a\,b^2}{35}+10b^3 e^{16dx+16c}+\frac{8a^3}{3}+108a^2 b\,e^{14dx+14c}+24a\,b^2 e^{16dx+16c}+18a^2 b\,e^{16d\ldots}}{}$

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(35*tanh(d*x+c)^9*b^3+135*a*b^2*tanh(d*x+c)^7+45*tanh(d*x+c)^7*b^3+1
89*a^2*b*tanh(d*x+c)^5+189*tanh(d*x+c)^5*a*b^2+63*b^3*tanh(d*x+c)^5+105*a^3
*tanh(d*x+c)^3+315*tanh(d*x+c)^3*a^2*b+315*a*b^2*tanh(d*x+c)^3+105*b^3*tanh
(d*x+c)^3-315*a^3*d*x-945*a^2*b*d*x-945*a*b^2*d*x-315*b^3*d*x+315*a^3*tanh(
d*x+c)+945*a^2*b*tanh(d*x+c)+945*a*b^2*tanh(d*x+c)+315*b^3*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(106) = 212$.

Time = 0.27 (sec) , antiderivative size = 1563, normalized size of antiderivative = 13.71

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/315*((420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2
+ 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x
 + c)^8 - (420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*sinh(d*x + c)^9 + 9
*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^7 - 9*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^
3 + 4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^7 + 21*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(420*a^3 + 1449*a^2*b +
1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^6 + 36*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 9*(14*(420*a^3 + 14
49*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^4 + 700*a^3 + 2016*a^2*b + 2
136*a*b^2 + 852*b^3 + 21*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*cosh(d
```

```
*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563
*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 35*(420*a
^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*x)*cosh(d*x + c)^3 + 20*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 +
315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 8
4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 3*(28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)
*cosh(d*x + c)^4 + 2660*a^3 + 8232*a^2*b + 8232*a*b^2 + 1764*b^3 + 120*(175
*a^3 + 504*a^2*b + 534*a*b^2 + 213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
9*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 21*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 40*(4
20*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^3 + 28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^
3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2
 + 126*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 9*((420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3)*cosh(d*x + c)^8 + 7*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*c
osh(d*x + c)^6 + 20*(175*a^3 + 504*a^2*b + 534*a*b^2 + 213*b^3)*cosh(d*x +
c)^4 + 420*a^3 + 1386*a^2*b + 1176*a*b^2 + 882*b^3 + 28*(95*a^3 + 294*a^2*b
 + 294*a*b^2 + 63*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 +
 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14
*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x +
c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5
 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(
d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(99) = 198$.

Time = 0.30 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.28

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \begin{cases} a^3x - \frac{a^3\tanh^3(c+dx)}{3d} - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{3a^2b\tanh^5(c+dx)}{5d} - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3}{} \\ x\left(a+b\tanh^2(c)\right)^3\tanh^4(c) \end{cases}$$

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)**3/(3*d) - a**3*tanh(c + d*x)/d + 3*
a**2*b*x - 3*a**2*b*tanh(c + d*x)**5/(5*d) - a**2*b*tanh(c + d*x)**3/d - 3*
```

```
a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**7/(7*d) - 3*a
*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c
+ d*x)/d + b**3*x - b**3*tanh(c + d*x)**9/(9*d) - b**3*tanh(c + d*x)**7/(7*
d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(
c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(106) = 212$.

Time = 0.21 (sec) , antiderivative size = 583, normalized size of antiderivative = 5.11

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{315}b^3\left(315\,x + \frac{315\,c}{d} - \frac{2\left(3492\,e^{(-2\,dx-2\,c)} + 13968\,e^{(-4\,dx-4\,c)} + 26292\,e^{(-6\,dx-6\,c)} + 39438\,e^{(-8\,dx-8\,c)} + 315\right.}{d(9\,e^{(-2\,dx-2\,c)} + 36\,e^{(-4\,dx-4\,c)} + 84\,e^{(-6\,dx-6\,c)} + 126\,e^{(-8\,dx-8\,c)} + 126\,e^{(-10\,d}}\right.$$
$$+ \frac{1}{35}ab^2\left(105\,x + \frac{105\,c}{d} - \frac{8\left(203\,e^{(-2\,dx-2\,c)} + 609\,e^{(-4\,dx-4\,c)} + 770\,e^{(-6\,dx-6\,c)} + 770\,e^{(-8\,dx-8\,c)} + 315\,e^{(-}\right.}{d(7\,e^{(-2\,dx-2\,c)} + 21\,e^{(-4\,dx-4\,c)} + 35\,e^{(-6\,dx-6\,c)} + 35\,e^{(-8\,dx-8\,c)} + 21\,e^{(-10\,dx-10\,c)}}\right.$$
$$+ \frac{1}{5}a^2b\left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right)$$
$$+ \frac{1}{3}a^3\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/315*b^3*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4
*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x -
10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*
x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x -
12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
 1))) + 1/35*a*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d
*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x -
10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a^2*b*(15*x + 15*c
/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) +
45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10
*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^
3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(106) = 212$.

Time = 0.51 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.68

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{315\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)+\frac{2\,(630\,a^3e^{(16\,dx+16\,c)}+2835\,a^2be^{(16\,dx+16\,c)}+3780\,ab^2e^{(16\,dx+16\,c)}+1575\,b^3e^{(16\,dx+16\,c)}+\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/315*(315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(630*a^3*e^(16*d*x
 + 16*c) + 2835*a^2*b*e^(16*d*x + 16*c) + 3780*a*b^2*e^(16*d*x + 16*c) + 15
75*b^3*e^(16*d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) + 17010*a^2*b*e^(14*d
*x + 14*c) + 18900*a*b^2*e^(14*d*x + 14*c) + 6300*b^3*e^(14*d*x + 14*c) + 1
3650*a^3*e^(12*d*x + 12*c) + 48510*a^2*b*e^(12*d*x + 12*c) + 54180*a*b^2*e^
(12*d*x + 12*c) + 21000*b^3*e^(12*d*x + 12*c) + 24570*a^3*e^(10*d*x + 10*c)
 + 85050*a^2*b*e^(10*d*x + 10*c) + 94500*a*b^2*e^(10*d*x + 10*c) + 31500*b^
3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) + 97524*a^2*b*e^(8*d*x + 8*
c) + 105084*a*b^2*e^(8*d*x + 8*c) + 39438*b^3*e^(8*d*x + 8*c) + 21630*a^3*e
^(6*d*x + 6*c) + 73206*a^2*b*e^(6*d*x + 6*c) + 78876*a*b^2*e^(6*d*x + 6*c)
+ 26292*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) + 35154*a^2*b*e^(4*
d*x + 4*c) + 38124*a*b^2*e^(4*d*x + 4*c) + 13968*b^3*e^(4*d*x + 4*c) + 3150
*a^3*e^(2*d*x + 2*c) + 10206*a^2*b*e^(2*d*x + 2*c) + 10476*a*b^2*e^(2*d*x +
 2*c) + 3492*b^3*e^(2*d*x + 2*c) + 420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*
b^3)/(e^(2*d*x + 2*c) + 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right) - \frac{\tanh(c+d\,x)\,\left(a+b\right)^3}{d} - \frac{\tanh(c+d\,x)^5\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d} - \frac{\tanh(c+d\,x)^7\,\left(b^3+3\,a\,b^2\right)}{7\,d} - \frac{b^3\tanh(c+d\,x)^9}{9\,d} - \frac{\tanh(c+d\,x)^3\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{3\,d}$$

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c +
 d*x)^5*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) - (tanh(c + d*x)^7*(3*a*b^2 + b^3)
)/(7*d) - (b^3*tanh(c + d*x)^9)/(9*d) - (tanh(c + d*x)^3*(3*a*b^2 + 3*a^2*b
 + a^3 + b^3))/(3*d)
```

3.157 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{(a+b)^3\tanh^2(c+dx)}{2d} - \frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d} - \frac{b^2(3a+b)\tanh^6(c+dx)}{6d} - \frac{b^3\tanh^8(c+dx)}{8d}$$

```
[Out] (a+b)^3*ln(cosh(d*x+c))/d-1/2*(a+b)^3*tanh(d*x+c)^2/d-1/4*b*(3*a^2+3*a*b+b^
2)*tanh(d*x+c)^4/d-1/6*b^2*(3*a+b)*tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 78}

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d} - \frac{b^2(3a+b)\tanh^6(c+dx)}{6d} - \frac{(a+b)^3\tanh^2(c+dx)}{2d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{b^3\tanh^8(c+dx)}{8d}$$

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - ((a + b)^3*Tanh[c + d*x]^2)/(2*d) - (b*(
3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/(4*d) - (b^2*(3*a + b)*Tanh[c + d*x]^
6)/(6*d) - (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \frac{x^3(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x(a+bx)^3}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\operatorname{Subst}\left(\int \left(-(a+b)^3-\frac{(a+b)^3}{-1+x}-b(3a^2+3ab+b^2)\,x-b^2(3a+b)x^2-b^3x^3\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\tanh^2(c+dx)}{2d}\\
&\quad-\frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d}-\frac{b^2(3a+b)\tanh^6(c+dx)}{6d}-\frac{b^3\tanh^8(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2(a+b)^3\log(\cosh(c+dx))-(a+b)^3\tanh^2(c+dx)-\frac{1}{2}b(3a^2+3ab+b^2)\tanh^4(c+dx)-\frac{1}{3}b^2(3a+b)\,\text{ta}}{2d}$$

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] - (a + b)^3*Tanh[c + d*x]^2 - (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3 - (b^3*T
anh[c + d*x]^8)/4)/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(99) = 198.

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^6 a\,b^2}{2}-\frac{3\tanh(dx+c)^4 a^2 b}{4}-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6 b^3}{6}-\frac{b^3\tanh(d}{4}}{}$
default	$\frac{-\frac{\tanh(dx+c)^6 a\,b^2}{2}-\frac{3\tanh(dx+c)^4 a^2 b}{4}-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6 b^3}{6}-\frac{b^3\tanh(d}{4}}{}$
parts	$\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^8}{8}-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+}{2}\right.}{d}$
parallelrisch	$-\frac{3\tanh(dx+c)^8 b^3+12\tanh(dx+c)^6 a\,b^2+4\tanh(dx+c)^6 b^3+18\tanh(dx+c)^4 a^2 b+18\tanh(dx+c)^4 a\,b^2+6b^3\tanh(dx+c)}{}$
risch	$-a^3x-3b\,a^2x-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6bc\,a^2}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}+\frac{2\,\mathrm{e}^{2dx+2c}\left(27a\,b^2+3a^3+18a^2b\,\mathrm{e}^{12c}\right.}{}$

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*tanh(d*x+c)^6*a*b^2-3/4*tanh(d*x+c)^4*a^2*b-3/4*tanh(d*x+c)^4*a*b
^2-3/2*tanh(d*x+c)^2*a^2*b-3/2*a*b^2*tanh(d*x+c)^2-1/6*tanh(d*x+c)^6*b^3-1/
4*b^3*tanh(d*x+c)^4-1/2*tanh(d*x+c)^2*a^3-1/2*b^3*tanh(d*x+c)^2-1/8*tanh(d*
x+c)^8*b^3+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)*ln(tanh(d*x+c)+1)-1/2*(a^3+3*a^2*
b+3*a*b^2+b^3)*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7502 vs. 2(99) = 198.

Time = 0.34 (sec) , antiderivative size = 7502, normalized size of antiderivative = 70.11

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(94) = 188.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.61

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^4(c+dx)}{4d} - \frac{3a^2b\tanh^2(c+dx)}{2d} - \\ x\left(a+b\tanh^2(c)\right)^3\tanh^3(c) \end{cases}$$

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(
2*d) + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d
*x)**4/(4*d) - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*log(
tanh(c + d*x) + 1)/d - a*b**2*tanh(c + d*x)**6/(2*d) - 3*a*b**2*tanh(c + d*
x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c +
d*x) + 1)/d - b**3*tanh(c + d*x)**8/(8*d) - b**3*tanh(c + d*x)**6/(6*d) - b
**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a
+ b*tanh(c)**2)**3*tanh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.05

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=ab^2\left(3\,x+\frac{3\,c}{d}+\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\left(9\,e^{(-2\,dx-2\,c)}+18\,e^{(-4\,dx-4\,c)}+34\,e^{(-6\,dx-6\,c)}+18\,e^{(-}\right.}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+}\right.$$
$$+\frac{1}{3}\,b^3\left(3\,x+\frac{3\,c}{d}+\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{8\left(3\,e^{(-2\,dx-2\,c)}+9\,e^{(-4\,dx-4\,c)}+25\,e^{(-6\,dx-6\,c)}+26\right.}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c}}\right.$$
$$+3\,a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c)
+ 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(-1
0*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x
- 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) +
1))) + 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 8*(3*e^(-2*d*
x - 2*c) + 9*e^(-4*d*x - 4*c) + 25*e^(-6*d*x - 6*c) + 26*e^(-8*d*x - 8*c) +
 25*e^(-10*d*x - 10*c) + 9*e^(-12*d*x - 12*c) + 3*e^(-14*d*x - 14*c))/(d*(8
*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*
x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 1
4*c) + e^(-16*d*x - 16*c) + 1))) + 3*a^2*b*(x + c/d + log(e^(-2*d*x - 2*c)
+ 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e
^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c
) + 1))) + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(99) = 198.

Time = 0.46 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.89

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c)-3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\left(3\left(a^3+6\,a^2b+9\,ab^2+4\right.\right.}{}}{}$$

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
)*e^(14*d*x + 14*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*e^(12*d*x + 12*c
) + (45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3)*e^(10*d*x + 10*c) + 4*(15*a^
3 + 63*a^2*b + 78*a*b^2 + 26*b^3)*e^(8*d*x + 8*c) + (45*a^3 + 198*a^2*b + 2
37*a*b^2 + 100*b^3)*e^(6*d*x + 6*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*
e^(4*d*x + 4*c) + 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/(e^(
2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)^4\left(3a^2b+3ab^2+b^3\right)}{4d}$$
$$-\frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d}-\frac{\tanh(c+dx)^6\left(b^3+3ab^2\right)}{6d}$$
$$-\frac{b^3\tanh(c+dx)^8}{8d}-\frac{\tanh(c+dx)^2\left(a^3+3a^2b+3ab^2+b^3\right)}{2d}$$

```
[In] int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^4*(3*a*b^2 + 3*a^2*b + b
^3))/(4*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (
tanh(c + d*x)^6*(3*a*b^2 + b^3))/(6*d) - (b^3*tanh(c + d*x)^8)/(8*d) - (tan
h(c + d*x)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*d)
```

3.158 $\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/5
*b^2*(3*a+b)*tanh(d*x+c)^5/d-1/7*b^3*tanh(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 212}

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\tanh^5(c+dx)}{5d} - \frac{(a+b)^3\tanh(c+dx)}{d} + x(a+b)^3 - \frac{b^3\tanh^7(c+dx)}{7d}$$

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c
 + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*
x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3 - b(3a^2+3ab+b^2)\,x^2 - b^2(3a+b)x^4 - b^3x^6 + \frac{a^3+3a^2b+3ab^2+b^3}{1-x^2}\right)\,dx, x, \tanh(c\right.}{d}$$

$$= -\frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^7(c+dx)}{7d} + \frac{(a+b)^3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{\tanh(c+dx)\Big(-105(a+b)^3-35b(3a^2+3ab+b^2)\tanh^2(c+dx)-21b^2(3a+b)\tanh^4(c+dx)-15b^3\,\text{ta}}{105d}$$

[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (Tanh[c + d*x]*(-105*(a + b)^3 - 35*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^2 - 21*b^2*(3*a + b)*Tanh[c + d*x]^4 - 15*b^3*Tanh[c + d*x]^6 + (105*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2]))/(105*d)

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

method	result
parallelrisch	$-\frac{15\tanh(dx+c)^7b^3+63\tanh(dx+c)^5ab^2+21b^3\tanh(dx+c)^5+105\tanh(dx+c)^3a^2b+105ab^2\tanh(dx+c)^3+35b^3\tanh(}{}$
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3ab^2\tanh(dx+c)-\frac{3\tanh(dx+c)^5ab^2}{5}-\tanh(dx+c)^3a^2b-ab^2\tanh(dx+c)^3-\frac{b^3\tanh(dx+c)^5}{5}-\frac{b^3\tanh}{}}{}$
default	$\frac{-3a^2b\tanh(dx+c)-3ab^2\tanh(dx+c)-\frac{3\tanh(dx+c)^5ab^2}{5}-\tanh(dx+c)^3a^2b-ab^2\tanh(dx+c)^3-\frac{b^3\tanh(dx+c)^5}{5}-\frac{b^3\tanh}{}}{}$
parts	$\frac{a^3\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+\right.}{d}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x+\frac{\frac{46ab^2}{5}+2a^3+12a^2b\,e^{12dx+12c}+72ab^2e^{10dx+10c}+60a^2b\,e^{10dx+10c}+18ab^2e^{12}}{}$

[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] -1/105*(15*tanh(d*x+c)^7*b^3+63*tanh(d*x+c)^5*a*b^2+21*b^3*tanh(d*x+c)^5+105*tanh(d*x+c)^3*a^2*b+105*a*b^2*tanh(d*x+c)^3+35*b^3*tanh(d*x+c)^3-105*a^3*d*x-315*a^2*b*d*x-315*a*b^2*d*x-105*b^3*d*x+105*a^3*tanh(d*x+c)+315*a^2*b*tanh(d*x+c)+315*a*b^2*tanh(d*x+c)+105*b^3*tanh(d*x+c))/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 1036, normalized size of antiderivative = 11.02

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 + 17
6*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c
)^6 - (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*sinh(d*x + c)^7 + 7*(105*
a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^5 - 7*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3 + 3*(105
*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
35*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 + (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3
 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4
+ 21*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 7*(5*(105*a^3 + 420*a^2*b + 483*a*b^2 + 1
76*b^3)*cosh(d*x + c)^4 + 135*a^3 + 360*a^2*b + 369*a*b^2 + 168*b^3 + 10*(7
5*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
7*(3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 10*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176
*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 9*(105*a^
3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3 + 420*a^2*b + 483*a*b^2 +
176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 7*((105*
a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 + 240*a^
2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 75*a^3 + 180*a^2*b + 225*a*b^2
+ 9*(45*a^3 + 120*a^2*b + 123*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x +
c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh
(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x
+ c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(82) = 164.

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.04

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\begin{cases} a^3x-\frac{a^3\tanh(c+dx)}{d}+3a^2bx-\frac{a^2b\tanh^3(c+dx)}{d}-\frac{3a^2b\tanh(c+dx)}{d}+3ab^2x-\frac{3ab^2\tanh^5(c+dx)}{5d}-\frac{ab^2\tanh^3(c+dx)}{d}- \\ x\left(a+b\tanh^2(c)\right)^3\tanh^2(c) \end{cases}$$

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)/d + 3*a**2*b*x - a**2*b*tanh(c + d*x
)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**5/
(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**
3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)
**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(
c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(88) = 176.

Time = 0.21 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.26

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{105}b^3\left(105x+\frac{105c}{d}-\frac{8\left(203e^{(-2dx-2c)}+609e^{(-4dx-4c)}+770e^{(-6dx-6c)}+770e^{(-8dx-8c)}+315e^{(-10}\right.}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+}\right.$$
$$+\frac{1}{5}ab^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$
$$+a^2b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+a^3\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/105*b^3*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
```

```
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a*b^2*(15*x + 15*c/d - 2*(7
0*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*
d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*b*(3*x + 3*
c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*(x + c/d - 2/(d*(e^(
-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(88) = 176$.

Time = 0.43 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.45

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{105\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)+\frac{2\left(105a^3e^{(12dx+12c)}+630a^2be^{(12dx+12c)}+945ab^2e^{(12dx+12c)}+420b^3e^{(12dx+12c)}+630a^3\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] 1/105*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(105*a^3*e^(12*d*x
 + 12*c) + 630*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) + 420*
b^3*e^(12*d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) + 3150*a^2*b*e^(10*d*x +
10*c) + 3780*a*b^2*e^(10*d*x + 10*c) + 1260*b^3*e^(10*d*x + 10*c) + 1575*a^
3*e^(8*d*x + 8*c) + 6720*a^2*b*e^(8*d*x + 8*c) + 7665*a*b^2*e^(8*d*x + 8*c)
 + 3080*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x + 6*c) + 7980*a^2*b*e^(6*d*
x + 6*c) + 9240*a*b^2*e^(6*d*x + 6*c) + 3080*b^3*e^(6*d*x + 6*c) + 1575*a^3
*e^(4*d*x + 4*c) + 5670*a^2*b*e^(4*d*x + 4*c) + 6363*a*b^2*e^(4*d*x + 4*c)
+ 2436*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c) + 2310*a^2*b*e^(2*d*x
+ 2*c) + 2436*a*b^2*e^(2*d*x + 2*c) + 812*b^3*e^(2*d*x + 2*c) + 105*a^3 + 4
20*a^2*b + 483*a*b^2 + 176*b^3)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(a^3+3a^2b+3ab^2+b^3\right) - \frac{\tanh(c+dx)\,(a+b)^3}{d} - \frac{\tanh(c+dx)^3\,(3a^2b+3ab^2+b^3)}{3d} - \frac{\tanh(c+dx)^5\,(b^3+3ab^2)}{5d} - \frac{b^3\tanh(c+dx)^7}{7d}$$

```
[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c +
 d*x)^3*(3*a*b^2 + 3*a^2*b + b^3))/(3*d) - (tanh(c + d*x)^5*(3*a*b^2 + b^3)
)/(5*d) - (b^3*tanh(c + d*x)^7)/(7*d)
```

3.159 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b(a+b)^2\tanh^2(c+dx)}{2d}-\frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d}-\frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}$$

```
[Out] (a+b)^3*ln(cosh(d*x+c))/d-1/2*b*(a+b)^2*tanh(d*x+c)^2/d-1/4*(a+b)*(a+b*tanh
(d*x+c)^2)^2/d-1/6*(a+b*tanh(d*x+c)^2)^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 455, 45}

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{b(a+b)^2\tanh^2(c+dx)}{2d}-\frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d}-\frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - (b*(a + b)^2*Tanh[c + d*x]^2)/(2*d) - ((
a + b)*(a + b*Tanh[c + d*x]^2)^2)/(4*d) - (a + b*Tanh[c + d*x]^2)^3/(6*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-b(a+b)^2+\frac{(a+b)^3}{1-x}-b(a+b)(a+bx)-b(a+bx)^2\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b(a+b)^2\tanh^2(c+dx)}{2d}\\
&\quad -\frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d}-\frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{-2(a+b)^3\log(\cosh(c+dx))+b(a+b)^2\tanh^2(c+dx)+\frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2+\frac{1}{3}\left(a+b\tan\right.}{2d}$$

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(-2*(a + b)^3*Log[Cosh[c + d*x]] + b*(a + b)^2*Tanh[c + d*x]^2 + ((a +
 b)*(a + b*Tanh[c + d*x]^2)^2)/2 + (a + b*Tanh[c + d*x]^2)^3/3)/d
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2a^2b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}-\frac{b^3\tanh(dx+c)^4}{4}-\frac{b^3\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6b^3}{6}+\frac{(-a^3-3a^2b-3}{}}{d}$
default	$\frac{-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2a^2b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}-\frac{b^3\tanh(dx+c)^4}{4}-\frac{b^3\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6b^3}{6}+\frac{(-a^3-3a^2b-3}{}}{d}$
parts	$\frac{a^3\ln(\cosh(dx+c))}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{3a^2b\left(-\right.}{}$
parallelrisch	$-\frac{2\tanh(dx+c)^6b^3+9\tanh(dx+c)^4a\,b^2+3b^3\tanh(dx+c)^4+12a^3dx+36a^2bdx+36a\,b^2dx+12b^3dx+18\tanh(dx+c)^2a^2b+1}{}$
risch	$-a^3x-3b\,a^2x-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6bc\,a^2}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}+\frac{2b\,e^{2dx+2c}(9a^2e^{8dx+8c}+18ab\,e^{8dx+}}{}$

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3/4*tanh(d*x+c)^4*a*b^2-3/2*tanh(d*x+c)^2*a^2*b-3/2*a*b^2*tanh(d*x+c)
^2-1/4*b^3*tanh(d*x+c)^4-1/2*b^3*tanh(d*x+c)^2-1/6*tanh(d*x+c)^6*b^3+1/2*(-
a^3-3*a^2*b-3*a*b^2-b^3)*ln(tanh(d*x+c)+1)-1/2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln
(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4298 vs. $2(77) = 154$.

Time = 0.30 (sec) , antiderivative size = 4298, normalized size of antiderivative = 51.78

$$\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^2*b - 2*a*b^2 - b^3 + (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(8*a^2*b + 12*a*b^2 + 4
*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 8*a^2*b - 12*a*b^2 - 4*b^
3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 90*(a^2*b + 2*a*b^2 + b^3 - (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 30*(a^2*b + 2*a*b^2
+ b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (8*a^2*b + 1
2*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^7 - 4*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^6 - 945*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^4 - 27*a^2*b - 36*a*b^2 - 17*b^3 + 15*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 63*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 189*(a^2*b + 2*a*b^2 + b^3 - (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(8*a^2*b + 12*a*b^2 +
 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (27*a^2*b
 + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x*cosh(d*x + c)^8 - 1260*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 24*a^2*b - 36*a*b^2 - 12*b^3
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 20*(27*a^2*b + 36*a*b^2 + 17*b^
3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^9 - 540*(a^2*b
 + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 1
26*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
```

h(d*x + c)^5 - 20*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x*cosh(d*x + c)^10 - 135*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*x)*cosh(d*x + c)^8 - 42*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 10*(27*a^2*b + 36*a*b^2 + 17*b
^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a^2*b - 6*
a*b^2 - 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 9*(8*a^2*b + 12*a*b
^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d
x + c)^2 - 3((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*sinh(d*x + c)^12 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
x + c)^10 + 6(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 15*(
33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 30*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 315*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 + 15*a^2*b + 15*a*b^2 +
 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 63*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*(33*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^9 + 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 42*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*
x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^2 + 6*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10
+ 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 70*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^6 + 50*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^11 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 10*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2

```
 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3
 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*cosh(d*x + c)^11 - 15*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 6*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 2*(27*a^2*b + 36*a*b^2
 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 3*(8*
a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d
*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x +
 c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))
*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(
d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*
(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*
d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d
*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^
4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4
+ 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36
*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10
 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(
d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(71) = 142$.

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.54

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\begin{cases} a^3x-\frac{a^3\log(\tanh(c+dx)+1)}{d}+3a^2bx-\frac{3a^2b\log(\tanh(c+dx)+1)}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}+3ab^2x-\frac{3ab^2\log(\tanh(c+dx)+1)}{d} \\ x\left(a+b\tanh^2(c)\right)^3\tanh(c) \end{cases}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d + 3*a**2*b*x - 3*a**2*b*l
og(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*
a*b**2*log(tanh(c + d*x) + 1)/d - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*b**
2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**3*ta
```

```
nh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/
(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.23

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{1}{3}b^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-\cdots}\right.}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+\cdots}\right.$$
$$+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$
$$+\frac{a^3\log\left(\cosh\left(dx+c\right)\right)}{d}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c
) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) + 3*a*b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a^2*b*(x + c/
d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + 1))) + a^3*log(cosh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(77) = 154$.

Time = 0.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.60

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right)-\frac{2\left(9\left(a^2b+2ab^2+b^3\right)e^{(10d\cdots}\right.}{\cdots}}{3a\cdots}$$

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(9*(a^2*b + 2*a*b^2 + b^3)*e^(10*d*
x + 10*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(8*d*x + 8*c) + 2*(27*a^2*b + 36
*a*b^2 + 17*b^3)*e^(6*d*x + 6*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(4*d*x +
4*c) + 9*(a^2*b + 2*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^6)/
d
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)^2\left(3a^2b+3ab^2+b^3\right)}{2d} - \frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d} - \frac{\tanh(c+dx)^4\left(b^3+3ab^2\right)}{4d}-\frac{b^3\tanh(c+dx)^6}{6d}$$

```
[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b + b
^3))/(2*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (
tanh(c + d*x)^4*(3*a*b^2 + b^3))/(4*d) - (b^3*tanh(c + d*x)^6)/(6*d)
```

3.160 $\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx=(a+b)^3x-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+b)*tanh(d*x+c)^3/d
-1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3742, 398, 212}

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}+x(a+b)^3-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + b)*Ta
nh[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-b(3a^2+3ab+b^2)-b^2(3a+b)x^2-b^3x^4+\frac{(a+b)^3}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d} \\
&\quad -\frac{b^3\tanh^5(c+dx)}{5d}+\frac{(a+b)^3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= (a+b)^3x-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$
\int \left(a+b\tanh^2(c+dx)\right)^3\,dx
$$

$$
= \frac{\tanh(c+dx)\left(\frac{15(a+b)^3\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(45a^2+15ab\left(3+\tanh^2(c+dx)\right)+b^2\left(15+5\tanh^2(c+\right.\right.\right.}{15d}
$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^3,x]
[Out] (Tanh[c + d*x]*((15*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
 d*x]^2] - b*(45*a^2 + 15*a*b*(3 + Tanh[c + d*x]^2) + b^2*(15 + 5*Tanh[c +
d*x]^2 + 3*Tanh[c + d*x]^4))))/(15*d)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{3b^3\tanh(dx+c)^5+15a\,b^2\tanh(dx+c)^3+5b^3\tanh(dx+c)^3-15a^3dx-45a^2bdx-45a\,b^2dx-15b^3dx+45a^2b\tanh(dx+c)+4}{15d}$
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-a\,b^2\tanh(dx+c)^3-\frac{(a^3+3a^2b+3a\,b^2+b^3)\ln(\tanh(dx+c)-1)}{2}-\frac{b^3\tanh(dx+c)^3}{3}-b^3\tanh}{d}$
default	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-a\,b^2\tanh(dx+c)^3-\frac{(a^3+3a^2b+3a\,b^2+b^3)\ln(\tanh(dx+c)-1)}{2}-\frac{b^3\tanh(dx+c)^3}{3}-b^3\tanh}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{3a^2b\left(-\tanh(dx+c)-\right.}{}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x+\frac{2b\left(45a^2e^{8dx+8c}+90ab\,e^{8dx+8c}+45b^2e^{8dx+8c}+180a^2e^{6dx+6c}+270ab\,e^{6dx+6c}+\right.}{}$

```
[In] int((a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(3*b^3*tanh(d*x+c)^5+15*a*b^2*tanh(d*x+c)^3+5*b^3*tanh(d*x+c)^3-15*a^
3*d*x-45*a^2*b*d*x-45*a*b^2*d*x-15*b^3*d*x+45*a^2*b*tanh(d*x+c)+45*a*b^2*ta
nh(d*x+c)+15*b^3*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 567, normalized size of antiderivative = 7.66

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{\left(45\,a^2b+60\,ab^2+23\,b^3+15\,(a^3+3\,a^2b+3\,ab^2+b^3)dx\right)\cosh\left(dx+c\right)^5+5\left(45\,a^2b+60\,ab^2+23\,b^3+15\,(a\right.}{}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x)*cosh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 - (45*a^2*b + 60*a*b^2 +
23*b^3)*sinh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 5*(27*a^2*b + 24*a*b^2 + 5*b^3 +
 2*(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(
45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh
(d*x + c)^3 + 3*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(45*a^2*b + 60*a*b^2 + 23*
b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 5*((45*a^2*b
+ 60*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 18*a^2*b + 12*a*b^2 + 10*b^3 + 3*(27
```

```
*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)
^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\begin{cases} a^3x+3a^2bx-\frac{3a^2b\tanh{(c+dx)}}{d}+3ab^2x-\frac{ab^2\tanh^3{(c+dx)}}{d}-\frac{3ab^2\tanh{(c+dx)}}{d}+b^3x-\frac{b^3\tanh^5{(c+dx)}}{5d}-\frac{b^3\tanh^3{(c+d} \\ x\left(a+b\tanh^2{(c)}\right)^3 \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - a*
b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c +
 d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d,
0)), (x*(a + b*tanh(c)**2)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(70) = 140$.

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.23

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{15}\,b^3\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)$$
$$+ab^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+3\,a^2b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^3x$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4
*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1
))) + 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(70) = 140.

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.26

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c)+\frac{2\left(45\,a^2be^{(8\,dx+8\,c)}+90\,ab^2e^{(8\,dx+8\,c)}+45\,b^3e^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}+270\,ab^2e^{(6\,dx+6\,c)}\ldots\right.}{}}{}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(45*a^2*b*e^(8*d*x +
 8*c) + 90*a*b^2*e^(8*d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*
d*x + 6*c) + 270*a*b^2*e^(6*d*x + 6*c) + 90*b^3*e^(6*d*x + 6*c) + 270*a^2*b
*e^(4*d*x + 4*c) + 330*a*b^2*e^(4*d*x + 4*c) + 140*b^3*e^(4*d*x + 4*c) + 18
0*a^2*b*e^(2*d*x + 2*c) + 210*a*b^2*e^(2*d*x + 2*c) + 70*b^3*e^(2*d*x + 2*c
) + 45*a^2*b + 60*a*b^2 + 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)-\frac{\tanh(c+d\,x)^3\left(b^3+3\,a\,b^2\right)}{3\,d}-\frac{b^3\tanh(c+d\,x)^5}{5\,d}-\frac{b\tanh(c+d\,x)\,\left(3\,a^2+3\,a\,b+b^2\right)}{d}$$

```
[In] int((a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^3*(3*a*b^2 + b^3))/(3*d)
 - (b^3*tanh(c + d*x)^5)/(5*d) - (b*tanh(c + d*x)*(3*a*b + 3*a^2 + b^2))/d
```

3.161 $\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^3\log(\tanh(c+dx))}{d} - \frac{b^2(3a+b)\tanh^2(c+dx)}{2d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

```
[Out] (a+b)^3*ln(cosh(d*x+c))/d+a^3*ln(tanh(d*x+c))/d-1/2*b^2*(3*a+b)*tanh(d*x+c)
^2/d-1/4*b^3*tanh(d*x+c)^4/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 457, 84}

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{a^3\log(\tanh(c+dx))}{d}-\frac{b^2(3a+b)\tanh^2(c+dx)}{2d} +\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^3*Log[Tanh[c + d*x]])/d - (b^2*(3*a +
 b)*Tanh[c + d*x]^2)/(2*d) - (b^3*Tanh[c + d*x]^4)/(4*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-b^2(3a+b) - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x} - b^3x\right)\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a^3\log(\tanh(c+dx))}{d} \\
&\quad - \frac{b^2(3a+b)\tanh^2(c+dx)}{2d} - \frac{b^3\tanh^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2(a+b)^3\log(\cosh(c+dx)) + 2a^3\log(\tanh(c+dx)) - b^2(3a+b)\tanh^2(c+dx) - \frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] + 2*a^3*Log[Tanh[c + d*x]] - b^2*(3*a + b)*
Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{-4(a+b)^3 \ln(1-\tanh(dx+c))+4a^3 \ln(\tanh(dx+c))-b^3 \tanh(dx+c)^4+\left(-6a\,b^2-2b^3\right)\tanh(dx+c)^2-4dx(a+b)^3}{4d}$
derivativedivides	$-\frac{\frac{b^3 \tanh(dx+c)^4}{4}+\frac{3a\,b^2 \tanh(dx+c)^2}{2}+\frac{b^3 \tanh(dx+c)^2}{2}+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)-a^3 \ln(\tanh(dx+c)}{d}$
default	$-\frac{\frac{b^3 \tanh(dx+c)^4}{4}+\frac{3a\,b^2 \tanh(dx+c)^2}{2}+\frac{b^3 \tanh(dx+c)^2}{2}+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)-a^3 \ln(\tanh(dx+c)}{d}$
risch	$-a^3x-3b\,a^2x-3a\,b^2x-b^3x-\frac{6bc\,a^2}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}+\frac{2b^2 e^{2dx+2c}\left(3a\,e^{4dx+4c}+2b\,e^{4dx+4}\right.}{d\left(e^{2dx}\right.}$

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-4*(a+b)^3*ln(1-tanh(d*x+c))+4*a^3*ln(tanh(d*x+c))-b^3*tanh(d*x+c)^4+(
-6*a*b^2-2*b^3)*tanh(d*x+c)^2-4*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 2381, normalized size of antiderivative = 33.07

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^2 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3
 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^4 - 6*a*b^2 - 2*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 15*(3*a*
b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 5*(
3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 -
(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(3*a*b^2 + 2*b^3 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*
```

```
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x - 6*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^
7 + (3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^
2 + b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)*sinh(d*x
+ c)^7 + a^3*sinh(d*x + c)^8 + 4*a^3*cosh(d*x + c)^6 + 6*a^3*cosh(d*x + c)^
4 + 4*(7*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 8*(7*a^3*cosh(d*x + c
)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^3*cosh(d*x + c)^2 + 2*(35*
a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + 3*a^3)*sinh(d*x + c)^4 + 8*(
7*a^3*cosh(d*x + c)^5 + 10*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(
d*x + c)^3 + a^3 + 4*(7*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4 + 9*a^
3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 8*(a^3*cosh(d*x + c)^7 + 3*a^3*c
osh(d*x + c)^5 + 3*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (3*a*b^2 + 2*b^3 - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x +
c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
+ c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*co
sh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5
+ 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2(c+dx)\right)^3\coth(c+dx)\,dx$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(68)=136$.

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.97

$$\begin{aligned}&\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)\\&\quad+3\,ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&\quad+\frac{3\,a^2b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^3\log\left(\sinh\left(dx+c\right)\right)}{d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a*b^2*(x + c/d + log(e^(-2
*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + 1))) + 3*a^2*b*log(e^(d*x + c) + e^(-d*x - c))/d + a^3*log(sinh(d*
x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(68)=136$.

Time = 0.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.71

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{2\,a^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+2\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{9\,a^2b(e^{(2\,dx+2\,\cdots}}{\cdots}}{\cdots}$$

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + 2*(3*a^2*b + 3*a*b
^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (9*a^2*b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 36*a^2*b*(e^(2*d*x + 2*
c) + e^(-2*d*x - 2*c)) + 12*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*
b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36*a^2*b - 12*a*b^2 - 4*b^3)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 380, normalized size of antiderivative = 5.28

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(e^{4c+4dx}-1\right)\left(a^3d+d\left(3a^2b+3ab^2+b^3\right)\right)}{2d^2} - x(a+b)^3 + \frac{2\left(2b^3+3ab^2\right)}{d\left(e^{2c+2dx}+1\right)} + \frac{8b^3}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)} + \frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}\left(b^3\sqrt{-d^2}-a^3\sqrt{-d^2}+3ab^2\sqrt{-d^2}+3a^2b\sqrt{-d^2}\right)}{d\sqrt{a^6-6a^5b+3a^4b^2+16a^3b^3+15a^2b^4+6ab^5+b^6}}\right)\sqrt{a^6-6a^5b+3a^4b^2+16a^3b^3+15a^2b^4+6ab^5+}}{\sqrt{-d^2}} - \frac{2\left(4b^3+3ab^2\right)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{4b^3}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

```
[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(a^3*d + d*(3*a*b^2 + 3*a^2*b + b^3)))/(2*d^2) -
 x*(a + b)^3 + (2*(3*a*b^2 + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^3)/(
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (atan
((exp(2*c)*exp(2*d*x)*(b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15*a^2*
b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2)))*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15*a
^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a*b^2 + 4*b^3)
)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3)/(d*(4*exp(2*c +
 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.162 $\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = (a+b)^3x - \frac{a^3\coth(c+dx)}{d} - \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)^3*x-a^3*coth(d*x+c)/d-b^2*(3*a+b)*tanh(d*x+c)/d-1/3*b^3*tanh(d*x+c)^3
/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = -\frac{a^3\coth(c+dx)}{d} - \frac{b^2(3a+b)\tanh(c+dx)}{d} + x(a+b)^3 - \frac{b^3\tanh^3(c+dx)}{3d}$$

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a^3*Coth[c + d*x])/d - (b^2*(3*a + b)*Tanh[c + d*x])/d - (b^
3*Tanh[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b^2(3a+b)+\frac{a^3}{x^2}-b^3x^2-\frac{(a+b)^3}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}\\
&\quad -\frac{(a+b)^3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= (a+b)^3x-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{\tanh(c+dx)\left(-3a^3\coth^2(c+dx)+3(a+b)^3\text{arctanh}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c+dx)}-b^2(9a+3b+\right.}{3d}$$

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

[Out] (Tanh[c + d*x]*(-3*a^3*Coth[c + d*x]^2 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2] - b^2*(9*a + 3*b + b*Tanh[c + d*x]^2)))/(3*d
)

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-b^3 \tanh(dx+c)^3+(-9a\,b^2-3b^3)\tanh(dx+c)-3\coth(dx+c)a^3+3dx(a+b)^3}{3d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^3}{3}+3a\,b^2\tanh(dx+c)+b^3\tanh(dx+c)+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}\right.}{d}$
default	$-\frac{\frac{b^3\tanh(dx+c)^3}{3}+3a\,b^2\tanh(dx+c)+b^3\tanh(dx+c)+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}\right.}{d}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x-\frac{2\left(3a^3e^{6dx+6c}-9a\,b^2e^{6dx+6c}-6\,e^{6dx+6c}b^3+9a^3e^{4dx+4c}-9a\,b^2e^{4dx+4c}+9a^3e\right.}{3d\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}\right.}$

[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/3*(-b^3*tanh(d*x+c)^3+(-9*a*b^2-3*b^3)*tanh(d*x+c)-3*coth(d*x+c)*a^3+3*d*x*(a+b)^3)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(57) = 114.

Time = 0.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{(3\,a^3+9\,ab^2+4\,b^3)\cosh\left(dx+c\right)^4-4\left(3\,a^3+9\,ab^2+4\,b^3+3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\right)\cosh\left(dx+c\right.}{}$$

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/12*((3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 4*(3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3)*sinh(d*x + c)^4 + 9*a^3 - 9*a*b^2 + 4*(3*a^3 - b^3)*cosh(d*x + c)^2 + 2*(6*a^3 - 2*b^3 + 3*(3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))

Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2{(c+dx)}\right)^3\coth^2{(c+dx)}\,dx$$

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.49

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{3}b^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+3ab^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+a^3\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+3a^2bx$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/3*b^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 3*a*b^2
*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*(x + c/d + 2/(d*(e^(-2*d*x
- 2*c) - 1))) + 3*a^2*b*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-\frac{6a^3}{e^{(2dx+2c)}-1}+\frac{2\left(9ab^2e^{(4dx+4c)}+6b^3e^{(4dx+4c)}+18ab^2e^{(2dx+2c)}+6b^3e^{(2dx+2c)}+9ab^2+4\right.}{\left(e^{(2dx+2c)}+1\right)^3}}{3d}$$

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] 1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 6*a^3/(e^(2*d*x + 2*c) -
 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 6*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*
x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 4*b^3)/(e^(2*d*x + 2*c) + 1)^3
)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.69

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\,(a+b)^3+\frac{\frac{2\,a\,b^2}{d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{4\,a\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}$$
$$-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

```
[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(a + b)^3 + ((2*a*b^2)/d + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))/(3*d))/
(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(3*a*b^2 + 2*b^3))/(3*d)
+ (2*exp(4*c + 4*d*x)*(3*a*b^2 + 2*b^3))/(3*d) + (4*a*b^2*exp(2*c + 2*d*x))
/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a
^3)/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 2*b^3))/(3*d*(exp(2*c + 2*d*
x) + 1))
```

3.163 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^2(a+3b)\log(\tanh(c+dx))}{d}-\frac{b^3\tanh^2(c+dx)}{2d}$$

```
[Out] -1/2*a^3*coth(d*x+c)^2/d+(a+b)^3*ln(cosh(d*x+c))/d+a^2*(a+3*b)*ln(tanh(d*x+
c))/d-1/2*b^3*tanh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx)}{2d}+\frac{a^2(a+3b)\log(\tanh(c+dx))}{d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^2(c+dx)}{2d}$$

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a^3*Coth[c + d*x]^2)/d + ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^2*(a +
 3*b)*Log[Tanh[c + d*x]])/d - (b^3*Tanh[c + d*x]^2)/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^3(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^2}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-b^3 - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x^2} + \frac{a^2(a+3b)}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= -\frac{a^3\coth^2(c+dx)}{2d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} \\
&\quad + \frac{a^2(a+3b)\log(\tanh(c+dx))}{d} - \frac{b^3\tanh^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx)-2(a+b)^3\log(\cosh(c+dx))-2a^2(a+3b)\log(\tanh(c+dx))+b^3\tanh^2(c+dx)}{2d}$$

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a^3*Coth[c + d*x]^2 - 2*(a + b)^3*Log[Cosh[c + d*x]] - 2*a^2*(a + 3*b
)*Log[Tanh[c + d*x]] + b^3*Tanh[c + d*x]^2)/d
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-2(a+b)^3\ln(1-\tanh(dx+c))+2a^2(a+3b)\ln(\tanh(dx+c))-\coth(dx+c)^2a^3-b^3\tanh(dx+c)^2-2dx(a+b)^3}{2d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^2}{2}+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)+\frac{a^3}{2\tanh(dx+c)^2}-a^2(a+3b)\ln(\tanh(dx+c))+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b\right.}{d}$
default	$-\frac{\frac{b^3\tanh(dx+c)^2}{2}+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)+\frac{a^3}{2\tanh(dx+c)^2}-a^2(a+3b)\ln(\tanh(dx+c))+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b\right.}{d}$
risch	$-a^3x-3b\,a^2x-3a\,b^2x-b^3x-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}-\frac{6bc\,a^2}{d}-\frac{2\,e^{2dx+2c}\left(a^3e^{4dx+4c}-e^{4dx+4c}b^3+2\right.}{d\left(e^{2dx+2c}+1\right)^2\left(\right.}$

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*(a+b)^3*ln(1-tanh(d*x+c))+2*a^2*(a+3*b)*ln(tanh(d*x+c))-coth(d*x+c)
^2*a^3-b^3*tanh(d*x+c)^2-2*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 1686, normalized size of antiderivative = 23.42

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 + 2*(a^3 - b^3)*cosh(d*x + c)^6 + 2*(14*(a^3 +
```

3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 - b^3)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(a^3 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(a^3 - b^3)*cosh(d*x + c)^3 + (2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(a^3 - b^3)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(a^3 - b^3)*cosh(d*x + c)^4 + a^3 - b^3 + 6*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + b^3)*cosh(d*x + c)^8 + 56*(3*a*b^2 + b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*b^2 + b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a*b^2 + b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3)*sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^5 - (3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*b^2 + b^3 + 4*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^6 - 3*(3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a*b^2 + b^3)*cosh(d*x + c)^7 - (3*a*b^2 + b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 56*(a^3 + 3*a^2*b)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 + 3*a^2*b)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(a^3 + 3*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b)*sinh(d*x + c)^8 - 2*(a^3 + 3*a^2*b)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b)*cosh(d*x + c)^4 - a^3 - 3*a^2*b)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b)*cosh(d*x + c)^5 - (a^3 + 3*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 4*(7*(a^3 + 3*a^2*b)*cosh(d*x + c)^6 - 3*(a^3 + 3*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b)*cosh(d*x + c)^7 - (a^3 + 3*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 3*(a^3 - b^3)*cosh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (a^3 - b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)

Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2(c+dx)\right)^3\coth^3(c+dx)\,dx$$

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(68)=136$.

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.82

$$\begin{aligned}\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&+b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&+\frac{3\,ab^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,a^2b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*(x + c/d
+ log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*log(e^(d*x + c) + e^(-d*x - c))/d + 3*a^
2*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(68)=136$.

Time = 0.47 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.81

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{2\left(3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+2\left(a^3+3\,a^2b\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^3\left(e^{(2\,dx+2\,c)}\ldots\right.}{\ldots}}{\ldots}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^3
 + 3*a^2*b)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^3*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c))^2 + 3*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2
 + 3*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + b^3*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 8*b^3*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a^3 - 12*a^2*b - 12*a*b^2 + 12*b^3)
/((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 - 4))/d
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.54

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(e^{4c+4dx}-1\right)\left(d\,a^3+3\,d\,a^2\,b+3\,d\,a\,b^2+d\,b^3\right)}{2\,d^2} - \frac{\frac{4\,(a^3+b^3)}{d}+\frac{2\,e^{2c+2dx}\,(a^3-b^3)}{d}}{e^{4c+4dx}-1} - \frac{\frac{4\,(a^3+b^3)}{d}+\frac{4\,e^{2c+2dx}\,(a^3-b^3)}{d}}{e^{8c+8dx}-2\,e^{4c+4dx}+1} - \frac{\operatorname{atan}\left(\frac{e^{2c}\,e^{2dx}\,\left(a^3\,\sqrt{-d^2}-b^3\,\sqrt{-d^2}-3\,a\,b^2\,\sqrt{-d^2}+3\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\,\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+}}{\sqrt{-d^2}} - x\,(a+b)^3$$

```
[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(a^3*d + b^3*d + 3*a*b^2*d + 3*a^2*b*d))/(2*d^2)
 - ((4*(a^3 + b^3))/d + (2*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(4*c + 4*d*
x) - 1) - ((4*(a^3 + b^3))/d + (4*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(8*c
 + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (atan((exp(2*c)*exp(2*d*x)*(a^3*(-d^2
)^(1/2) - b^3*(-d^2)^(1/2) - 3*a*b^2*(-d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/
(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^2)^(1/
2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^2)^(
1/2))/(-d^2)^(1/2) - x*(a + b)^3
```

3.164 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=(a+b)^3x-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] (a+b)^3*x-a^2*(a+3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-b^3*tanh(d*x+c)
/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\coth^3(c+dx)}{3d}-\frac{a^2(a+3b)\coth(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh(c+dx)}{d}$$

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d)
 - (b^3*Tanh[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b^3+\frac{a^3}{x^4}+\frac{a^2(a+3b)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^3\tanh(c+dx)}{d}\\
&\quad -\frac{(a+b)^3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= (a+b)^3x-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^3\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{\left(-3b^3-3a^2(a+3b)\coth^2(c+dx)-a^3\coth^4(c+dx)+3(a+b)^3\text{arctanh}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c}\right.}{3d}$$

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

[Out] ((-3*b^3 - 3*a^2*(a + 3*b)*Coth[c + d*x]^2 - a^3*Coth[c + d*x]^4 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2])*Tanh[c + d*x])/(3*d)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{-\coth(dx+c)^3 a^3-3a^2(a+3b)\coth(dx+c)-3b^3\tanh(dx+c)+3dx(a+b)^3}{3d}$
derivativedivides	$-\frac{\frac{\coth(dx+c)^3 a^3}{3}+\coth(dx+c)a^3+3a^2b\coth(dx+c)+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2-\frac{1}{2}b^3\right)\ln(\coth(dx+c)+1)+\frac{b^3}{\coth(dx+c)}+\left(\frac{1}{2}a^3+\right.}{d}$
default	$-\frac{\frac{\coth(dx+c)^3 a^3}{3}+\coth(dx+c)a^3+3a^2b\coth(dx+c)+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2-\frac{1}{2}b^3\right)\ln(\coth(dx+c)+1)+\frac{b^3}{\coth(dx+c)}+\left(\frac{1}{2}a^3+\right.}{d}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x-\frac{2\left(6a^3e^{6dx+6c}+9a^2b\,e^{6dx+6c}-3\,e^{6dx+6c}b^3-9a^2b\,e^{4dx+4c}+9\,e^{4dx+4c}b^3-2a^3e^{2d}\right.}{3d\left(e^{2dx+2c}-1\right)^3\left(e^{2dx+2c}+1\right.}$

[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/3*(-coth(d*x+c)^3*a^3-3*a^2*(a+3*b)*coth(d*x+c)-3*b^3*tanh(d*x+c)+3*d*x*(a+b)^3)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(57) = 114.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.78

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{(4\,a^3+9\,a^2b+3\,b^3)\cosh(dx+c)^4-4\left(4\,a^3+9\,a^2b+3\,b^3+3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\right)\cosh(dx+c)}{}$$

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/12*((4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^4 - 4*(4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 9*a^2*b + 9*b^3 + 4*(a^3 - 3*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 - 6*b^3 + 3*(4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))

Sympy [F]

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \coth^4\left(c+dx\right) dx$$

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.49

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{1}{3}a^3\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + b^3\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + 3a^2b\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + 3ab^2x$$

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^3*(x
+ c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 3*a^2*b*(x + c/d + 2/(d*(e^(-2*d*x
- 2*c) - 1))) + 3*a*b^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c) + \frac{6b^3}{e^{(2dx+2c)}+1} - \frac{2\left(6a^3e^{(4dx+4c)}+9a^2be^{(4dx+4c)}-6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}+4a^3+\cdots\right.}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] 1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 6*b^3/(e^(2*d*x + 2*c) +
 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x +
 2*c) - 18*a^2*b*e^(2*d*x + 2*c) + 4*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3
)/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.71

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\,(a+b)^3 + \frac{\frac{2\,a^2\,b}{d} - \frac{2\,e^{2\,c+2\,d\,x}\,(2\,a^3+3\,b\,a^2)}{3\,d}}{e^{4\,c+4\,d\,x} - 2\,e^{2\,c+2\,d\,x} + 1} - \frac{\frac{2\,(2\,a^3+3\,b\,a^2)}{3\,d} + \frac{2\,e^{4\,c+4\,d\,x}\,(2\,a^3+3\,b\,a^2)}{3\,d} - \frac{4\,a^2\,b\,e^{2\,c+2\,d\,x}}{d}}{3\,e^{2\,c+2\,d\,x} - 3\,e^{4\,c+4\,d\,x} + e^{6\,c+6\,d\,x} - 1}$$
$$+ \frac{2\,b^3}{d\,(e^{2\,c+2\,d\,x}+1)} - \frac{2\,(2\,a^3+3\,b\,a^2)}{3\,d\,(e^{2\,c+2\,d\,x}-1)}$$

```
[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(a + b)^3 + ((2*a^2*b)/d - (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a^3))/(3*d))/
(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*b + 2*a^3))/(3*d)
+ (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3))/(3*d) - (4*a^2*b*exp(2*c + 2*d*x))
/d)/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (2*b
^3)/(d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a^2*b + 2*a^3))/(3*d*(exp(2*c + 2*d*
x) - 1))
```

3.165 $\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{a^2(a+3b)\coth^2(c+dx)}{2d} - \frac{a^3\coth^4(c+dx)}{4d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a(a^2+3ab+3b^2)\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*a^2*(a+3*b)*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d+(a+b)^3*ln(cosh(d*
x+c))/d+a*(a^2+3*a*b+3*b^2)*ln(tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{a^3\coth^4(c+dx)}{4d} + \frac{a(a^2+3ab+3b^2)\log(\tanh(c+dx))}{d} - \frac{a^2(a+3b)\coth^2(c+dx)}{2d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a^2*(a + 3*b)*Coth[c + d*x]^2)/d - (a^3*Coth[c + d*x]^4)/(4*d) + ((a
+ b)^3*Log[Cosh[c + d*x]])/d + (a*(a^2 + 3*a*b + 3*b^2)*Log[Tanh[c + d*x]])
/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^5(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{a^3}{x^3}+\frac{a^2(a+3b)}{x^2}+\frac{a(a^2+3ab+3b^2)}{x}\right)dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= -\frac{a^2(a+3b)\coth^2(c+dx)}{2d}-\frac{a^3\coth^4(c+dx)}{4d}\\
&\quad+\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a(a^2+3ab+3b^2)\log(\tanh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-a^2(a+3b)\coth^2(c+dx)-\frac{1}{2}a^3\coth^4(c+dx)+2(a+b)^3\log(\sinh(c+dx))-2b^3\log(\tanh(c+dx))}{2d}$$

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-(a^2*(a + 3*b)*Coth[c + d*x]^2) - (a^3*Coth[c + d*x]^4)/2 + 2*(a + b)^3*L
og[Sinh[c + d*x]] - 2*b^3*Log[Tanh[c + d*x]])/(2*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{-4(a+b)^3\ln(1-\tanh(dx+c))+4(a^3+3a^2b+3a\,b^2)\ln(\tanh(dx+c))-\coth(dx+c)^4a^3-2\coth(dx+c)^2a^2(a+3b)-4dx(a+b}{4d}$
derivativedivides	$-\frac{\frac{a^3}{4\tanh(dx+c)^4}-a(a^2+3ab+3b^2)\ln(\tanh(dx+c))+\frac{a^2(a+3b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a}{d}$
default	$-\frac{\frac{a^3}{4\tanh(dx+c)^4}-a(a^2+3ab+3b^2)\ln(\tanh(dx+c))+\frac{a^2(a+3b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a}{d}$
risch	$-a^3x-3b\,a^2x-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6bc\,a^2}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a^2\mathrm{e}^{2dx+2c}(2a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+}}{d(\mathrm{e}^{2dx}}$

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-4*(a+b)^3*ln(1-tanh(d*x+c))+4*(a^3+3*a^2*b+3*a*b^2)*ln(tanh(d*x+c))-c
oth(d*x+c)^4*a^3-2*coth(d*x+c)^2*a^2*(a+3*b)-4*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. $2(79) = 158$.

Time = 0.29 (sec) , antiderivative size = 2393, normalized size of antiderivative = 28.83

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
```

+ b^3)*d*x*sinh(d*x + c)^8 + 2*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^2 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3
+ 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^4 - 2*a^3 - 6*a^2*b + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(2*a^
3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(
2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 -
(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(2*a^3 + 3*a^2*b -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x - 6*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - (b^3*cosh(d*x + c)^8 + 8*b
^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4*b^3*cosh(d*x + c
)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)
^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c))*sinh(d*x + c)^5 - 4*b^
3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cosh(d*x + c)^2 + 3*
b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*cosh(d*x + c)^3 +
3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh(d*x + c)^6 - 15*
b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 8*(b^3
*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x + c)^3 - b^3*cosh
(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - ((a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b + 3*a*b
^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2)*sinh(d*x + c)
^8 - 4*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^6 - 4*(a^3 + 3*a^2*b + 3*a*b
^2 - 7*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a
^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 2
(35(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2
- 30*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3
+ 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 - 10*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d
x + c)^3 + 3(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + a^
3 + 3*a^2*b + 3*a*b^2 - 4*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2 + 4*(7*
(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^6 - 15*(a^3 + 3*a^2*b + 3*a*b^2)*co
sh(d*x + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 + 9*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^7
- 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2)
*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*

```
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x +
c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3
 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
+ c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*co
sh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*
x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5
+ 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(79) = 158$.

Time = 0.21 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

$$\begin{aligned}&\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\cdots}}\right.\\&\quad+3\,a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&\quad+\frac{b^3\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,ab^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a^2*
b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^(d*x
+ c) + e^(-d*x - c))/d + 3*a*b^2*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(79) = 158$.

Time = 0.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.22

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{2\,b^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+2\left(a^3+3\,a^2b+3\,ab^2\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{3\,a^3\left(e^{(2\,dx+2\,c)}+\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^3 + 3*a^2*b +
 3*a*b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (3*a^3*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c))^2 + 9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2
 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) - 12*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 36*a*
b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 - 12*a^2*b + 36*a*b^2)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.59

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(e^{4\,c+4\,d\,x}-1\right)\left(b^3\,d+d\left(a^3+3\,a^2\,b+3\,a\,b^2\right)\right)}{2\,d^2}$$
$$-x\,(a+b)^3-\frac{2\left(2\,a^3+3\,b\,a^2\right)}{d\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{8\,a^3}{d\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{\mathrm{atan}\left(\frac{e^{2\,c}\,e^{2\,d\,x}\left(a^3\sqrt{-d^2}-b^3\sqrt{-d^2}+3\,a\,b^2\sqrt{-d^2}+3\,a^2\,b\sqrt{-d^2}\right)}{d\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+16\,a^3\,b^3+3\,a^2\,b^4-6\,a\,b^5+b^6}}\right)\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+16\,a^3\,b^3+3\,a^2\,b^4-6\,a\,b^5+\cdots}}{\sqrt{-d^2}}$$
$$-\frac{2\left(4\,a^3+3\,b\,a^2\right)}{d\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{4\,a^3}{d\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(b^3*d + d*(3*a*b^2 + 3*a^2*b + a^3)))/(2*d^2) -
 x*(a + b)^3 - (2*(3*a^2*b + 2*a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*a^3)/(
d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (atan
((exp(2*c)*exp(2*d*x)*(a^3*(-d^2)^(1/2) - b^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*a^2*b
^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*a^
2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a^2*b + 4*a^3)
)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (4*a^3)/(d*(6*exp(4*c +
 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.166 $\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = (a+b)^3x - \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} - \frac{a^2(a+3b)\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d}$$

```
[Out] (a+b)^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a^2*(a+3*b)*coth(d*x+c)^3/d
-1/5*a^3*coth(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{a^3\coth^5(c+dx)}{5d} - \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} - \frac{a^2(a+3b)\coth^3(c+dx)}{3d} + x(a+b)^3$$

```
[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a^2*(a + 3*b)*Co
th[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^6}+\frac{a^2(a+3b)}{x^4}+\frac{a(a^2+3ab+3b^2)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d} \\
&\quad -\frac{a^3\coth^5(c+dx)}{5d}-\frac{(a+b)^3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= (a+b)^3x-\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\begin{aligned}
&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= -\frac{a\coth(c+dx)\left(15(a^2+3ab+3b^2)+5a(a+3b)\coth^2(c+dx)+3a^2\coth^4(c+dx)\right)}{15d} \\
&\quad +\frac{(a+b)^3\text{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)\tanh(c+dx)}{d\sqrt{\tanh^2(c+dx)}}
\end{aligned}$$

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/15*(a*Coth[c + d*x]*(15*(a^2 + 3*a*b + 3*b^2) + 5*a*(a + 3*b)*Coth[c + d
*x]^2 + 3*a^2*Coth[c + d*x]^4))/d + ((a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2
]]*Tanh[c + d*x])/(d*Sqrt[Tanh[c + d*x]^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{-3\coth(dx+c)^5a^3-5a^2\coth(dx+c)^3(a+3b)+(-15a^3-45a^2b-45a\,b^2)\coth(dx+c)+15dx(a+b)^3}{15d}$
derivativedivides	$-\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)-1)+\frac{a^3}{5\tanh(dx+c)^5}+\frac{a\left(a^2+3ab+3b^2\right)}{\tanh(dx+c)}+\frac{a^2(a+3b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2\right.}{d}$
default	$-\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)-1)+\frac{a^3}{5\tanh(dx+c)^5}+\frac{a\left(a^2+3ab+3b^2\right)}{\tanh(dx+c)}+\frac{a^2(a+3b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2\right.}{d}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x-\frac{2a\left(45a^2e^{8dx+8c}+90ab\,e^{8dx+8c}+45b^2e^{8dx+8c}-90a^2e^{6dx+6c}-270ab\,e^{6dx+6c}\right.}{}$

```
[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-3*coth(d*x+c)^5*a^3-5*a^2*coth(d*x+c)^3*(a+3*b)+(-15*a^3-45*a^2*b-45
*a*b^2)*coth(d*x+c)+15*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 557, normalized size of antiderivative = 7.53

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{(23\,a^3+60\,a^2b+45\,ab^2)\cosh\left(dx+c\right)^5+5\left(23\,a^3+60\,a^2b+45\,ab^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-(23}{}$$

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^5 + 5*(23*a^3 + 60*a^2*
b + 45*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (23*a^3 + 60*a^2*b + 45*a*b^2
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^5 - 5*(5*a^3 + 24*
a^2*b + 27*a*b^2)*cosh(d*x + c)^3 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*
(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^3 - 3*(5*a^3 + 24*a^2*b + 27*a
*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 6*a^2*b + 9*a*b^2)*cosh(
```

```
d*x + c) - 5*((23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^4 + 46*a^3 + 120*a^2*b + 90*a*b^2 + 30*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x - 3*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^
5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*
d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(70) = 140$.

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.23

$$\begin{aligned}
&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{1}{15}a^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1)}\right) \\
&\quad + a^2b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right) \\
&\quad + 3ab^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+b^3x
\end{aligned}$$

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + a^2*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4
*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1
))) + 3*a*b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(70) = 140$.

Time = 0.57 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.26

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c) - \frac{2\,(45\,a^3e^{(8\,dx+8\,c)}+90\,a^2be^{(8\,dx+8\,c)}+45\,ab^2e^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}-270\,a^2be^{(6\,dx+6\,c)}}{}}{}$$

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 2*(45*a^3*e^(8*d*x + 8
*c) + 90*a^2*b*e^(8*d*x + 8*c) + 45*a*b^2*e^(8*d*x + 8*c) - 90*a^3*e^(6*d*x
 + 6*c) - 270*a^2*b*e^(6*d*x + 6*c) - 180*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e
^(4*d*x + 4*c) + 330*a^2*b*e^(4*d*x + 4*c) + 270*a*b^2*e^(4*d*x + 4*c) - 70
*a^3*e^(2*d*x + 2*c) - 210*a^2*b*e^(2*d*x + 2*c) - 180*a*b^2*e^(2*d*x + 2*c
) + 23*a^3 + 60*a^2*b + 45*a*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 568, normalized size of antiderivative = 7.68

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\,(a+b)^3$$
$$-\frac{\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{6\,e^{8\,c+8\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{24\,e^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{24\,e^{6\,c+6\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}$$
$$-\frac{\frac{2\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{15\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{12\,e^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1}$$
$$+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,e^{6\,c+6\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{18\,e^{4\,c+4\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{5\,d}}{6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,e^{2\,c+2\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}-\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d\,(e^{2\,c+2\,d\,x}-1)}$$

```
[In] int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(a + b)^3 - ((6*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*b
^2 + 2*a^2*b + a^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) -
(24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)*(9*a*b^2
+ 6*a^2*b + 5*a^3))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(9*a*b
^2 + 6*a^2*b + 5*a^3))/(15*d) + (6*exp(4*c + 4*d*x)*(a*b^2 + 2*a^2*b + a^3)
)/(5*d) - (12*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d))/(3*exp(2*c + 2*d*x)
```

```
- 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + ((6*(a*b^2 + a^2*b))/(5*d) -
 (6*exp(6*c + 6*d*x)*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (18*exp(4*c + 4*d*x)*
(a*b^2 + a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 + 6*a^2*b + 5*a^3))/(
5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) + ((6*(a*b^2 + a^2*b))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2
+ 2*a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (6*(
a*b^2 + 2*a^2*b + a^3))/(5*d*(exp(2*c + 2*d*x) - 1))
```

3.167 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d} - \frac{a^2(a+3b)\coth^4(c+dx)}{4d} - \frac{a^3\coth^6(c+dx)}{6d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{(a+b)^3\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*a*(a^2+3*a*b+3*b^2)*coth(d*x+c)^2/d-1/4*a^2*(a+3*b)*coth(d*x+c)^4/d-1/
6*a^3*coth(d*x+c)^6/d+(a+b)^3*ln(cosh(d*x+c))/d+(a+b)^3*ln(tanh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^6(c+dx)}{6d} - \frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d} - \frac{a^2(a+3b)\coth^4(c+dx)}{4d} + \frac{(a+b)^3\log(\tanh(c+dx))}{d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

```
[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]^2)/d - (a^2*(a + 3*b)*Coth[c +
d*x]^4)/(4*d) - (a^3*Coth[c + d*x]^6)/(6*d) + ((a + b)^3*Log[Cosh[c + d*x]]
)/d + ((a + b)^3*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^7(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^4}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{a^3}{x^4}+\frac{a^2(a+3b)}{x^3}+\frac{a(a^2+3ab+3b^2)}{x^2}+\frac{(a+b)^3}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= -\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d}-\frac{a^2(a+3b)\coth^4(c+dx)}{4d}\\
&\quad -\frac{a^3\coth^6(c+dx)}{6d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{(a+b)^3\log(\tanh(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a(a+b)^2\coth^2(c+dx)+\frac{1}{2}(a+b)\left(b+a\coth^2(c+dx)\right)^2+\frac{1}{3}\left(b+a\coth^2(c+dx)\right)^3-2(a+b)^3\log(\sin}{2d}$$

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
[Out] -1/2*(a*(a + b)^2*Coth[c + d*x]^2 + ((a + b)*(b + a*Coth[c + d*x]^2)^2)/2 +
 (b + a*Coth[c + d*x]^2)^3/3 - 2*(a + b)^3*Log[Sinh[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{-12(a+b)^3\ln(1-\tanh(dx+c))+12(a+b)^3\ln(\tanh(dx+c))-2\coth(dx+c)^6a^3-3a^2\coth(dx+c)^4(a+3b)+(-6a^3-18a^2b-}{12d}$
derivativedivides	$-\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)-1)+\left(-a^3-3a^2b-3a\,b^2-b^3\right.}{d}$
default	$-\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)+1)+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)-1)+\left(-a^3-3a^2b-3a\,b^2-b^3\right.}{d}$
risch	$-a^3x-3b\,a^2x-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6bc\,a^2}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+18ab\,e^{8dx}\right.}{}$

```
[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/12*(-12*(a+b)^3*ln(1-tanh(d*x+c))+12*(a+b)^3*ln(tanh(d*x+c))-2*coth(d*x+c
)^6*a^3-3*a^2*coth(d*x+c)^4*(a+3*b)+(-6*a^3-18*a^2*b-18*a*b^2)*coth(d*x+c)^
2-12*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4305 vs. 2(97) = 194.

Time = 0.31 (sec) , antiderivative size = 4305, normalized size of antiderivative = 41.80

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(4*a^3 + 12*a^2*b + 8*a
b^2 - 5(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 4*a^3 - 12*a^2*b - 8*a*b^
2 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 90*(a^3 + 2*a^2*b + a*b^2 - (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 30*(a^3 + 2*a^2*b +
a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 12*
a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^7 + 4*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^6 + 945*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^4 + 17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 63*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 189*(a^3 + 2*a^2*b + a*b^2 - (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(4*a^3 + 12*a^2*b + 8
*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (17*a^3 +
 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x*cosh(d*x + c)^8 + 1260*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 12*a^3 - 36*a^2*b - 24*a*b^2
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 20*(17*a^3 + 36*a^2*b + 27*a*b^
2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^9 + 540*(a^3 +
 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 1
26*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^5 + 20*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x*cosh(d*x + c)^10 + 135*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*x)*cosh(d*x + c)^8 - 42*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 10*(17*a^3 + 36*a^2*b + 27*a*b
^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*a^3 + 6*a^
2*b + 3*a*b^2 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 9*(4*a^3 + 12*a^2*b

+ 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d
x + c)^2 - 3((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b +
3*a*b^2 + b^3)*sinh(d*x + c)^12 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
x + c)^10 - 6(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 11*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 15*(
33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 - 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 30*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
+ c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 315*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 5*a^3 - 15*a^2*b - 15*a*b^2 -
5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 63*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*(33*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 +
a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
+ c)^9 - 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 42*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*
x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^2 + 6*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10
- 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 70*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^6 - 50*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^11 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 10*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 10*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3
- (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*x*cosh(d*x + c)^11 + 15*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 6*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 2*(17*a^3 + 36*a^2*b +
27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 3*(4*
a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d

```
*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x +
 c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))
*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(
d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*
(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*
d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d
*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^
4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4
- 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36
*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10
 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*
d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(
d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(97)=194$.

Time = 0.21 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.08

$$\begin{aligned}
&\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&=\frac{1}{3}a^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+3\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-}}\right. \\
&\quad+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-}\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e}\right. \\
&\quad+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right) \\
&\quad+\frac{b^3\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}
\end{aligned}$$

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 3*a^2*b*(x + c/d + log(e^(-d*x - c) + 1)/d
 + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*
c) - e^(-8*d*x - 8*c) - 1))) + 3*a*b^2*(x + c/d + log(e^(-d*x - c) + 1)/d +
 log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) - 1))) + b^3*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(97) = 194$.

Time = 0.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.11

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)+\frac{2\left(9\left(a^3+2a^2b+ab^2\right)e\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*(9*(a^3 + 2*a^2*b + a*b^2)*e^(
10*d*x + 10*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(8*d*x + 8*c) + 2*(17*a^3 +
 36*a^2*b + 27*a*b^2)*e^(6*d*x + 6*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(4*d
*x + 4*c) + 9*(a^3 + 2*a^2*b + a*b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1
)^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.69

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d}$$
$$-\frac{4\,(11\,a^3+3\,b\,a^2)}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{32\,a^3}{3\,d\,\left(15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$
$$-\frac{6\,(3\,a^3+4\,a^2\,b+a\,b^2)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$
$$-\frac{8\,(13\,a^3+9\,b\,a^2)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{32\,a^3}{d\,\left(5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1\right)}-x\,(a+b)^3$$

```
[In] int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (4*(3*a^
2*b + 11*a^3))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*
d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^3)/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(
2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*
d*x) + exp(12*c + 12*d*x) + 1)) - (6*(a*b^2 + 4*a^2*b + 3*a^3))/(d*(exp(4*c
 + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*(a*b^2 + 2*a^2*b + a^3))/(d*(exp(
2*c + 2*d*x) - 1)) - (8*(9*a^2*b + 13*a^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (32*a^3)/(d*(5*exp(2*c + 2*d*x) -
 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c
+ 10*d*x) - 1)) - x*(a + b)^3
```

3.168 $\int \left(a+b\tanh^2(c+dx)\right)^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx=(a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}-\frac{b^4\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+4*a*b+b^
2)*tanh(d*x+c)^3/d-1/5*b^3*(4*a+b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3742, 398, 212}

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx=-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}+x(a+b)^4-\frac{b^4\tanh^7(c+dx)}{7d}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^4,x]
```

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6
*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/(3*d) - (b^3*(4*a + b)*Tanh[c + d*x]^5
)/(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^4}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)\left(2a^2+2ab+b^2\right)-b^2\left(6a^2+4ab+b^2\right)x^2-b^3(4a+b)x^4-b^4x^6+\frac{(a+b)^4}{1-x^2}\right)\,dx\right.}{d}$$

$$= -\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d} - \frac{b^3(4a+b)\tanh^5(c+dx)}{5d}-\frac{b^4\tanh^7(c+dx)}{7d}+\frac{(a+b)^4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= (a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}-\frac{b^4\tanh^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx$$
$$=\frac{\tanh(c+dx)\left(\frac{105(a+b)^4\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\big(105(4a^3+6a^2b+4ab^2+b^3)+35b(6a^2+4ab+b^2)\tan\right.}{105d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^4,x]
```

```
[Out] (Tanh[c + d*x]*((105*(a + b)^4*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(105*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3) + 35*b*(6*a^2 + 4*a*b
+ b^2)*Tanh[c + d*x]^2 + 21*b^2*(4*a + b)*Tanh[c + d*x]^4 + 15*b^3*Tanh[c +
 d*x]^6)))/(105*d)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.63

method	result
parallelrisch	$-\frac{15b^4\tanh(dx+c)^7+84a\,b^3\tanh(dx+c)^5+21b^4\tanh(dx+c)^5+210a^2b^2\tanh(dx+c)^3+140a\,b^3\tanh(dx+c)^3+35b^4\tanh}{}$
derivativedivides	$\frac{-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-4a\,b^3\tanh(dx+c)-\frac{4a\,b^3\tanh(dx+c)^5}{5}-2a^2b^2\tanh(dx+c)^3-\frac{4a\,b^3\tanh(dx+c)^3}{3}-}{}$
default	$\frac{-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-4a\,b^3\tanh(dx+c)-\frac{4a\,b^3\tanh(dx+c)^5}{5}-2a^2b^2\tanh(dx+c)^3-\frac{4a\,b^3\tanh(dx+c)^3}{3}-}{}$
parts	$x\,a^4+\frac{b^4\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{4a\,b^3}{}$
risch	$x\,a^4+4b\,a^3x+6a^2b^2x+4a\,b^3x+b^4x+\frac{8b(161a\,b^2+105a^3+315a^2b\,e^{12dx+12c}+1575a^2b\,e^{10dx+10c}+315}{}$

```
[In] int((a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(15*b^4*tanh(d*x+c)^7+84*a*b^3*tanh(d*x+c)^5+21*b^4*tanh(d*x+c)^5+21
0*a^2*b^2*tanh(d*x+c)^3+140*a*b^3*tanh(d*x+c)^3+35*b^4*tanh(d*x+c)^3-105*a^
4*d*x-420*a^3*b*d*x-630*a^2*b^2*d*x-420*a*b^3*d*x-105*b^4*d*x+420*a^3*b*tan
h(d*x+c)+630*a^2*b^2*tanh(d*x+c)+420*a*b^3*tanh(d*x+c)+105*b^4*tanh(d*x+c))
/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 1176, normalized size of antiderivative = 10.69

$$\int \left(a + b\tanh^2(c + dx)\right)^4\, dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*
b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*d*x)*cosh(d*x + c)*sinh(d*x + c)^6 - 4*(105*a^3*b + 210*a^2*b^2 + 161*a*b^
3 + 44*b^4)*sinh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*
b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^5
- 28*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4 + 3*(105*a^3*b + 210*a^2*b
^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((420*a^3*b
+ 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 17
6*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))
*sinh(d*x + c)^4 + 21*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 - 28*(5*(1
05*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^4 + 135*a^3*b +
180*a^2*b^2 + 123*a*b^3 + 42*b^4 + 10*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 +
14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(420*a^3*b + 840*a^2*b^2 +
644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c)^5 + 10*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + 9*(420*a^
3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(420*a^3*b + 840*a^
2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d*x)*cosh(d*x + c) - 28*((105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*
cosh(d*x + c)^6 + 5*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4)*cosh(d*x +
 c)^4 + 75*a^3*b + 90*a^2*b^2 + 75*a*b^3 + 9*(45*a^3*b + 60*a^2*b^2 + 41*a*
b^3 + 14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh
(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x +
c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cos
h(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(99) = 198$.

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int \left(a + b\tanh^2(c + dx)\right)^4 dx$$
$$= \begin{cases} a^4x + 4a^3bx - \frac{4a^3b\tanh(c+dx)}{d} + 6a^2b^2x - \frac{2a^2b^2\tanh^3(c+dx)}{d} - \frac{6a^2b^2\tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3\tanh^5(c+dx)}{5d} - \frac{4ab}{} \\ x\left(a + b\tanh^2(c)\right)^4 \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*tanh(c + d*x)/d + 6*a**2*b**2*x -
 2*a**2*b**2*tanh(c + d*x)**3/d - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*x
- 4*a*b**3*tanh(c + d*x)**5/(5*d) - 4*a*b**3*tanh(c + d*x)**3/(3*d) - 4*a*b
**3*tanh(c + d*x)/d + b**4*x - b**4*tanh(c + d*x)**7/(7*d) - b**4*tanh(c +
d*x)**5/(5*d) - b**4*tanh(c + d*x)**3/(3*d) - b**4*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(104) = 208$.

Time = 0.21 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.73

$$\int \left(a + b\tanh^2(c + dx)\right)^4 dx$$
$$= \frac{1}{105}b^4\left(105\,x + \frac{105\,c}{d} - \frac{8\left(203\,e^{(-2\,dx-2\,c)} + 609\,e^{(-4\,dx-4\,c)} + 770\,e^{(-6\,dx-6\,c)} + 770\,e^{(-8\,dx-8\,c)} + 315\,e^{(-10}\right.}{d(7\,e^{(-2\,dx-2\,c)} + 21\,e^{(-4\,dx-4\,c)} + 35\,e^{(-6\,dx-6\,c)} + 35\,e^{(-8\,dx-8\,c)} + 21\,e^{(-10\,dx-10\,c)} +}\right.$$
$$+ \frac{4}{15}ab^3\left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right)$$
$$+ 2\,a^2b^2\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$
$$+ 4\,a^3b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)} + 1)}\right) + a^4x$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
```

```
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/15*a*b^3*(15*x + 15*c/d - 2*(
70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8
*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^2*b^2*(3*x
 + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x
- 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^3*b*(x + c/d -
2/(d*(e^(-2*d*x - 2*c) + 1))) + a^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(104) = 208$.

Time = 0.31 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.06

$$\int \left(a + b\tanh^2(c + dx)\right)^4 dx$$
$$= \frac{105\,(a^4 + 4\,a^3b + 6\,a^2b^2 + 4\,ab^3 + b^4)(dx + c) + \frac{8\,(105\,a^3be^{(12\,dx+12\,c)} + 315\,a^2b^2e^{(12\,dx+12\,c)} + 315\,ab^3e^{(12\,dx+12\,c)} + 105\,b^4e^{(12\,dx}}{}}{}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c) + 8*(105*a
^3*b*e^(12*d*x + 12*c) + 315*a^2*b^2*e^(12*d*x + 12*c) + 315*a*b^3*e^(12*d*
x + 12*c) + 105*b^4*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x + 10*c) + 1575*
a^2*b^2*e^(10*d*x + 10*c) + 1260*a*b^3*e^(10*d*x + 10*c) + 315*b^4*e^(10*d*
x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 3360*a^2*b^2*e^(8*d*x + 8*c) + 255
5*a*b^3*e^(8*d*x + 8*c) + 770*b^4*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d*x + 6
*c) + 3990*a^2*b^2*e^(6*d*x + 6*c) + 3080*a*b^3*e^(6*d*x + 6*c) + 770*b^4*e
^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 2835*a^2*b^2*e^(4*d*x + 4*c)
+ 2121*a*b^3*e^(4*d*x + 4*c) + 609*b^4*e^(4*d*x + 4*c) + 630*a^3*b*e^(2*d*x
 + 2*c) + 1155*a^2*b^2*e^(2*d*x + 2*c) + 812*a*b^3*e^(2*d*x + 2*c) + 203*b^
4*e^(2*d*x + 2*c) + 105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)/(e^(2*d*x
 + 2*c) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \left(a + b\tanh^2(c + dx)\right)^4 dx = x\left(a^4 + 4\,a^3\,b + 6\,a^2\,b^2 + 4\,a\,b^3 + b^4\right) - \frac{\tanh(c + d\,x)^3\,(6\,a^2\,b^2 + 4\,a\,b^3 + b^4)}{3\,d} - \frac{\tanh(c + d\,x)^5\,(b^4 + 4\,a\,b^3)}{5\,d} - \frac{b^4\tanh(c + d\,x)^7}{7\,d} - \frac{b\tanh(c + d\,x)\,\left(4\,a^3 + 6\,a^2\,b + 4\,a\,b^2 + b^3\right)}{d}$$

```
[In] int((a + b*tanh(c + d*x)^2)^4,x)
```

```
[Out] x*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) - (tanh(c + d*x)^3*(4*a*b^3 +
 b^4 + 6*a^2*b^2))/(3*d) - (tanh(c + d*x)^5*(4*a*b^3 + b^4))/(5*d) - (b^4*t
anh(c + d*x)^7)/(7*d) - (b*tanh(c + d*x)*(4*a*b^2 + 6*a^2*b + 4*a^3 + b^3))
/d
```

3.169 $\int (a+b\tanh^2(c+dx))^5\,dx$

Optimal result

Integrand size = 14, antiderivative size = 160

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx=(a+b)^5x-\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}-\frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d}-\frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^4(5a+b)\tanh^7(c+dx)}{7d}-\frac{b^5\tanh^9(c+dx)}{9d}$$

```
[Out] (a+b)^5*x-b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*(
10*a^3+10*a^2*b+5*a*b^2+b^3)*tanh(d*x+c)^3/d-1/5*b^3*(10*a^2+5*a*b+b^2)*tan
h(d*x+c)^5/d-1/7*b^4*(5*a+b)*tanh(d*x+c)^7/d-1/9*b^5*tanh(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3742, 398, 212}

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx=-\frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d}-\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}-\frac{b^4(5a+b)\tanh^7(c+dx)}{7d}+x(a+b)^5-\frac{b^5\tanh^9(c+dx)}{9d}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^5,x]
```

```
[Out] (a + b)^5*x - (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d
*x])/d - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^3)/(3*d) -
(b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + b)*Tanh[c
+ d*x]^7)/(7*d) - (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^5}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4) - b^2(10a^3+10a^2b+5ab^2+b^3)\,x^2 - b^3(10a^2+5\right.\right.}{d} \\
&= -\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d} \\
&\quad - \frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d} \\
&\quad - \frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^4(5a+b)\tanh^7(c+dx)}{7d} \\
&\quad - \frac{b^5\tanh^9(c+dx)}{9d} + \frac{(a+b)^5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}
\end{aligned}$$

$$= (a+b)^5x - \frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d} - \frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d} - \frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^4(5a+b)\tanh^7(c+dx)}{7d} - \frac{b^5\tanh^9(c+dx)}{9d}$$

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \left(a+b\tanh^2(c+dx)\right)^5 \, dx$$

$$= \frac{\tanh(c+dx)\left(\frac{315(a+b)^5\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(315(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)+105b(10a^3+10\right.\right.}{}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^5,x]
```

```
[Out] (Tanh[c + d*x]*((315*(a + b)^5*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(315*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4) + 105*b*
(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^2 + 63*b^2*(10*a^2 + 5*a*
b + b^2)*Tanh[c + d*x]^4 + 45*b^3*(5*a + b)*Tanh[c + d*x]^6 + 35*b^4*Tanh[c
 + d*x]^8)))/(315*d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.64

method	result
parallelrisch	$-\frac{3150a^3b^2\tanh(dx+c)+1575a\,b^4\tanh(dx+c)+525a\,b^4\tanh(dx+c)^3+3150a^2b^3\tanh(dx+c)+225a\,b^4\tanh(dx+c)^7+630}{}$
derivativedivides	$\frac{-5a^4b\tanh(dx+c)-10a^3b^2\tanh(dx+c)-10a^2b^3\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{5a\,b^4\tanh(dx+c)^7}{7}-b^5\tanh(dx+c)-\frac{b^5\,\mathrm{ta}}{}}{}$
default	$\frac{-5a^4b\tanh(dx+c)-10a^3b^2\tanh(dx+c)-10a^2b^3\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{5a\,b^4\tanh(dx+c)^7}{7}-b^5\tanh(dx+c)-\frac{b^5\,\mathrm{ta}}{}}{}$
parts	$a^5x+\frac{b^5\left(-\frac{\tanh(dx+c)^9}{9}-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$
risch	$a^5x+5b\,a^4x+10a^3b^2x+10b^3a^2x+5a\,b^4x+b^5x+\frac{2b\left(31500a^3b\,\mathrm{e}^{2dx+2c}+34020a^2b^2\mathrm{e}^{2dx+2c}+17460\right.}{}$

```
[In] int((a+b*tanh(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(3150*a^3*b^2*tanh(d*x+c)+1575*a*b^4*tanh(d*x+c)+525*a*b^4*tanh(d*x+
c)^3+3150*a^2*b^3*tanh(d*x+c)+225*a*b^4*tanh(d*x+c)^7+630*a^2*b^3*tanh(d*x+
```

```
c)^5+315*a*b^4*tanh(d*x+c)^5+1050*a^3*b^2*tanh(d*x+c)^3-3150*a^2*b^3*d*x-31
5*a^5*d*x-315*b^5*d*x+63*b^5*tanh(d*x+c)^5+105*b^5*tanh(d*x+c)^3+45*b^5*tan
h(d*x+c)^7+315*b^5*tanh(d*x+c)+35*b^5*tanh(d*x+c)^9-1575*a^4*b*d*x-3150*a^3
*b^2*d*x-1575*a*b^4*d*x+1050*a^2*b^3*tanh(d*x+c)^3+1575*a^4*b*tanh(d*x+c))/
d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. $2(152) = 304$.

Time = 0.30 (sec) , antiderivative size = 2133, normalized size of antiderivative = 13.33

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 3
15*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x
+ c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5
 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(
d*x + c)*sinh(d*x + c)^8 - (1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640
*a*b^4 + 563*b^5)*sinh(d*x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2
*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
+ 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 - 9*(1225*a^4*b + 2800*a^3*b^2 + 2730
*a^2*b^3 + 1240*a*b^4 + 213*b^5 + 4*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b
^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(1575*a
^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^3 + 3*(15
75*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 +
5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c))*sinh
(d*x + c)^6 + 36*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 5
63*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)
*cosh(d*x + c)^5 - 9*(3500*a^4*b + 7000*a^3*b^2 + 6720*a^2*b^3 + 3560*a*b^4
 + 852*b^5 + 14*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 56
3*b^5)*cosh(d*x + c)^4 + 21*(1225*a^4*b + 2800*a^3*b^2 + 2730*a^2*b^3 + 124
0*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(1575*a^4*b + 4
200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^5 + 35*(1575*a^4*
b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^3 + 20*(1575
*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c))*sinh(d
*x + c)^4 + 84*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563
*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*c
osh(d*x + c)^3 - 3*(28*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b
```

```
^4 + 563*b^5)*cosh(d*x + c)^6 + 14700*a^4*b + 26600*a^3*b^2 + 27440*a^2*b^3
 + 13720*a*b^4 + 1764*b^5 + 105*(1225*a^4*b + 2800*a^3*b^2 + 2730*a^2*b^3 +
 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 16
80*a^2*b^3 + 890*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(
1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5
+ 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 +
 21*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c
)^5 + 40*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 +
 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*
x + c)^3 + 28*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*
b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^2 + 126*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^
3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5
*a*b^4 + b^5)*d*x)*cosh(d*x + c) - 9*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2
*b^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^8 + 7*(1225*a^4*b + 2800*a^3*b^2
 + 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^6 + 2450*a^4*b + 4200
*a^3*b^2 + 4620*a^2*b^3 + 1960*a*b^4 + 882*b^5 + 20*(875*a^4*b + 1750*a^3*b
^2 + 1680*a^2*b^3 + 890*a*b^4 + 213*b^5)*cosh(d*x + c)^4 + 28*(525*a^4*b +
950*a^3*b^2 + 980*a^2*b^3 + 490*a*b^4 + 63*b^5)*cosh(d*x + c)^2)*sinh(d*x +
 c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x
+ c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*
d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*c
osh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)
^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh
(d*x + c)^2 + 126*d*cosh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(148) = 296$.

Time = 0.32 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.92

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx$$
$$= \begin{cases} a^5x + 5a^4bx - \frac{5a^4b\tanh(c+dx)}{d} + 10a^3b^2x - \frac{10a^3b^2\tanh^3(c+dx)}{3d} - \frac{10a^3b^2\tanh(c+dx)}{d} + 10a^2b^3x - \frac{2a^2b^3\tanh^5(c+dx)}{d} \\ x\left(a + b\tanh^2(c)\right)^5 \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c)**2)**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*tanh(c + d*x)/d + 10*a**3*b**2*x
- 10*a**3*b**2*tanh(c + d*x)**3/(3*d) - 10*a**3*b**2*tanh(c + d*x)/d + 10*a
**2*b**3*x - 2*a**2*b**3*tanh(c + d*x)**5/d - 10*a**2*b**3*tanh(c + d*x)**3
/(3*d) - 10*a**2*b**3*tanh(c + d*x)/d + 5*a*b**4*x - 5*a*b**4*tanh(c + d*x)
```

```
**7/(7*d) - a*b**4*tanh(c + d*x)**5/d - 5*a*b**4*tanh(c + d*x)**3/(3*d) - 5
*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*tanh(c + d*x)**9/(9*d) - b**5*tanh(
c + d*x)**7/(7*d) - b**5*tanh(c + d*x)**5/(5*d) - b**5*tanh(c + d*x)**3/(3*
d) - b**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(152) = 304$.

Time = 0.21 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.90

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx$$

$$= \frac{1}{315}b^5\left(315\,x + \frac{315\,c}{d} - \frac{2\left(3492\,e^{(-2\,dx-2\,c)} + 13968\,e^{(-4\,dx-4\,c)} + 26292\,e^{(-6\,dx-6\,c)} + 39438\,e^{(-8\,dx-8\,c)} + 31\cdots\right.}{d(9\,e^{(-2\,dx-2\,c)} + 36\,e^{(-4\,dx-4\,c)} + 84\,e^{(-6\,dx-6\,c)} + 126\,e^{(-8\,dx-8\,c)} + 126\,e^{(-10\cdots}}\right.$$

$$+ \frac{1}{21}ab^4\left(105\,x + \frac{105\,c}{d} - \frac{8\left(203\,e^{(-2\,dx-2\,c)} + 609\,e^{(-4\,dx-4\,c)} + 770\,e^{(-6\,dx-6\,c)} + 770\,e^{(-8\,dx-8\,c)} + 315\,e^{(\cdots}\right.}{d(7\,e^{(-2\,dx-2\,c)} + 21\,e^{(-4\,dx-4\,c)} + 35\,e^{(-6\,dx-6\,c)} + 35\,e^{(-8\,dx-8\,c)} + 21\,e^{(-10\,dx-10\,c)}\cdots}\right.$$

$$+ \frac{2}{3}a^2b^3\left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right)$$

$$+ \frac{10}{3}a^3b^2\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$

$$+ 5\,a^4b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)} + 1)}\right) + a^5x$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/315*b^5*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4
*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x -
10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*
x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x -
12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
 1))) + 1/21*a*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d
*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x -
10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/3*a^2*b^3*(15*x + 15
*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c)
+ 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 10/3
*a^3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 5*a^4*
b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(152) = 304$.

Time = 0.36 (sec) , antiderivative size = 721, normalized size of antiderivative = 4.51

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx$$
$$= \frac{315\,(a^5 + 5\,a^4b + 10\,a^3b^2 + 10\,a^2b^3 + 5\,ab^4 + b^5)(dx + c) + \frac{2\,(1575\,a^4be^{(16\,dx+16\,c)} + 6300\,a^3b^2e^{(16\,dx+16\,c)} + 9450\,a^2b^3e^{(16\,dx}}{}}{}$$

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(d*x +
 c) + 2*(1575*a^4*b*e^(16*d*x + 16*c) + 6300*a^3*b^2*e^(16*d*x + 16*c) + 94
50*a^2*b^3*e^(16*d*x + 16*c) + 6300*a*b^4*e^(16*d*x + 16*c) + 1575*b^5*e^(1
6*d*x + 16*c) + 12600*a^4*b*e^(14*d*x + 14*c) + 44100*a^3*b^2*e^(14*d*x + 1
4*c) + 56700*a^2*b^3*e^(14*d*x + 14*c) + 31500*a*b^4*e^(14*d*x + 14*c) + 63
00*b^5*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c) + 136500*a^3*b^2*e
^(12*d*x + 12*c) + 161700*a^2*b^3*e^(12*d*x + 12*c) + 90300*a*b^4*e^(12*d*x
 + 12*c) + 21000*b^5*e^(12*d*x + 12*c) + 88200*a^4*b*e^(10*d*x + 10*c) + 24
5700*a^3*b^2*e^(10*d*x + 10*c) + 283500*a^2*b^3*e^(10*d*x + 10*c) + 157500*
a*b^4*e^(10*d*x + 10*c) + 31500*b^5*e^(10*d*x + 10*c) + 110250*a^4*b*e^(8*d
*x + 8*c) + 283500*a^3*b^2*e^(8*d*x + 8*c) + 325080*a^2*b^3*e^(8*d*x + 8*c)
 + 175140*a*b^4*e^(8*d*x + 8*c) + 39438*b^5*e^(8*d*x + 8*c) + 88200*a^4*b*e
^(6*d*x + 6*c) + 216300*a^3*b^2*e^(6*d*x + 6*c) + 244020*a^2*b^3*e^(6*d*x +
 6*c) + 131460*a*b^4*e^(6*d*x + 6*c) + 26292*b^5*e^(6*d*x + 6*c) + 44100*a^
4*b*e^(4*d*x + 4*c) + 107100*a^3*b^2*e^(4*d*x + 4*c) + 117180*a^2*b^3*e^(4*
d*x + 4*c) + 63540*a*b^4*e^(4*d*x + 4*c) + 13968*b^5*e^(4*d*x + 4*c) + 1260
0*a^4*b*e^(2*d*x + 2*c) + 31500*a^3*b^2*e^(2*d*x + 2*c) + 34020*a^2*b^3*e^(
2*d*x + 2*c) + 17460*a*b^4*e^(2*d*x + 2*c) + 3492*b^5*e^(2*d*x + 2*c) + 157
5*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5)/(e^(2*d*x + 2
*c) + 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx = x\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right) - \frac{\tanh(c+d\,x)^3\,\left(10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{3\,d} - \frac{\tanh(c+d\,x)^5\,\left(10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{5\,d} - \frac{\tanh(c+d\,x)^7\,\left(b^5+5\,a\,b^4\right)}{7\,d} - \frac{b^5\tanh(c+d\,x)^9}{9\,d} - \frac{b\tanh(c+d\,x)\,\left(5\,a^4+10\,a^3\,b+10\,a^2\,b^2+5\,a\,b^3+b^4\right)}{d}$$

```
[In] int((a + b*tanh(c + d*x)^2)^5,x)
```

```
[Out] x*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2) - (tanh(c + d*x
)^3*(5*a*b^4 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(3*d) - (tanh(c + d*x)^5*(5*
a*b^4 + b^5 + 10*a^2*b^3))/(5*d) - (tanh(c + d*x)^7*(5*a*b^4 + b^5))/(7*d)
- (b^5*tanh(c + d*x)^9)/(9*d) - (b*tanh(c + d*x)*(5*a*b^3 + 10*a^3*b + 5*a^
4 + b^4 + 10*a^2*b^2))/d
```

3.170 $\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d}-\frac{\tanh^2(c+dx)}{2bd}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d+1/2*a^2*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)/d-1/2*tanh(
d*x+c)^2/b/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 84}

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\tanh^2(c+dx)}{2bd}$$

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + (a^2*Log[a + b*Tanh[c + d*x]^2])/(2*b^2*(a
 + b)*d) - Tanh[c + d*x]^2/(2*b*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}-\frac{1}{(a+b)(-1+x)}+\frac{a^2}{b(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d}-\frac{\tanh^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{-\frac{2\log(\cosh(c+dx))}{a+b}-\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{b^2(a+b)}+\frac{\tanh^2(c+dx)}{b}}{2d}$$

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
[Out] -1/2*((-2*Log[Cosh[c + d*x]])/(a + b) - (a^2*Log[a + b*Tanh[c + d*x]^2])/(b
^2*(a + b)) + Tanh[c + d*x]^2/b)/d
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

method	result
parallelrisch	$-\frac{2b^2dx+\tanh(dx+c)^2ab+b^2\tanh(dx+c)^2+2\ln(1-\tanh(dx+c))b^2-a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2b^2d(a+b)}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^2}{2b}+\frac{a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b^2}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^2}{2b}+\frac{a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b^2}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2a^2x}{b^2(a+b)}-\frac{2a^2c}{b^2d(a+b)}+\frac{2ax}{b^2}+\frac{2ac}{b^2d}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2\,e^{2dx+2c}}{bd\left(e^{2dx+2c}+1\right)^2}+\frac{a^2\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right.}{2b^2d(a+b)}$

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*(2*b^2*d*x+tanh(d*x+c)^2*a*b+b^2*tanh(d*x+c)^2+2*ln(1-tanh(d*x+c))*b^2
-a^2*ln(a+b*tanh(d*x+c)^2))/b^2/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 742, normalized size of antiderivative = 11.24

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx =$$

$$-\frac{2\,b^2dx\cosh\left(dx+c\right)^4+8\,b^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,b^2dx\sinh\left(dx+c\right)^4+2\,b^2dx+4\left(b^2dx-a\right.}{}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x - a*b - b^2)*cosh(d*x +
 c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x - a*b - b^2)*sinh(d*x + c)^2
 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
 + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*
log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x
 + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a^2 - b^2
)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^
2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d
```

```
*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x - a
*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^2 + b^3)*d*cosh(d*x + c)^4 +
4*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2 + b^3)*d*sinh(d*x
+ c)^4 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(3*(a*b^2 + b^3)*d*cosh(d*x
+ c)^2 + (a*b^2 + b^3)*d)*sinh(d*x + c)^2 + (a*b^2 + b^3)*d + 4*((a*b^2 + b
^3)*d*cosh(d*x + c)^3 + (a*b^2 + b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(53) = 106$.

Time = 10.07 (sec) , antiderivative size = 415, normalized size of antiderivative = 6.29

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \tilde{\infty}x\tanh^3(c) \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}-\frac{\tanh^4(c+dx)}{4d}-\frac{\tanh^2(c+dx)}{2d}}{a} \\ \frac{4dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{4dx}{2bd\tanh^2(c+dx)-2bd}-\frac{4\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{4\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd}-\frac{\tanh^4(c+dx)}{2bd\tanh^2(c+dx)-2} \\ \frac{x\tanh^5(c)}{a+b\tanh^2(c)} \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d}+\frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d}-\frac{ab\tanh^2(c+dx)}{2ab^2d+2b^3d}+\frac{2b^2dx}{2ab^2d+2b^3d}-\frac{2b^2\log(\tanh(c+dx)+1)}{2ab^2d+2b^3d}-\frac{b^2\tan}{2ab} \end{cases}$$

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tan
h(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a, Eq(
b, 0)), (4*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*d*x/(2
*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/
(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 4*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c +
 d*x)**2 - 2*b*d) - tanh(c + d*x)**4/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 2/(
2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**5/(a + b*tanh(c)**
2), Eq(d, 0)), (a**2*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*
d) + a**2*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*d) - a*b*tan
h(c + d*x)**2/(2*a*b**2*d + 2*b**3*d) + 2*b**2*d*x/(2*a*b**2*d + 2*b**3*d)
- 2*b**2*log(tanh(c + d*x) + 1)/(2*a*b**2*d + 2*b**3*d) - b**2*tanh(c + d*x
)**2/(2*a*b**2*d + 2*b**3*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.02

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(ab^2+b^3)d} + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,be^{(-2\,dx-2\,c)}+be^{(-4\,dx-4\,c)}+b)d} - \frac{(a-b)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a*b^2 + b^3)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*
d*x - 2*c) + b*e^(-4*d*x - 4*c) + b)*d) - (a - b)*log(e^(-2*d*x - 2*c) + 1)
/(b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{a^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab^2+b^3} - \frac{2\,(dx+c)}{a+b} - \frac{2\,(a-b)\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2} + \frac{4\,e^{(2\,dx+2\,c)}}{b\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a*b^2 + b^3) - 2*(d*x + c)/(a + b) - 2*(a - b
)*log(e^(2*d*x + 2*c) + 1)/b^2 + 4*e^(2*d*x + 2*c)/(b*(e^(2*d*x + 2*c) + 1)
^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= -\frac{b^2\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x+\frac{\tanh(c+d\,x)^2}{2}\right)-\frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+\frac{a\,b\tanh(c+d\,x)^2}{2}}{b^2\,d\,(a+b)}$$

```
[In] int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2),x)
[Out] -(b^2*(log(tanh(c + d*x) + 1) - d*x + tanh(c + d*x)^2/2) - (a^2*log(a + b*t
anh(c + d*x)^2))/2 + (a*b*tanh(c + d*x)^2)/2)/(b^2*d*(a + b))
```

3.171 $\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd}$$

```
[Out] x/(a+b)+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/(a+b)/d-tanh(d*
x+c)/b/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 490, 536, 212, 211}

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)} + \frac{x}{a+b} - \frac{\tanh(c+dx)}{bd}$$

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(b^(3/2)*(a +
 b)*d) - Tanh[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{bd}$$

$$= -\frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{b(a+b)d}$$

$$= \frac{x}{a+b} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{c+dx}{(a+b)d} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd}$$

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
[Out] (c + d*x)/((a + b)*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)b\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	87
default	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)b\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	87
risch	$\frac{x}{a+b}+\frac{2}{bd\left(e^{2dx+2c}+1\right)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2b^2(a+b)d}-\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2b^2(a+b)d}$	131

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b*tanh(d*x+c)+1/(a+b)/b*a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^
(1/2))-1/(2*a+2*b)*ln(tanh(d*x+c)-1)+1/(2*a+2*b)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 777, normalized size of antiderivative = 13.17

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{2\,bdx\cosh(dx+c)^2 + 4\,bdx\cosh(dx+c)\sinh(dx+c) + 2\,bdx\sinh(dx+c)^2 + 2\,bdx + \left(a\cosh(dx+c)^2\right.}{}\right.$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d
*x*sinh(d*x + c)^2 + 2*b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(
d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^
2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c) + 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*s
inh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + 4*a + 4*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b
 + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*
b + b^2)*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*x + c)*sinh(d*x + c) +
 b*d*x*sinh(d*x + c)^2 + b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x
+ c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(a/b)/a) + 2*a + 2*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b +
b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*b +
 b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(48) = 96$.

Time = 4.76 (sec) , antiderivative size = 428, normalized size of antiderivative = 7.25

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \tilde{\infty} x \tanh^2(c) \\ \frac{x - \frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d}}{a} \\ \frac{x - \frac{\tanh(c+dx)}{d}}{b} \\ \frac{3dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{3dx}{2bd\tanh^2(c+dx)-2bd} - \frac{2\tanh^3(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{3\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} \\ \frac{x\tanh^4(c)}{a+b\tanh^2(c)} \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{2ab\sqrt{-\frac{a}{b}}\tanh(c+dx)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} + \frac{2b^2dx\sqrt{-\frac{a}{b}}}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{2b^2\sqrt{-\frac{a}{b}}\tanh}{2ab^2d\sqrt{-\frac{a}{b}}+2} \end{cases}$$

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
+ d*x)**3/(3*d) - tanh(c + d*x)/d)/a, Eq(b, 0)), ((x - tanh(c + d*x)/d)/b,
```

```
Eq(a, 0)), (3*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 3*d*x
/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*tanh(c + d*x)**3/(2*b*d*tanh(c + d*x)
**2 - 2*b*d) + 3*tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b))
, (x*tanh(c)**4/(a + b*tanh(c)**2), Eq(d, 0)), (a**2*log(-sqrt(-a/b) + tanh
(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - a**2*log(sqrt(-a
/b) + tanh(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - 2*a*b*
sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) + 2*
b**2*d*x*sqrt(-a/b)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - 2*b**2*
sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(51) = 102$.

Time = 0.39 (sec) , antiderivative size = 509, normalized size of antiderivative = 8.63

$$\begin{aligned}\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = & -\frac{(a-b)\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{8\,(ab+b^2)d} \\ & +\frac{(a-b)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{8\,(ab+b^2)d} \\ & +\frac{(a^2-6\,ab+b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{16\,(ab+b^2)\sqrt{ab}d} \\ & +\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}bd} \\ & -\frac{(a^2-6\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{16\,(ab+b^2)\sqrt{ab}d} \\ & -\frac{3\,(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}bd} \\ & -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}bd} \\ & -\frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,bd} \\ & +\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,bd} \\ & +\frac{3\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{4\,bd}-\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{4\,bd} \\ & +\frac{5}{8\,(be^{(2\,dx+2\,c)}+b)d}-\frac{11}{8\,(be^{(-2\,dx-2\,c)}+b)d}\end{aligned}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a - b)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a +
b)/((a*b + b^2)*d) + 1/8*(a - b)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e
^(-4*d*x - 4*c) + a + b)/((a*b + b^2)*d) + 1/16*(a^2 - 6*a*b + b^2)*arctan(
1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)*d)
+ 1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*b*d) - 1/16*(a^2 - 6*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c)
+ a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)*d) - 3/8*(a + b)*arctan(1/2*((a
+ b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b*d) - 1/4*(a - b)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b*d) - 1/4
*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(b*d) + 1
/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(b*d)
 + 3/4*log(e^(2*d*x + 2*c) + 1)/(b*d) - 3/4*log(e^(-2*d*x - 2*c) + 1)/(b*d)
 + 5/8/((b*e^(2*d*x + 2*c) + b)*d) - 11/8/((b*e^(-2*d*x - 2*c) + b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(ab+b^2)\sqrt{ab}}+\frac{dx+c}{a+b}+\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
((a*b + b^2)*sqrt(a*b)) + (d*x + c)/(a + b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/
d
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}-\frac{\tanh(c+d\,x)}{b\,d}+\frac{a^2\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{b\,d\,\sqrt{a\,b}\,(a+b)}$$

```
[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) - tanh(c + d*x)/(b*d) + (a^2*atan((b*tanh(c + d*x))/(a*b)^(1/2)))
/(b*d*(a*b)^(1/2)*(a + b))
```

3.172 $\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d-1/2*a*ln(a+b*tanh(d*x+c)^2)/b/(a+b)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 78}

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{d(a+b)} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2bd(a+b)}$$

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b
)*d)
```

<u>Rule 78</u>

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} - \frac{a}{(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{\log(\cosh(c+dx))}{a+b} - \frac{a\log(a+b\tanh^2(c+dx))}{2b(a+b)}}{d}$$

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
[Out] (Log[Cosh[c + d*x]]/(a + b) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b)))
/d
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$-\frac{2dxb+2\ln(1-\tanh(dx+c))b+a\ln\left(a+b\tanh(dx+c)^2\right)}{2bd(a+b)}$	49
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b}}{d}$	70
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b}}{d}$	70
risch	$\frac{x}{a+b}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2ax}{b(a+b)}+\frac{2ac}{bd(a+b)}+\frac{\ln(\mathrm{e}^{2dx+2c}+1)}{bd}-\frac{a\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a-b)\mathrm{e}^{2dx+2c}}{a+b}+1\right)}{2bd(a+b)}$	117

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*(2*d*x*b+2*ln(1-tanh(d*x+c))*b+a*ln(a+b*tanh(d*x+c)^2))/b/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.57

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,bdx+a\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(ab+b^2)d}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -1/2*(2*b*d*x + a*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
+ a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a*b
+ b^2)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(36) = 72.

Time = 2.86 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.65

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \tilde{\infty}x\tanh(c) \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}-\frac{\tanh^2(c+dx)}{2d}}{a} \\ \frac{2dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{2dx}{2bd\tanh^2(c+dx)-2bd}-\frac{2\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{2\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd}+\frac{1}{2bd\tanh^2(c+dx)-\ldots} \\ \frac{x\tanh^3(c)}{a+b\tanh^2(c)} \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}-\frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}+\frac{2bdx}{2abd+2b^2d}-\frac{2b\log(\tanh(c+dx)+1)}{2abd+2b^2d} \end{cases}$$

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a, Eq(b, 0)), (2*d*x*tanh(c + d*x)
**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*
d) - 2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*
b*d) + 2*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 1/(2*b*d
*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**3/(a + b*tanh(c)**2), E
q(d, 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) - a*log
(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) + 2*b*d*x/(2*a*b*d + 2*b*
*2*d) - 2*b*log(tanh(c + d*x) + 1)/(2*a*b*d + 2*b**2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{a\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(ab+b^2)d}+\frac{dx+c}{(a+b)d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{bd}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(
(a*b + b^2)*d) + (d*x + c)/((a + b)*d) + log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(44) = 88.

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab+b^2}+\frac{2\,(dx+c)}{a+b}-\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b}}{2\,d}$$

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] -1/2*(a*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)/(a*b + b^2) + 2*(d*x + c)/(a + b) - 2*log(e^(2*
d*x + 2*c) + 1)/b)/d

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+b\,\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x\right)}{b\,d\,\left(a+b\right)}$$

[In] int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)

[Out] -((a*log(a + b*tanh(c + d*x)^2))/2 + b*(log(tanh(c + d*x) + 1) - d*x))/(b*d
*(a + b))

3.173 $\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{x}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)d}$$

```
[Out] x/(a+b)-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/(a+b)/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3751, 492, 212, 211}

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{x}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}d(a+b)}$$

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} \\
&= \frac{x}{a+b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \operatorname{arctanh}(\tanh(c+dx))}{(a+b)d}$$

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[b]) + ArcTanh[Tan
h[c + d*x]])/((a + b)*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	72
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	72
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2b(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2b(a+b)d}$	108

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(2*a+2*b)*ln(tanh(d*x+c)+1)-a/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)
/(a*b)^(1/2))-1/(2*a+2*b)*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

Time = 0.29 (sec) , antiderivative size = 486, normalized size of antiderivative = 10.57

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{2\,dx+\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+}\right.}{}\right.$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b
 + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b
+ b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)))/((a + b)*d), (d*x - sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
a/b)/a))/((a + b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(37) = 74$.

Time = 2.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.50

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \tilde{\infty}x & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{x-\frac{\tanh(c+dx)}{d}}{a} & \text{for}\ b=0 \\ \frac{x}{b} & \text{for}\ a=0 \\ \frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{dx}{2bd\tanh^2(c+dx)-2bd}+\frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for}\ a=-b \\ \frac{x\tanh^2(c)}{a+b\tanh^2(c)} & \text{for}\ d=0 \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}}+\frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}}+\frac{2bdx\sqrt{-\frac{a}{b}}}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)/d)/a
, Eq(b, 0)), (x/b, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2
 - 2*b*d) - d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*tan
h(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**2/(a + b*tanh(c)**2), Eq(d,
 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 2*b**2*d*s
qrt(-a/b)) + a*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 2*b**2
*d*sqrt(-a/b)) + 2*b*d*x*sqrt(-a/b)/(2*a*b*d*sqrt(-a/b) + 2*b**2*d*sqrt(-a/
b)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(38) = 76$.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 4.67

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}(a+b)d} + \frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}d} + \frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}(a+b)d} + \frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a+b)d} - \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a+b)d}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(
a*b)*(a + b)*d) + 1/2*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*
b))/(sqrt(a*b)*d) + 1/4*(a - b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/4*log((a + b)*e^(4*d*x + 4*c) + 2*(
a - b)*e^(2*d*x + 2*c) + a + b)/((a + b)*d) - 1/4*log(2*(a - b)*e^(-2*d*x -
 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a + b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{dx+c}{a+b}}{d}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*(a + b)) - (d*x + c)/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{a\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\sqrt{a\,b}\,(a+b)}$$

```
[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
[Out] x/(a + b) - (a*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b))
```

3.174 $\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3751, 455, 36, 31}

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)} + \frac{\log(\cosh(c+dx))}{d(a+b)}$$

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2(a+b)d} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \tanh^2(c+dx)\right)}{2(a+b)d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\log(\cosh(c+dx)) + \log\left(a+b\tanh^2(c+dx)\right)}{2ad+2bd}$$

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2])/(2*a*d + 2*b*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{2dx+2\ln(1-\tanh(dx+c))-\ln\left(a+b\tanh(dx+c)^{2}\right)}{2d(a+b)}$	44
risch	$-\frac{x}{a+b}-\frac{2c}{d(a+b)}+\frac{\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a-b)\mathrm{e}^{2dx+2c}}{a+b}+1\right)}{2d(a+b)}$	64
derivativedivides	$\frac{\frac{\ln\left(a+b\tanh(dx+c)^{2}\right)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	66
default	$\frac{\frac{\ln\left(a+b\tanh(dx+c)^{2}\right)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	66

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*(2*d*x+2*ln(1-tanh(d*x+c))-ln(a+b*tanh(d*x+c)^2))/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,dx-\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)}{2\,(a+b)d}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] -1/2*(2*d*x - log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a
- b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/
((a + b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(32) = 64.

Time = 1.93 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\tanh{(c)}} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x-\frac{\log{(\tanh{(c+dx)}+1)}}{d}}{a} & \text{for } b=0 \\ \frac{1}{2bd\tanh^2{(c+dx)}-2bd} & \text{for } a=-b \\ \frac{x\tanh{(c)}}{a+b\tanh^2{(c)}} & \text{for } d=0 \\ \frac{2dx}{2ad+2bd}+\frac{\log{\left(-\sqrt{-\frac{a}{b}}+\tanh{(c+dx)}\right)}}{2ad+2bd}+\frac{\log{\left(\sqrt{-\frac{a}{b}}+\tanh{(c+dx)}\right)}}{2ad+2bd}-\frac{2\log{(\tanh{(c+dx)}+1)}}{2ad+2bd} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d)/a, Eq(b, 0)), (1/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b
)), (x*tanh(c)/(a + b*tanh(c)**2), Eq(d, 0)), (2*d*x/(2*a*d + 2*b*d) + log(
-sqrt(-a/b) + tanh(c + d*x))/(2*a*d + 2*b*d) + log(sqrt(-a/b) + tanh(c + d*
x))/(2*a*d + 2*b*d) - 2*log(tanh(c + d*x) + 1)/(2*a*d + 2*b*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{dx+c}{(a+b)d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a+b)d}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a + b)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a + b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\,(a+b)d}$$

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b))/((a + b)*d)

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\ln\left(\tanh(c+d\,x)+1\right) - \frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}}{d\,\left(a+b\right)}$$

[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2),x)

[Out] x/(a + b) - (log(tanh(c + d*x) + 1) - log(a + b*tanh(c + d*x)^2)/2)/(d*(a + b))

3.175 $\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)d}$$

```
[Out] x/(a+b)+arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/(a+b)/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3741, 3756, 211}

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)} + \frac{x}{a+b}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a +
 b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3741

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{a+b} + \frac{b\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx}{a+b}\\ &= \frac{x}{a+b} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}\\ &= \frac{x}{a+b} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - \log(1-\tanh(c+dx)) + \log(1+\tanh(c+dx))}{2ad+2bd}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

```
[Out] ((2*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - Log[1 - Tanh
[c + d*x]] + Log[1 + Tanh[c + d*x]])/(2*a*d + 2*b*d)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{\frac{b \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	71
default	$\frac{\frac{b \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	71
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a(a+b)d}$	108

```
[In] int(1/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))-1/(2*a+2*b)*ln(t
anh(d*x+c)-1)+1/(2*a+2*b)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 484, normalized size of antiderivative = 10.76

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{2\,dx+\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(dx\ldots}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c\ldots}\right.}{}\right.$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] [1/2*(2*d*x + sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)))/((a + b)*d), (d*x + sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
b/a)/b))/((a + b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

Time = 2.00 (sec) , antiderivative size = 240, normalized size of antiderivative = 5.33

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\tanh^2(c)} & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{x}{a} & \text{for}\ b=0 \\ \frac{x-\frac{1}{d\tanh(c+dx)}}{b} & \text{for}\ a=0 \\ -\frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{dx}{2bd\tanh^2(c+dx)-2bd}+\frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for}\ a=-b \\ \frac{x}{a+b\tanh^2(c)} & \text{for}\ d=0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}}+\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}}-\frac{\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0
)), ((x - 1/(d*tanh(c + d*x)))/b, Eq(a, 0)), (-d*x*tanh(c + d*x)**2/(2*b*d*
tanh(c + d*x)**2 - 2*b*d) + d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c +
 d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x/(a + b*tanh(c)**2),
Eq(d, 0)), (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) + log(-s
qrt(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) - log(sqrt
(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)d}+\frac{dx+c}{(a+b)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a +
 b)*d) + (d*x + c)/((a + b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{dx+c}{a+b}}{d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(s
qrt(a*b)*(a + b)) + (d*x + c)/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}+\frac{b\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,(a+b)}$$

```
[In] int(1/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) + (b*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b))
```

3.176 $\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{\log(\tanh(c+dx))}{ad}-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d+ln(tanh(d*x+c))/a/d-1/2*b*ln(a+b*tanh(d*x+c)^2)/a/(
a+b)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 457, 84}

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2ad(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}+\frac{\log(\tanh(c+dx))}{ad}$$

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[Tanh[c + d*x]]/(a*d) - (b*Log[a + b*Ta
nh[c + d*x]^2])/(2*a*(a + b)*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)}+\frac{1}{ax}-\frac{b^2}{a(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{\log(\tanh(c+dx))}{ad}-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{\log(\cosh(c+dx))}{a+b}+\frac{\log(\tanh(c+dx))}{a}-\frac{b\log(a+b\tanh^2(c+dx))}{2a(a+b)}}{d}$$

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] (Log[Cosh[c + d*x]]/(a + b) + Log[Tanh[c + d*x]]/a - (b*Log[a + b*Tanh[c +
d*x]^2])/(2*a*(a + b)))/d
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-b\ln\left(a+b\tanh(dx+c)^2\right)-2\ln(1-\tanh(dx+c))a+(2a+2b)\ln(\tanh(dx+c))-2adx}{2da(a+b)}$	65
derivativedivides	$-\frac{\frac{b\ln\left(a+b\tanh(dx+c)^2\right)}{2a(a+b)}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c))}{a}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	81
default	$-\frac{\frac{b\ln\left(a+b\tanh(dx+c)^2\right)}{2a(a+b)}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c))}{a}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	81
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{da}+\frac{2bx}{a(a+b)}+\frac{2bc}{da(a+b)}+\frac{\ln\left(e^{2dx+2c}-1\right)}{da}-\frac{b\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2da(a+b)}$	117

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*ln(a+b*tanh(d*x+c)^2)-2*ln(1-tanh(d*x+c))*a+(2*a+2*b)*ln(tanh(d*x+c
))-2*a*d*x)/d/a/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(58) = 116.

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,adx+b\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(a^2+ab)d}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x + b*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
+ a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2
+ a*b)*d)
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+ab)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(
(a^2 + a*b)*d) + (d*x + c)/((a + b)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(
e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2+ab}+\frac{2\,(dx+c)}{a+b}-\frac{2\,\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a}}{2\,d}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -1/2*(b*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)/(a^2 + a*b) + 2*(d*x + c)/(a + b) - 2*log(abs(e
^(2*d*x + 2*c) - 1))/a)/d
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.23

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\ln\left(12\,a\,b^2+4\,a^2\,b+9\,b^3-9\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-12\,a\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-4\,a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{a\,d}$$
$$-\frac{b\,\ln\left(5\,a\,b+2\,a^2+3\,b^2+4\,a^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-6\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+3\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+2\,a\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+5\,a}{2\,d\,a^2+2\,b\,d\,a}$$
$$-\frac{x}{a+b}$$

```
[In] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] log(12*a*b^2 + 4*a^2*b + 9*b^3 - 9*b^3*exp(2*c)*exp(2*d*x) - 12*a*b^2*exp(2
*c)*exp(2*d*x) - 4*a^2*b*exp(2*c)*exp(2*d*x))/(a*d) - (b*log(5*a*b + 2*a^2
+ 3*b^2 + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)*exp(4*d*x) - 6*b^2*exp
(2*c)*exp(2*d*x) + 3*b^2*exp(4*c)*exp(4*d*x) + 2*a*b*exp(2*c)*exp(2*d*x) +
5*a*b*exp(4*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d) - x/(a + b)
```

3.177 $\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

```
[Out] x/(a+b)-b^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)/d-coth(d*
x+c)/a/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 491, 536, 212, 211}

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)} + \frac{x}{a+b} - \frac{\coth(c+dx)}{ad}$$

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a +
 b)*d) - Coth[c + d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{a(a+b)d}\\
&= \frac{x}{a+b} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{c+dx}{(a+b)d} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
[Out] (c + d*x)/((a + b)*d) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
a^(3/2)*(a + b)*d) - Coth[c + d*x]/(a*d)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$-\frac{\frac{1}{a\tanh(dx+c)}+\frac{b^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{a(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	89
default	$-\frac{\frac{1}{a\tanh(dx+c)}+\frac{b^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{a(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	89
risch	$\frac{x}{a+b}-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^2(a+b)d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2(a+b)d}$	131

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/a/tanh(d*x+c)+b^2/a/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1
/2))-1/(2*a+2*b)*ln(tanh(d*x+c)+1)+1/(2*a+2*b)*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(52) = 104.

Time = 0.31 (sec) , antiderivative size = 784, normalized size of antiderivative = 13.07

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{2\,adx\cosh(dx+c)^2 + 4\,adx\cosh(dx+c)\sinh(dx+c) + 2\,adx\sinh(dx+c)^2 - 2\,adx + \left(b\cosh(dx+c)\right.}{}\right.$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x*cosh(d*x + c)^2 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*a*d
*x*sinh(d*x + c)^2 - 2*a*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^
2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*s
inh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) - 4*a - 4*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2
 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^
2 + a*b)*d), (a*d*x*cosh(d*x + c)^2 + 2*a*d*x*cosh(d*x + c)*sinh(d*x + c) +
 a*d*x*sinh(d*x + c)^2 - a*d*x - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x
+ c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(b/a)/b) - 2*a - 2*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 +
a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 +
 a*b)*d)]
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^2{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(52) = 104$.

Time = 0.35 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.48

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+ab)d} + \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+ab)d} + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+ab)\sqrt{ab}d} - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+ab)\sqrt{ab}d} + \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}ad} + \frac{\log\left(e^{(2\,dx+2\,c)}-1\right)}{2\,ad} - \frac{\log\left(e^{(-2\,dx-2\,c)}-1\right)}{2\,ad} - \frac{1}{2\,(ae^{(2\,dx+2\,c)}-a)d} + \frac{3}{2\,(ae^{(-2\,dx-2\,c)}-a)d}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^2 + a*b)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + a*b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x
 + 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqrt(a*b)*d) - 1/4*(a*b - b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqrt(a*b
)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*a*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/(a*d) - 1/2*log(e^(-2*d*x - 2*c)
- 1)/(a*d) - 1/2/((a*e^(2*d*x + 2*c) - a)*d) + 3/2/((a*e^(-2*d*x - 2*c) - a
)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+ab)\sqrt{ab}} - \frac{dx+c}{a+b} + \frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))
/((a^2 + a*b)*sqrt(a*b)) - (d*x + c)/(a + b) + 2/(a*(e^(2*d*x + 2*c) - 1)))
/d
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 6.70

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\operatorname{atan}\left(\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}\left(\frac{4b^2}{a\,d\,(a+b)^3\,(a^2+b\,a)\,\sqrt{b^3}} + \frac{\left(a^3\,d\,\sqrt{b^3}-a\,b^2\,d\,\sqrt{b^3}\right)(a-b)}{b^2\,(a+b)^2\,(a^2+b\,a)\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}\,\sqrt{a^3\,d^2\,(a+b)^2}}\right) + \frac{(a-b)\left(a^3 \cdots\right.}{b^2\,(a+b)^2\,(a^2+b\,a) \cdots}\right.\right.}{\sqrt{a \cdots}} - \frac{2}{a\,d\,\left(\mathrm{e}^{2c+2dx}-1\right)}$$

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) - (atan((exp(2*c)*exp(2*d*x)*((4*b^2)/(a*d*(a + b)^3*(a*b + a^2)*
(b^3)^(1/2)) + ((a^3*d*(b^3)^(1/2) - a*b^2*d*(b^3)^(1/2))*(a - b))/(b^2*(a
+ b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)*(a^3*d^2*(a
+ b)^2)^(1/2))) + ((a - b)*(a^3*d*(b^3)^(1/2) + a*b^2*d*(b^3)^(1/2) + 2*a^2
*b*d*(b^3)^(1/2)))/(b^2*(a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*
b^2*d^2)^(1/2)*(a^3*d^2*(a + b)^2)^(1/2)))*((a^3*(a^5*d^2 + 2*a^4*b*d^2 + a
^3*b^2*d^2)^(1/2))/2 + (a*b^2*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2))/
2 + a^2*b*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2))/(a^5*d
^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) - 1))
```

3.178 $\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\coth^2(c+dx)}{2ad}+\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)d}$$

```
[Out] -1/2*coth(d*x+c)^2/a/d+ln(cosh(d*x+c))/(a+b)/d+(a-b)*ln(tanh(d*x+c))/a^2/d+
1/2*b^2*ln(a+b*tanh(d*x+c)^2)/a^2/(a+b)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 84}

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\coth^2(c+dx)}{2ad}$$

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*Coth[c + d*x]^2/(a*d) + Log[Cosh[c + d*x]]/((a + b)*d) + ((a - b)*Log[
Tanh[c + d*x]])/(a^2*d) + (b^2*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)*d
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)}+\frac{1}{ax^2}+\frac{a-b}{a^2x}+\frac{b^3}{a^2(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= -\frac{\coth^2(c+dx)}{2ad}+\frac{\log(\cosh(c+dx))}{(a+b)d}\\
&\quad+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{\coth^2(c+dx)}{a}-\frac{b^2\log(b+a\coth^2(c+dx))}{a^2(a+b)}-\frac{2\log(\sinh(c+dx))}{a+b}}{2d}$$

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
[Out] -1/2*(Coth[c + d*x]^2/a - (b^2*Log[b + a*Coth[c + d*x]^2])/(a^2*(a + b)) -
(2*Log[Sinh[c + d*x]])/(a + b))/d
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)-2\ln(1-\tanh(dx+c))a^2+(2a^2-2b^2)\ln(\tanh(dx+c))-\coth(dx+c)^2a(a+b)-2a^2dx}{2d\,a^2(a+b)}$
derivativedivides	$-\frac{-\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)a^2}+\frac{(-a+b)\ln(\tanh(dx+c))}{a^2}+\frac{1}{2a\tanh(dx+c)^2}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
default	$-\frac{-\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)a^2}+\frac{(-a+b)\ln(\tanh(dx+c))}{a^2}+\frac{1}{2a\tanh(dx+c)^2}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{da}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}-\frac{2b^2x}{a^2(a+b)}-\frac{2b^2c}{d\,a^2(a+b)}-\frac{2\,e^{2dx+2c}}{da\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)}{da}-\frac{\ln\left(e^{2dx+2c}-\right.}{d\,a^2}$

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*(b^2*ln(a+b*tanh(d*x+c)^2)-2*ln(1-tanh(d*x+c))*a^2+(2*a^2-2*b^2)*ln(tan
h(d*x+c))-coth(d*x+c)^2*a*(a+b)-2*a^2*d*x)/d/a^2/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(81) = 162.

Time = 0.34 (sec) , antiderivative size = 747, normalized size of antiderivative = 8.79

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx =$$

$$-\frac{2\,a^2dx\cosh(dx+c)^4+8\,a^2dx\cosh(dx+c)\sinh(dx+c)^3+2\,a^2dx\sinh(dx+c)^4+2\,a^2dx-4\,(a^2dx-}{}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^4 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*a^2*d*x*sinh(d*x + c)^4 + 2*a^2*d*x - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x +
 c)^2 + 4*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^2
 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x
 + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*
log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x
 + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^2 - b^2
)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^
2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(
d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d
*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a
^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x + c)^4 +
4*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + a^2*b)*d*sinh(d*x
+ c)^4 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + a^2*b)*d*cosh(d*x
+ c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^2 + (a^3 + a^2*b)*d + 4*((a^3 + a^2
*b)*d*cosh(d*x + c)^3 - (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^3\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+a^2b)d} + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,ae^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}-a)d} + \frac{(a-b)\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{(a-b)\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + a^2*b)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*a*e^(-2*
d*x - 2*c) - a*e^(-4*d*x - 4*c) - a)*d) + (a - b)*log(e^(-d*x - c) + 1)/(a^
2*d) + (a - b)*log(e^(-d*x - c) - 1)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{b^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^3+a^2b}-\frac{2\,(dx+c)}{a+b}+\frac{2\,(a-b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2}-\frac{4\,e^{(2\,dx+2\,c)}}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a^3 + a^2*b) - 2*(d*x + c)/(a + b) + 2*(a - b
)*log(abs(e^(2*d*x + 2*c) - 1))/a^2 - 4*e^(2*d*x + 2*c)/(a*(e^(2*d*x + 2*c)
 - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.68

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\,\ln\left(3\,a\,b^2-2\,a^2\,b-2\,a^3+3\,b^3-4\,a^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-2\,a^3\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-6\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+3\,b^3\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+6\,a\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2}\right.}{2\,d\,a^3+2\,b\,d\,a^2} - \frac{x}{a+b} - \frac{2}{a\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(4\,a^4\,b+9\,b^5-12\,a^2\,b^3-9\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-4\,a^4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+12\,a^2\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a-b)}{a^2\,d} - \frac{2\,\left(a^2+b\,a\right)}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,(a+b)}$$

```
[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (b^2*log(3*a*b^2 - 2*a^2*b - 2*a^3 + 3*b^3 - 4*a^3*exp(2*c)*exp(2*d*x) - 2*
a^3*exp(4*c)*exp(4*d*x) - 6*b^3*exp(2*c)*exp(2*d*x) + 3*b^3*exp(4*c)*exp(4*
d*x) + 6*a*b^2*exp(2*c)*exp(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 3*a*b^2*
exp(4*c)*exp(4*d*x) - 2*a^2*b*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a^2*b*d) -
```

```
 x/(a + b) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(4*a
^4*b + 9*b^5 - 12*a^2*b^3 - 9*b^5*exp(2*c)*exp(2*d*x) - 4*a^4*b*exp(2*c)*ex
p(2*d*x) + 12*a^2*b^3*exp(2*c)*exp(2*d*x))*(a - b))/(a^2*d) - (2*(a*b + a^2
))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

3.179 $\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] x/(a+b)+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)/d-(a-b)*c
oth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3751, 491, 597, 536, 212, 211}

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d(a+b)} - \frac{(a-b)\coth(c+dx)}{a^2d}+\frac{x}{a+b}-\frac{\coth^3(c+dx)}{3ad}$$

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a +
 b)*d) - ((a - b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{3(a-b)+3bx^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{3ad} \\
&= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} - \frac{\text{Subst}\left(\int \frac{-3(a^2-ab+b^2)-3(a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{3a^2d} \\
&= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} \\
&\quad + \frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{a^2(a+b)d} \\
&= \frac{x}{a+b} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{6\left(c+dx+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}\right)}{a+b} - \frac{(-2a+3b+(4a-3b)\cosh(2(c+dx)))\coth(c+dx)\text{csch}^2(c+dx)}{a^2}}{6d}$$

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((6*(c + d*x + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2)))/
(a + b) - ((-2*a + 3*b + (4*a - 3*b)*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[
c + d*x]^2)/a^2)/(6*d)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{-\frac{b^3 \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a^2\sqrt{ab}}-\frac{-a+b}{a^2\tanh(dx+c)}+\frac{1}{3a\tanh(dx+c)^3}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
default	$-\frac{-\frac{b^3 \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a^2\sqrt{ab}}-\frac{-a+b}{a^2\tanh(dx+c)}+\frac{1}{3a\tanh(dx+c)^3}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2(6a\,\mathrm{e}^{4dx+4c}-3b\,\mathrm{e}^{4dx+4c}-6\,\mathrm{e}^{2dx+2c}a+6b\,\mathrm{e}^{2dx+2c}+4a-3b)}{3d\,a^2\left(\mathrm{e}^{2dx+2c}-1\right)^3}+\frac{\sqrt{-ab}\,b^2\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^3(a+b)d}-\frac{\sqrt{-a}}{}$

[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] -1/d*(-b^3/(a+b)/a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))-1/a^2*(-a+b)/tanh(d*x+c)+1/3/a/tanh(d*x+c)^3-1/(2*a+2*b)*ln(tanh(d*x+c)+1)+1/(2*a+2*b)*ln(tanh(d*x+c)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. 2(72) = 144.

Time = 0.31 (sec) , antiderivative size = 2368, normalized size of antiderivative = 28.88

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/6*(6*a^2*d*x*cosh(d*x + c)^6 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 6*a^2*d*x*sinh(d*x + c)^6 - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 6*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^2)*sinh(d*x + c)^4 - 6*a^2*d*x + 24*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c)^2 + 6*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2

```
+ a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*
b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*s
qrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 16*a^2 - 4*a*b + 12*b^2 + 12*(3*
a^2*d*x*cosh(d*x + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x +
c)^3 + (3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^
2*b)*d*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 +
(a^3 + a^2*b)*d*sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*
(a^3 + a^2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3
 + a^2*b)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3
 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x
+ c)^4 - 6*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)
^2 - (a^3 + a^2*b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)
*d*cosh(d*x + c)^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*
a^2*d*x*cosh(d*x + c)^6 + 18*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 3*a^2*
d*x*sinh(d*x + c)^6 - 3*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4
 + 3*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^2)*sinh(
d*x + c)^4 - 3*a^2*d*x + 12*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x + 4*a^2
 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(3*a^2*d*x + 4*a^2 - 4
*b^2)*cosh(d*x + c)^2 + 3*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 6*(3*a^
2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sinh(d*x +
c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*s
inh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*si
nh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x +
c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*c
osh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
 c)^2 + a - b)*sqrt(b/a)/b) - 8*a^2 - 2*a*b + 6*b^2 + 6*(3*a^2*d*x*cosh(d*x
 + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (3*a^2*d*
x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x
+ c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*d*
sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*(a^3 + a^2*b)*d*
cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3 + a^2*b)*d*cosh
(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3 + a^2*b)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^4 - 6*(a^3
+ a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)^2 - (a^3 + a^2*
b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)
^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(72) = 144.

Time = 0.42 (sec) , antiderivative size = 1038, normalized size of antiderivative = 12.66

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] -1/8*(a*b - b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) +
a + b)/((a^3 + a^2*b)*d) + 1/8*(a*b - b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) +
 (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + a^2*b)*d) + 1/16*(a^2*b - 6*a*b^
2 + b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3 + a^
2*b)*sqrt(a*b)*d) - 1/16*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*
d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + a^2*b)*sqrt(a*b)*d) - 1/24*(3*(12*a
- b)*e^(4*d*x + 4*c) - 6*(9*a - b)*e^(2*d*x + 2*c) + 22*a - 3*b)/((a^2*e^(6
*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - a^2)*d) - 1/6
*(3*(4*a - b)*e^(4*d*x + 4*c) - 6*(2*a - b)*e^(2*d*x + 2*c) + 4*a - 3*b)/((
a^2*e^(6*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - a^2)*
d) - 1/24*(6*(9*a - b)*e^(-2*d*x - 2*c) - 3*(12*a - b)*e^(-4*d*x - 4*c) - 2
2*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) + a^2*e^(-6*d*
x - 6*c) - a^2)*d) - 1/6*(6*(2*a - b)*e^(-2*d*x - 2*c) - 3*(4*a - b)*e^(-4*
d*x - 4*c) - 4*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) +
 a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*(6*(a + b)*e^(-2*d*x - 2*c) - 3*b*e^(
-4*d*x - 4*c) - 2*a - 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c
) + a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*
(a - b)*e^(2*d*x + 2*c) + a + b)/(a^2*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x -
2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^2*d) + 1/4*(2*a - b)*log(e^(2*d
*x + 2*c) - 1)/(a^2*d) - 1/2*b*log(e^(2*d*x + 2*c) - 1)/(a^2*d) - 1/4*(2*a
- b)*log(e^(-2*d*x - 2*c) - 1)/(a^2*d) + 1/2*b*log(e^(-2*d*x - 2*c) - 1)/(a
^2*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a
*b))/(sqrt(a*b)*a^2*d) - 3/8*(a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*
c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/4*(a*b - b^2)*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.79

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+a^2b)\sqrt{ab}}+\frac{3\,(dx+c)}{a+b}-\frac{2\,\left(6\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+4\,a-3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^3 + a^2*b)*sqrt(a*b)) + 3*(d*x + c)/(a + b) - 2*(6*a*e^(4*d*x + 4
*c) - 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 4*a
 - 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 519, normalized size of antiderivative = 6.33

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}$$
$$+\frac{\mathrm{atan}\left(\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{4\,b^3}{a^2\,d\,(a+b)^3\,(a^3+b\,a^2)\,\sqrt{b^5}}+\frac{\left(a^4\,d\,\sqrt{b^5}-a^2\,b^2\,d\,\sqrt{b^5}\right)(a-b)}{b^3\,(a+b)^2\,(a^3+b\,a^2)\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}\,\sqrt{a^5\,d^2\,(a+b)^2}}\right)+\frac{(a-b)\,(a\cdots}{b^3\,(a+b)^2\,(a^3+b\,a\cdots}\right.\right.}{\sqrt{\cdots}}$$
$$-\frac{8}{3\,a\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{4\,(a^2+b\,a)}{a^2\,d\,(a+b)\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,(2\,a^2+a\,b-b^2)}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,(a+b)}$$

```
[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) + (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^2*d*(a + b)^3*(a^2*b + a
^3)*(b^5)^(1/2)) + ((a^4*d*(b^5)^(1/2) - a^2*b^2*d*(b^5)^(1/2))*(a - b))/(b
^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2)*(a^5
*d^2*(a + b)^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)^(1/
2) + a^2*b^2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*
b*d^2 + a^5*b^2*d^2)^(1/2)*(a^5*d^2*(a + b)^2)^(1/2)))*((a^4*(a^7*d^2 + 2*a
^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2 + a^3*b*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d
^2)^(1/2) + (a^2*b^2*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2))*(b^5)
^(1/2))/(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2) - 8/(3*a*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(a*b + a^2))/(a^
2*d*(a + b)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*(a*b + 2*a^2
- b^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

3.180 $\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2d} - \frac{a^2}{2b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d-1/2*a*(a+2*b)*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)^2/d
-1/2*a^2/b^2/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{a^2}{2b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) - (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])
/(2*b^2*(a + b)^2*d) - a^2/(2*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^2} - \frac{a(a+2b)}{b(a+b)^2(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^2d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2d} - \frac{a^2}{2b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-2\log(\cosh(c+dx)) + \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{b^2} + \frac{a^2(a+b)}{b^2\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2d}$$

[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/2*(-2*Log[Cosh[c + d*x]] + (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/b^2 + (a^2*(a + b))/(b^2*(a + b*Tanh[c + d*x]^2)))/((a + b)^2*d)

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{(a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b^2}+\frac{a(a+b)}{b^2\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{a\left(\frac{(a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b^2}+\frac{a(a+b)}{b^2\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{2x\tanh(dx+c)^2b^3d+2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^2a\,b^2+\ln\left(a+b\tanh(dx+c)^2\right)a^3+a^3+a^2b+2a\,b^2dx+2\ln(1-}{2\left(a+b\tanh\right.}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{b^2}-\frac{2c}{b^2d}+\frac{2a^2x}{b^2(a^2+2ab+b^2)}+\frac{2a^2c}{b^2d(a^2+2ab+b^2)}+\frac{4ax}{b(a^2+2ab+b^2)}+\frac{4ac}{bd(a^2+2ab+b^2)}-\frac{}{bd(a+}$

[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2*a/(a+b)^2*((a+2*b)/b^2*ln(a+b*tanh(d*x+c)^2)+a*(a+b)/b^2/(a+b*tanh(d*x+c)^2))-1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/(a+b)^2*ln(tanh(d*x+c)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.75

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 8*(a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b^2 + b^3)*d*x*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*d*x + 4*(a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^2*b + (a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - 2*a*b^2 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4

```
*((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (a^3 + a^2*b - 2*a*b^2)*cosh(
d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x
 + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 8*((a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + (a^2*b + (a*b^2 - b^3)*d*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x +
 c)^4 + 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(a^3*b^2
 + a^2*b^3 - a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3
*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*d)*sinh
(d*x + c)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d + 4*((a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 74.91 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{a^2\left(\begin{cases}\frac{\tanh^2(c+dx)}{a^2} & \text{for } b=0\\ -\frac{1}{b(a+b\tanh^2(c+dx))} & \text{otherwise}\end{cases}\right)}{2bd(a+b)} - \frac{a(a+2b)\left(\begin{cases}\frac{\tanh^2(c+dx)}{a} & \text{for } b=0\\ \frac{\log(a+b\tanh^2(c+dx))}{b} & \text{otherwise}\end{cases}\right)}{2bd(a+b)^2} - \frac{\log\left(\tanh^2(c+dx)-1\right)}{2d(a+b)^2}$$

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] a**2*Piecewise((tanh(c + d*x)**2/a**2, Eq(b, 0)), (-1/(b*(a + b*tanh(c + d*
x)**2)), True))/(2*b*d*(a + b)) - a*(a + 2*b)*Piecewise((tanh(c + d*x)**2/a
, Eq(b, 0)), (log(a + b*tanh(c + d*x)**2)/b, True))/(2*b*d*(a + b)**2) - lo
g(tanh(c + d*x)**2 - 1)/(2*d*(a + b)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.61

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2\,a^2e^{(-2\,dx-2\,c)}}{(a^3b+3\,a^2b^2+3\,ab^3+b^4+2\,(a^3b+a^2b^2-ab^3-b^4)e^{(-2\,dx-2\,c)}+(a^3b+3\,a^2b^2+3\,ab^3+b^4)e^{(-4\,dx-4\,c)})} - \frac{(a^2+2\,ab)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2b^2+2\,ab^3+b^4)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2*a^2*e^(-2*d*x - 2*c)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + 2*(a^3*b + a^
2*b^2 - a*b^3 - b^4)*e^(-2*d*x - 2*c) + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*e^(-4*d*x - 4*c))*d) - 1/2*(a^2 + 2*a*b)*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*d) + (d*x + c)
/((a^2 + 2*a*b + b^2)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(79) = 158$.

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.34

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(a^2+2\,ab)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2b^2+2\,ab^3+b^4} + \frac{2\,(dx+c)}{a^2+2\,ab+b^2} + \frac{4\,a^2e^{(2\,dx+2\,c)}}{(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2}}{2\,d}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a^2 + 2*a*b)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^2*b^2 + 2*a*b^3 + b^4) + 2*(d*x
+ c)/(a^2 + 2*a*b + b^2) + 4*a^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b*e^
(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)^
2*b) - 2*log(e^(2*d*x + 2*c) + 1)/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= -\frac{a^2}{2\,\left(d\,a^2\,b^2+d\,a\,b^3\,\tanh\left(c+d\,x\right)^2+d\,a\,b^3+d\,b^4\,\tanh\left(c+d\,x\right)^2\right)} - \frac{\ln\left(\tanh(c+d\,x)^2-1\right)}{2\,\left(d\,a^2+2\,d\,a\,b+d\,b^2\right)} - \frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,\left(d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4\right)} - \frac{a\,b\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4}$$

```
[In] int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] - a^2/(2*(a^2*b^2*d + b^4*d*tanh(c + d*x)^2 + a*b^3*d + a*b^3*d*tanh(c + d*
x)^2)) - log(tanh(c + d*x)^2 - 1)/(2*(a^2*d + b^2*d + 2*a*b*d)) - (a^2*log(
a + b*tanh(c + d*x)^2))/(2*(b^4*d + a^2*b^2*d + 2*a*b^3*d)) - (a*b*log(a +
b*tanh(c + d*x)^2))/(b^4*d + a^2*b^2*d + 2*a*b^3*d)
```

3.181 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} \, dx = \frac{x}{(a+b)^2} - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2d} + \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2-1/2*(a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b^(3/2)/(
a+b)^2/d+1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 481, 536, 212, 211}

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} \, dx = -\frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}d(a+b)^2} + \frac{a\tanh(c+dx)}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)} + \frac{x}{(a+b)^2}$$

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*b^(3/2)*(a + b)^2*d) + (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2b(a+b)d}$$

$$= \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} - \frac{(a(a+3b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2b(a+b)^2d}$$

$$= \frac{x}{(a+b)^2} - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2d} + \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2(c+dx) - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{a(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{2(a+b)^2d}$$

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
b^(3/2) + (a*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x
)])))/(2*(a + b)^2*d)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2} - \frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\tanh(dx+c)^2\right)} + \frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2} + \frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2} - \frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\tanh(dx+c)^2\right)} + \frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2} + \frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2} - \frac{a\left(e^{2dx+2c}a-b\,e^{2dx+2c}+a+b\right)}{d(a+b)^2b\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)} + \frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)a}{4b^2(a+b)^2d}$

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-a/(a+b)^2*(-1/2*(a+b)/b*tanh(d*x+c)/(a+
b*tanh(d*x+c)^2)+1/2*(a+3*b)/b/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)
))+1/2/(a+b)^2*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(77) = 154$.

Time = 0.32 (sec) , antiderivative size = 1950, normalized size of antiderivative = 21.91

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b + b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b + b^2)*d*x + 4
*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 4*(6*(a*b + b^2)*d*x*cos
h(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c)^2 + ((a^2 + 4*a
*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b - 3*b^2)*
cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
- 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 + 4*a*b + 3*b^2)*c
osh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)
*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c
)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)
^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a^2 - 4*a*b +
 8*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^2)*d*x - a^2 + a*b)*cos
h(d*x + c))*sinh(d*x + c))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x
+ c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b + a^
2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*d*cosh(d*x + c)^2 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d)*sinh(d*x + c)
^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*d*cosh(d*x + c)^3 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c
))*sinh(d*x + c)), 1/2*(2*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 2*(a
*b + b^2)*d*x + 2*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 2*(6*(a
*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c
)^2 - ((a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2
+ 2*a*b - 3*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)
^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 +
```

```
 4*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a/b)/a) - 2*a
^2 - 2*a*b + 4*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^2)*d*x - a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*cosh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2
*(a^3*b + a^2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d)*s
inh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d + 4*((a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(77) = 154$.

Time = 0.52 (sec) , antiderivative size = 1010, normalized size of antiderivative = 11.35

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) +
 a - b)/sqrt(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/32*(a^3 +
9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/16*(a^3 - 5*a^2*b - 5*
a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^(2*d*x + 2*c))/((a^4*b +
3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*e^(
4*d*x + 4*c) + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*e^(2*d*x + 2*c))*d) +
1/16*(a^3 - 5*a^2*b - 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^(
-2*d*x - 2*c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + 2*(a^4*b + a^3*b^2
 - a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b
^4)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(2*d*x +
```

```
2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + (a^3*b + 2*a^2*b^2 + a*b^3)*e^(4*d*x +
4*c) + 2*(a^3*b - a*b^3)*e^(2*d*x + 2*c))*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*
b + b^2)*e^(-2*d*x - 2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + 2*(a^3*b - a*b^3)*
e^(-2*d*x - 2*c) + (a^3*b + 2*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) + 3/8*(
(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(-2
*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log((a + b)*e^(4*d
*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/
4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2
+ 2*a*b + b^2)*d) - 1/8*(a + b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b
)/sqrt(a*b))/(sqrt(a*b)*a*b*d) + 1/8*(a + b)*arctan(1/2*((a + b)*e^(-2*d*x
- 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d) + 3/16*(a - b)*arctan(1/2*((a
+ b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(a^2+3\,ab)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2b+2\,ab^2+b^3)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2+2\,ab+b^2}+\frac{2\,\left(a^2e^{(2\,dx+2\,c)}-abe^{(2\,dx+2\,c)}+a^2+ab\right)}{(a^2b+2\,ab^2+b^3)\big(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}}}{2\,d}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a^2 + 3*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
 b)/sqrt(a*b))/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a
*b + b^2) + 2*(a^2*e^(2*d*x + 2*c) - a*b*e^(2*d*x + 2*c) + a^2 + a*b)/((a^2
*b + 2*a*b^2 + b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 1655, normalized size of antiderivative = 18.60

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^2*d + 2*b^2*d + 4*a*b*d) - log(tanh(c + d*x) -
1)/(2*d*(a + b)^2) - (atan((((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^
```

```
3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) + (
(a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8
*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^
2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2
 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(
b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b
^5*d + a^2*b^3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + (
(a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2
))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a + 3*b)*(-a*b^3)^(1/2)*((2*
a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)
/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) + (tanh(c + d*x)*(a +
3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^
5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d
)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)
))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))/((3*a*b^2 + (5*a^2*b)/2 + a^3/2
)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - ((a + 3*b)*(-a*b^3)
^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2
*a*b^2*d^2 + a^2*b*d^2)) + ((a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*
b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^
3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2
)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4
*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b
^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d))))/(4*(b^5*d + a^
2*b^3*d + 2*a*b^4*d)) + ((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b
+ a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a +
 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4
*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^
3) + (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 3
2*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*
d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d
 + a^2*b^3*d + 2*a*b^4*d))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d))))*(a + 3*b
)*(-a*b^3)^(1/2)*1i)/(2*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + (a*tanh(c + d*x)
)/(2*b*(a + b)*(a*d + b*d*tanh(c + d*x)^2))
```

3.182 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}+\frac{a}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d+1/2*a/b/(a+b)
/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 78}

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{a}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) + a/(2*b*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}-\frac{a}{(a+b)(a+bx)^2}+\frac{b}{(a+b)^2(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}+\frac{a}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2\log(\cosh(c+dx))+\log\left(a+b\tanh^2(c+dx)\right)+\frac{a(a+b)}{b(a+b\tanh^2(c+dx))}}{2(a+b)^2d}$$

[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2] + (a*(a + b))/(b*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{2abdx-ab+2\ln(1-\tanh(dx+c))ab-a^2-b^2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^2-\ln\left(a+b\tanh(dx+c)^2\right)ab+2x\tanh(dx}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2b}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}+\frac{2a\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}+\frac{2(}{}\right.}{2d(a^2+}$

[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/2/(a+b)^2*(-a*(a+b)/b/(a+b*tanh(d*x+c)^2)-ln(a+b*tanh(d*x+c)^2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 629, normalized size of antiderivative = 8.74

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx =$$

$$-\frac{2\,(a+b)dx\cosh\left(dx+c\right)^4+8\,(a+b)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,(a+b)dx\sinh\left(dx+c\right)^4+2\,(a}{2\,\left((a^3+3\,a^2b+\right.}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
+ c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x - a
)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x - a)*sin
h(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
 - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2
+ (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*
x - a)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos
h(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b
- a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 42.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{a\left(\begin{cases}\frac{\tanh^2(c+dx)}{a^2} & \text{for}\ b=0\\ -\frac{1}{b(a+b\tanh^2(c+dx))} & \text{otherwise}\end{cases}\right)}{2d\,(a+b)} + \frac{b\left(\begin{cases}\frac{\tanh^2(c+dx)}{a} & \text{for}\ b=0\\ \frac{\log(a+b\tanh^2(c+dx))}{b} & \text{otherwise}\end{cases}\right)}{2d\,(a+b)^2} - \frac{\log\left(\tanh^2(c+dx)-1\right)}{2d\,(a+b)^2}$$

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] -a*Piecewise((tanh(c + d*x)**2/a**2, Eq(b, 0)), (-1/(b*(a + b*tanh(c + d*x)
**2)), True))/(2*d*(a + b)) + b*Piecewise((tanh(c + d*x)**2/a, Eq(b, 0)), (
log(a + b*tanh(c + d*x)**2)/b, True))/(2*d*(a + b)**2) - log(tanh(c + d*x)*
*2 - 1)/(2*d*(a + b)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(68) = 136$.

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2\,ae^{(-2\,dx-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)}\right)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*a*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b
^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*
c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x -
2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2} - \frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)(a}}{2\,d}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) + e^
(-2*d*x - 2*c) - 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.92

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-a^2+a\,b\left(-1+\operatorname{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+d\,x)^2\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^2+b\tanh(c+d\,x)^2}\right)\,2\mathrm{i}\right)+b^2\tanh(c+d\,x)^2\operatorname{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c\text{-}}{2\,a-a\tanh(c+d\,x)^2+b\tanh}\right.}{2\,d\,a^3\,b+2\,d\,a^2\,b^2\tanh\left(c+d\,x\right)^2+4\,d\,a^2\,b^2+4\,d\,a\,b^3\tanh\left(c+d\,x\right)^2+2\,d\,a\,b^3+2\,d\,b^4\tanh\left(c+\right.}$$

```
[In] int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] -(a*b*(atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c +
 d*x)^2 + b*tanh(c + d*x)^2))*2i - 1) - a^2 + b^2*tanh(c + d*x)^2*atan((a*t
anh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh
(c + d*x)^2))*2i)/(4*a^2*b^2*d + 2*b^4*d*tanh(c + d*x)^2 + 2*a*b^3*d + 2*a^
3*b*d + 2*a^2*b^2*d*tanh(c + d*x)^2 + 4*a*b^3*d*tanh(c + d*x)^2)
```

3.183 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{x}{(a+b)^2}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2d}-\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2-1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)/b
^(1/2)-1/2*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 482, 536, 212, 211}

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d(a+b)^2}-\frac{\tanh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - ((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*Sqrt[a]*
Sqrt[b]*(a + b)^2*d) - Tanh[c + d*x]/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}\end{aligned}$$

$$= -\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^2 d}$$

$$= \frac{x}{(a+b)^2} - \frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2 d} - \frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2(c+dx) + \frac{(-a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} - \frac{(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2(a+b)^2 d}$$

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) + ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*
Sqrt[b]) - ((a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))
/(2*(a + b)^2*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2} - \frac{\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2} + \frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2} - \frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2} - \frac{\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2} + \frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2} - \frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2} + \frac{e^{2dx+2c}a - b\,e^{2dx+2c} + a + b}{d(a+b)^2\left(a\,e^{4dx+4c} + b\,e^{4dx+4c} + 2\,e^{2dx+2c}a - 2b\,e^{2dx+2c} + a + b\right)} - \frac{\ln\left(e^{2dx+2c} + \frac{a\sqrt{-ab} - b\sqrt{-ab} + 2ab}{(a+b)\sqrt{-ab}}\right)a}{4\sqrt{-ab}\,(a+b)^2 d} +$

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/(a+b)^2*((1/2*b+1/2*a)*tanh(d*x+c)/(a+
b*tanh(d*x+c)^2)+1/2*(a-b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))-1
/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(73) = 146$.

Time = 0.31 (sec) , antiderivative size = 2025, normalized size of antiderivative = 23.82

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b +
 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x
)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b - a*b^
2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 + ((a^2 - b^2)*cosh(d*x + c)^4 +
 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4
+ 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2
+ a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x
+ c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x
+ c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
- b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b
)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) + 8*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3
+ (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*
b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b
^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 -
a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*c
osh(d*x + c)^2 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^
3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^
2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh
(d*x + c)^4 + 2*a^2*b + 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b - a*b^2
+ 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*
x + c)^2 + a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 - ((a^2 -
 b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
- b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
- b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4
```

```
*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(
2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*
d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x +
c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^4*b + a^3*b^2 - a
^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b
 + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(73) = 146$.

Time = 0.42 (sec) , antiderivative size = 614, normalized size of antiderivative = 7.22

$$\begin{aligned}
\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx &= -\frac{(a^2-4\,ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d} \\
&+ \frac{(a^2-4\,ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d} \\
&+ \frac{a^2-b^2+(a^2-6\,ab+b^2)e^{(2\,dx+2\,c)}}{4\,(a^4+3\,a^3b+3\,a^2b^2+ab^3+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(4\,dx+4\,c)}+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(2\,dx+2\,c)})d} \\
&- \frac{a^2-b^2+(a^2-6\,ab+b^2)e^{(-2\,dx-2\,c)}}{4\,(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)}} \\
&- \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{2\,(a^3+2\,a^2b+ab^2+2\,(a^3-ab^2)e^{(-2\,dx-2\,c)}+(a^3+2\,a^2b+ab^2)e^{(-4\,dx-4\,c)})d} \\
&+ \frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\
&- \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} + \frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}ad}
\end{aligned}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2 - 4*a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + 1/8*(a^2 - 4*a*b - b^2)*arcta
n(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2
)*sqrt(a*b)*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(2*d*x + 2*c))/((a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(4*
d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(a
^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^
2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a
^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) - 1/2*((a - b)*e^(-2*d*x - 2
*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2)*e^(-2*d*x - 2*c) + (
a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log((a + b)*e^(4*d*x + 4*
c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*log(2
*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b
 + b^2)*d) + 1/4*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{2\,\left(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b\right)}{(a^2+2\,ab+b^2)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+\right.}}{2\,d}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sq
rt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2)
- 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*(a
*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{(a+b)^2}-\frac{\tanh(c+d\,x)}{2\,a\,d+2\,b\,d}+\frac{b\,x\,\tanh(c+d\,x)^2}{(a+b)^2}}{b\tanh\left(c+d\,x\right)^2+a} - \frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,(a-b)}{\sqrt{a\,b}\,\left(2\,d\,a^2+4\,d\,a\,b+2\,d\,b^2\right)}$$

```
[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(a + b)^2 - tanh(c + d*x)/(2*a*d + 2*b*d) + (b*x*tanh(c + d*x)^2)/(a
 + b)^2)/(a + b*tanh(c + d*x)^2) - (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(a
- b))/((a*b)^(1/2)*(2*a^2*d + 2*b^2*d + 4*a*b*d))
```

3.184 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d} - \frac{1}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d-1/2/(a+b)/d/(
a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 46}

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{1}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) - 1/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{b}{(a+b)(a+bx)^2}+\frac{b}{(a+b)^2(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}-\frac{1}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-2\log(\cosh(c+dx))-\log\left(a+b\tanh^2(c+dx)\right)+\frac{a+b}{a+b\tanh^2(c+dx)}}{2(a+b)^2d}$$

[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/2*(-2*Log[Cosh[c + d*x]] - Log[a + b*Tanh[c + d*x]^2] + (a + b)/(a + b*Tanh[c + d*x]^2))/((a + b)^2*d)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{b\left(-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{b\left(-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{-\tanh(dx+c)^2ab-a^2\ln\left(a+b\tanh(dx+c)^2\right)-b^2\tanh(dx+c)^2+2a^2dx-\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^2ab+2\ln(}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2a}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}-\frac{2b\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}+\right.}{2d(a^2}$

[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2*b/(a+b)^2*(-(a+b)/b/(a+b*tanh(d*x+c)^2)+1/b*ln(a+b*tanh(d*x+c)^2)) -1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/(a+b)^2*ln(tanh(d*x+c)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 623, normalized size of antiderivative = 9.16

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2\,(a+b)dx\cosh\left(dx+c\right)^4+8\,(a+b)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,(a+b)dx\sinh\left(dx+c\right)^4+2\,(a}{2\,\left((a^3+3\,a^2b+3\right.}$$

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x + b)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x

```
+ c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
 - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2
+ (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*
x + b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos
h(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b
- a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 58.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b\left(\begin{cases}\frac{\tanh^2(c+dx)}{a^2} & \text{for } b=0\\ -\frac{1}{b(a+b\tanh^2(c+dx))} & \text{otherwise}\end{cases}\right)}{2d(a+b)} + \frac{b\left(\begin{cases}\frac{\tanh^2(c+dx)}{a} & \text{for } b=0\\ \frac{\log(a+b\tanh^2(c+dx))}{b} & \text{otherwise}\end{cases}\right)}{2d(a+b)^2} - \frac{\log\left(\tanh^2(c+dx)-1\right)}{2d(a+b)^2}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] b*Piecewise((tanh(c + d*x)**2/a**2, Eq(b, 0)), (-1/(b*(a + b*tanh(c + d*x)*
*2)), True))/(2*d*(a + b)) + b*Piecewise((tanh(c + d*x)**2/a, Eq(b, 0)), (l
og(a + b*tanh(c + d*x)**2)/b, True))/(2*d*(a + b)**2) - log(tanh(c + d*x)**
2 - 1)/(2*d*(a + b)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(64) = 128$.

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.50

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2\,be^{(-2\,dx-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)}\right)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2*b*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*
b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4
*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x -
 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(64) = 128$.

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.19

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2} - \frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)}}{2\,d}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) + e^
(-2*d*x - 2*c) + 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{a^2+2\,a\,b+b^2}+\frac{b\,x\,\tanh(c+d\,x)^2}{a^2+2\,a\,b+b^2}+\frac{b\,\tanh(c+d\,x)^2}{2\,a\,d\,(a+b)}}{b\,\tanh\left(c+d\,x\right)^2+a} + \frac{\ln\left(b\,\tanh(c+d\,x)^2+a\right)}{2\,d\,\left(a^2+2\,a\,b+b^2\right)} - \frac{\ln\left(\tanh(c+d\,x)+1\right)}{d\,\left(a+b\right)^2}$$

```
[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(2*a*b + a^2 + b^2) + (b*x*tanh(c + d*x)^2)/(2*a*b + a^2 + b^2) + (b
*tanh(c + d*x)^2)/(2*a*d*(a + b)))/(a + b*tanh(c + d*x)^2) + log(a + b*tanh
(c + d*x)^2)/(2*d*(2*a*b + a^2 + b^2)) - log(tanh(c + d*x) + 1)/(d*(a + b)^
2)
```

3.185 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2}+\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2d}+\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2+1/2*(3*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/(
a+b)^2/d+1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3742, 425, 536, 212, 211}

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^2}+\frac{b\tanh(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a + b)^2 + (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*a^(3/2)*(a + b)^2*d) + (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} + \frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)^2 d}$$

$$= \frac{x}{(a+b)^2} + \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2 d} + \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \log(1-\tanh(c+dx)) + \log(1+\tanh(c+dx)) + \frac{b(a+b)\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2 d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

```
[Out] ((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) - Log[
1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]] + (b*(a + b)*Tanh[c + d*x])/(a*
(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{b\left(e^{2dx+2c}a-b\,e^{2dx+2c}+a+b\right)}{d(a+b)^2a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{4a(a+b)^2d}$

```
[In] int(1/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a+b)^2*ln(tanh(d*x+c)+1)+b/(a+b)^2*(1/2*(a+b)/a*tanh(d*x+c)/(a+b*
tanh(d*x+c)^2)+1/2*(3*a+b)/a/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))
-1/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(77) = 154$.

Time = 0.32 (sec) , antiderivative size = 1942, normalized size of antiderivative = 21.82

$$\int \frac{1}{(a + b\tanh^2(c + dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4
*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*cos
h(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c)^2 + ((3*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b - b^2)*
cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*
b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2 + 4*a*b + b^2)*c
osh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)
*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c
)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)
^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a*b - 4*b^2 +
 8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*b)*d*x - a*b + b^2)*cos
h(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x
+ c)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^4 + a^3*
b - a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*cosh(d*x + c)^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d)*sinh(d*x + c)
^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 4*((a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*d*cosh(d*x + c)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d*cosh(d*x + c
))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d
*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a
^2 + a*b)*d*x + 2*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 2*(6*(a
^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c
)^2 + ((3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^
2 - 2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)
^2 + 3*a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2
```

```
 + 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 2*a
*b - 2*b^2 + 4*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*b)*d*x - a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*d*cosh(d*x + c)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2
*(a^4 + a^3*b - a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d)*s
inh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 4*((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**(-2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(77) = 154.

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d} + \frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)}} + \frac{dx+c}{(a^2+2\,ab+b^2)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + (a*b + b^2 + (a*b - b^2)*e^(-2*d*x
 - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a
*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*
c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{1}{\left(a + b\tanh^2(c + dx)\right)^2}\,dx$$
$$= \frac{\frac{(3\,ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}} + \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{2\,\left(abe^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+ab+b^2\right)}{(a^3+2\,a^2b+ab^2)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 2*(d*x + c)/(a^2 + 2*a*
b + b^2) - 2*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)/((a^3
+ 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a + b\tanh^2(c + dx)\right)^2}\,dx = \frac{\frac{a\,x}{(a+b)^2} + \frac{b\,x\tanh(c+d\,x)^2}{(a+b)^2} + \frac{b\tanh(c+d\,x)}{2\,a\,d\,(a+b)}}{b\tanh\left(c + d\,x\right)^2 + a} + \frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(b^2 + 3\,a\,b\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d + a\,b\,\left(4\,a\,d + 2\,b\,d\right)\right)}$$

```
[In] int(1/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(a + b)^2 + (b*x*tanh(c + d*x)^2)/(a + b)^2 + (b*tanh(c + d*x))/(2*a
*d*(a + b)))/(a + b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))
*(3*a*b + b^2))/((a*b)^(1/2)*(2*a^3*d + a*b*(4*a*d + 2*b*d)))
```

3.186 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log(\tanh(c+dx))}{a^2d} - \frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)^2d} + \frac{b}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d+ln(tanh(d*x+c))/a^2/d-1/2*b*(2*a+b)*ln(a+b*tanh(d
*x+c)^2)/a^2/(a+b)^2/d+1/2*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 457, 84}

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)^2}+\frac{\log(\tanh(c+dx))}{a^2d} + \frac{b}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[Tanh[c + d*x]]/(a^2*d) - (b*(2*a + b
)*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)^2*d) + b/(2*a*(a + b)*d*(a + b
*Tanh[c + d*x]^2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{1}{a^2x} - \frac{b^2}{a(a+b)(a+bx)^2} - \frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{\log(\tanh(c+dx))}{a^2d} \\
&\quad - \frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)^2d} + \frac{b}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{2\log(\cosh(c+dx))}{(a+b)^2}+\frac{2\log(\tanh(c+dx))+\frac{b\left(-\left((2a+b)\log\left(a+b\tanh^2(c+dx)\right)\right)+\frac{a(a+b)}{a+b\tanh^2(c+dx)}\right)}{(a+b)^2}}{a^2}}{2d}$$

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((2*Log[Cosh[c + d*x]])/(a + b)^2 + (2*Log[Tanh[c + d*x]] + (b*(-((2*a + b)
*Log[a + b*Tanh[c + d*x]^2]) + (a*(a + b))/(a + b*Tanh[c + d*x]^2)))/(a + b
)^2)/a^2)/(2*d)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

method	result
derivativedivides	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c))}{a^2}+\frac{b^2\left(\frac{(2a+b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
default	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c))}{a^2}+\frac{b^2\left(\frac{(2a+b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
parallelrisch	$\frac{-2\left(a+b\tanh(dx+c)^2\right)b\left(a+\frac{b}{2}\right)\ln\left(a+b\tanh(dx+c)^2\right)+\left(-2\tanh(dx+c)^2a^2b-2a^3\right)\ln(1-\tanh(dx+c))+2(a+b)^2\left(a+b\right.}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2a^2}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{a^2}-\frac{2c}{a^2d}+\frac{4bx}{a(a^2+2ab+b^2)}+\frac{4bc}{da(a^2+2ab+b^2)}+\frac{2b^2x}{a^2(a^2+2ab+b^2)}+\frac{2b^2c}{d\,a^2(a^2+2ab+b^2)}+\frac{}{ad(a}$

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/a^2*ln(tanh(d*x+c))+1/2*b^2/(a+b)^2/a
^2*((2*a+b)/b*ln(a+b*tanh(d*x+c)^2)-a*(a+b)/b/(a+b*tanh(d*x+c)^2))+1/2/(a+b
)^2*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 1148, normalized size of antiderivative = 12.08

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 + 8*(a^3 + a^2*b)*d*x*cosh(d*x +
c)*sinh(d*x + c)^3 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^4 + 2*(a^3 + a^2*b)*
d*x - 4*(a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + a^2*b)*d*
x*cosh(d*x + c)^2 - a*b^2 + (a^3 - a^2*b)*d*x)*sinh(d*x + c)^2 + ((2*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)*sinh(d*x + c)^3 + (2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + 2*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b - a*b
^2 - b^3 + 3*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x
 + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 8*((a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x +
 c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^5 + a
^4*b - a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*sinh
(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2\,b^2 e^{(-2\,dx-2\,c)}}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)}\right)d} - \frac{(2\,ab+b^2)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^4+2\,a^3b+a^2b^2)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 2*b^2*e^(-2*d*x - 2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b
 - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
e^(-4*d*x - 4*c))*d) - 1/2*(2*a*b + b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (
a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + (d*x + c)/
((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) + 1)/(a^2*d) + log(e^(-d*x - c)
- 1)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.05

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(2\,ab+b^2)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^4+2\,a^3b+a^2b^2} + \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{4\,b^2e^{(2\,dx+2\,c)}}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\right.}}{2\,d}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^4 + 2*a^3*b + a^2*b^2) + 2*(d*x
+ c)/(a^2 + 2*a*b + b^2) - 4*b^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b*e^
(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)^
2*a) - 2*log(abs(e^(2*d*x + 2*c) - 1))/a^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.187 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{x}{(a+b)^2} - \frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2d} - \frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2-1/2*b^(3/2)*(5*a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)
/(a+b)^2/d-1/2*(2*a+3*b)*coth(d*x+c)/a^2/(a+b)/d+1/2*b*coth(d*x+c)/a/(a+b)/
d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 483, 597, 536, 212, 211}

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = -\frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^2} - \frac{(2a+3b)\coth(c+dx)}{2a^2d(a+b)} + \frac{b\coth(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)} + \frac{x}{(a+b)^2}$$

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

[Out] x/(a + b)^2 - (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(5/2)*(a + b)^2*d) - ((2*a + 3*b)*Coth[c + d*x])/(2*a^2*(a + b)*d) +
(b*Coth[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-3b+3bx^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d} \\
&= -\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{2a^2-2ab-3b^2+b(2a+3b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2a^2(a+b)d} \\
&= -\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} \\
&\quad - \frac{\left(b^2(5a+3b)\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2a^2(a+b)^2d} \\
&= \frac{x}{(a+b)^2} - \frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2d} \\
&\quad - \frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$
\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-\frac{2(c+dx)}{(a+b)^2} + \frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^2} + \frac{2\coth(c+dx)}{a^2} + \frac{b^2\sinh(2(c+dx))}{a^2(a+b)(a-b+(a+b)\cosh(2(c+dx)))}}{2d}
$$

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((-2*(c + d*x))/(a + b)^2 + (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^2) + (2*Coth[c + d*x])/a^2 + (b^2*Sinh
[2*(c + d*x)])/(a^2*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/d
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{1}{a^2\tanh(dx+c)}+\frac{b^2\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(5a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^2}}{d}$
default	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{1}{a^2\tanh(dx+c)}+\frac{b^2\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(5a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2a^3e^{4dx+4c}+6a^2b\,e^{4dx+4c}+5a\,b^2e^{4dx+4c}+3\,e^{4dx+4c}b^3+4a^3e^{2dx+2c}+4a^2b\,e^{2dx+2c}-4\,e^{2dx+2c}a\,b^2-6\,e^{2d\cdots}}{a^2d\left(e^{2dx+2c}-1\right)(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+\cdots\right.}$

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)+1)+1/2/(a+b)^2*ln(tanh(d*x+c)-1)+1/a^2/ta
nh(d*x+c)+b^2/(a+b)^2/a^2*((1/2*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/
2*(5*a+3*b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. $2(105) = 210$.

Time = 0.34 (sec) , antiderivative size = 3725, normalized size of antiderivative = 31.30

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 24*(a^3 + a^2*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^5 + 4*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 4*(2*a^3 + 6*a^
2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 +
 a^2*b)*d*x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3 - 3*
a^2*b)*d*x)*sinh(d*x + c)^4 + 16*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (2*
a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(
d*x + c)^3 - 8*a^3 - 24*a^2*b - 28*a*b^2 - 12*b^3 - 4*(a^3 + a^2*b)*d*x - 4
*(4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2
+ 4*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*b^2 + 6*b
^3 - (a^3 - 3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*
a^2*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 8*a*b^2 + 3*b^3)
*cosh(d*x + c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c
)^5 + (5*a^2*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b - 12*a*b^2 - 9
*b^3)*cosh(d*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a^2*b + 8*a*b^2
```

+ 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 - 5*a^2*b + 12*a*b^2 + 9*b^3 + 6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 2*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(a^3 + a^2*b)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^6 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^5 + 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 12*(a^3 + a^2*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 2*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 2*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3 - 3*a^2*b)*d*x)*sinh(d*x + c)^4 + 8*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a^3 - 12*a^2*b - 14*a*b^2 - 6*b^3 - 2*(a^3 + a^2*b)*d*x - 2*(4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2 + 2*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*b^2 + 6*b^3 - (a^3 - 3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2

```
*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d
*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a^2*b + 8*a*b^2 + 3*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x
+ c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^
2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2 + (15*
(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 - 5*a^2*b + 12*a*b^2 + 9*b^3 +
6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a
^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 2*(5*a^2*b - 12*a*b^2 - 9*b^3)*co
sh(d*x + c)^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sin
h(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(3*(a^3 + a^
2*b)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*
a^2*b)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3
*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*d*cosh(d*x + c)^6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x
+ c)^6 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 - a^4*b - 5*a^3*
b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)
*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x
+ c)^3 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x +
c)^3 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 6*(a^5
 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5 - a^4*b - 5*a^3*
b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*d + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^5 + 2*(a^5
- a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 - a^4*b - 5*a^3*b
^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth^2{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^2}\,dx$$

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(105) = 210$.

Time = 0.43 (sec) , antiderivative size = 976, normalized size of antiderivative = 8.20

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*b + b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + 1/4*(2*a*b + b^2)*log(2*(a - b)*e^
(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^
2)*d) + 1/8*(3*a^2*b - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c)
 + a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) - 1/8*(3*a^2*b
 - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) + 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2
+ 3*b^3 + (2*a^3 + 7*a^2*b + 3*b^3)*e^(4*d*x + 4*c) + 2*(2*a^3 + 2*a^2*b +
a*b^2 - 3*b^3)*e^(2*d*x + 2*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(6*d*x + 6*c) - (a^5 - a^4*b - 5*a^3*b
^2 - 3*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(
2*d*x + 2*c))*d) - 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2 + 3*b^3 + 2*(2*a^3 + 2*a^
2*b + a*b^2 - 3*b^3)*e^(-2*d*x - 2*c) + (2*a^3 + 7*a^2*b + 3*b^3)*e^(-4*d*x
 - 4*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 - a^4*b - 5*a^3*b^2 -
 3*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(-4*
d*x - 4*c) - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c))*d) - 1
/2*(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x - 2*c) + (2*a^2 + 3
*a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b
- 3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-4*d*x - 4*c
) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 3/4*b*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/2*log(e^(2*d
*x + 2*c) - 1)/(a^2*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(105) = 210$.

Time = 0.37 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.82

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{\frac{(5ab^2+3b^3)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{ab}} - \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{2\left(2a^3e^{(4dx+4c)}+6a^2be^{(4dx+4c)}+5ab^2e^{(4dx+4c)}+3b^3e^{(4dx+4c)}\cdots\right)}{(a^4+2a^3b+a^2b^2)\left(ae^{(6dx+6c)}+be^{(6dx+\cdots)}\cdots\right)}}{2d}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((5*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) +
 a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2
 + 2*a*b + b^2) + 2*(2*a^3*e^(4*d*x + 4*c) + 6*a^2*b*e^(4*d*x + 4*c) + 5*a*
b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 4*a^2
*b*e^(2*d*x + 2*c) - 4*a*b^2*e^(2*d*x + 2*c) - 6*b^3*e^(2*d*x + 2*c) + 2*a^
3 + 6*a^2*b + 7*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x + 6*c
) + b*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*
x + 2*c) + 3*b*e^(2*d*x + 2*c) - a - b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{{\mathrm{coth}\left(c+d\,x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.188 $\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{\coth^2(c+dx)}{2a^2d}+\frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{(a-2b)\log(\tanh(c+dx))}{a^3d} + \frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^2d} - \frac{b^2}{2a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*coth(d*x+c)^2/a^2/d+ln(cosh(d*x+c))/(a+b)^2/d+(a-2*b)*ln(tanh(d*x+c))/
a^3/d+1/2*b^2*(3*a+2*b)*ln(a+b*tanh(d*x+c)^2)/a^3/(a+b)^2/d-1/2*b^2/a^2/(a+
b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used

$= \{3751, 457, 90\}$

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^2} + \frac{(a-2b)\log(\tanh(c+dx))}{a^3d} - \frac{b^2}{2a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\coth^2(c+dx)}{2a^2d} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*Coth[c + d*x]^2/(a^2*d) + Log[Cosh[c + d*x]]/((a + b)^2*d) + ((a - 2*b
)*Log[Tanh[c + d*x]])/(a^3*d) + (b^2*(3*a + 2*b)*Log[a + b*Tanh[c + d*x]^2]
)/(2*a^3*(a + b)^2*d) - b^2/(2*a^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{1}{a^2x^2} + \frac{a-2b}{a^3x} + \frac{b^3}{a^2(a+b)(a+bx)^2} + \frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= -\frac{\coth^2(c+dx)}{2a^2d} + \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{(a-2b)\log(\tanh(c+dx))}{a^3d} + \frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^2d} - \frac{b^2}{2a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.75

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{\coth^2(c+dx)}{a^2} + \frac{b^3}{a^3(a+b)\left(b+a\coth^2(c+dx)\right)} + \frac{b^2(3a+2b)\log\left(b+a\coth^2(c+dx)\right)}{a^3(a+b)^2} + \frac{2\log(\sinh(c+dx))}{(a+b)^2}}{2d}$$

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(Coth[c + d*x]^2/a^2) + b^3/(a^3*(a + b)*(b + a*Coth[c + d*x]^2)) + (b^2*
(3*a + 2*b)*Log[b + a*Coth[c + d*x]^2])/(a^3*(a + b)^2) + (2*Log[Sinh[c + d
*x]])/(a + b)^2)/(2*d)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\frac{(-a+2b)\ln(\tanh(dx+c))}{a^3} + \frac{1}{2a^2\tanh(dx+c)^2} + \frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2} - \frac{b^3\left(-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)} + \frac{(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2a^3}}{d}$
default	$-\frac{\frac{(-a+2b)\ln(\tanh(dx+c))}{a^3} + \frac{1}{2a^2\tanh(dx+c)^2} + \frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2} - \frac{b^3\left(-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)} + \frac{(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2a^3}}{d}$
parallelrisch	$\frac{3\left(a+\frac{2b}{3}\right)\left(a+b\tanh(dx+c)^2\right)b^2\ln\left(a+b\tanh(dx+c)^2\right)+\left(-2\tanh(dx+c)^2a^3b-2a^4\right)\ln(1-\tanh(dx+c))+2(a-2b)(a+b \cdots}{2\left(a+b\tanh(dx+ \cdots\right)}$
risch	$\frac{x}{a^2+2ab+b^2} - \frac{2x}{a^2} - \frac{2c}{a^2d} + \frac{4bx}{a^3} + \frac{4bc}{a^3d} - \frac{6b^2x}{a^2(a^2+2ab+b^2)} - \frac{6b^2c}{d\,a^2(a^2+2ab+b^2)} - \frac{4b^3x}{a^3(a^2+2ab+b^2)} - \frac{4 \cdots}{a^3d(a^2 \cdots}$

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*((-a+2*b)/a^3*ln(tanh(d*x+c))+1/2/a^2/tanh(d*x+c)^2+1/2/(a+b)^2*ln(tan
h(d*x+c)-1)-1/2*b^3/(a+b)^2/a^3*(-a*(a+b)/b/(a+b*tanh(d*x+c)^2)+(3*a+2*b)/b
*ln(a+b*tanh(d*x+c)^2))+1/2/(a+b)^2*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. $2(118) = 236$.

Time = 0.45 (sec) , antiderivative size = 3468, normalized size of antiderivative = 27.97

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 + 16*(a^4 + a^3*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^7 + 2*(a^4 + a^3*b)*d*x*sinh(d*x + c)^8 - 4*(2*a^3*b*d*x
- a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 4*(2*a^3*b*d*x - 1
4*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*
sinh(d*x + c)^6 + 8*(14*(a^4 + a^3*b)*d*x*cosh(d*x + c)^3 - 3*(2*a^3*b*d*x
- a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(
2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^
4 + 4*(35*(a^4 + a^3*b)*d*x*cosh(d*x + c)^4 + 2*a^4 + 2*a^3*b - 2*a^2*b^2 -
 4*a*b^3 - (a^4 - 3*a^3*b)*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^
2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + a^3*b)*d*x*cos
h(d*x + c)^5 - 5*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d
*x + c)^3 + (2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + a^3*b)*d*x - 4*(2*a^3*b*d*x - a^4
- 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2 + 4*(14*(a^4 + a^3*b)*d*x*
cosh(d*x + c)^6 - 2*a^3*b*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2
 - 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 6*(2*a^
4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - ((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*
b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 5*a*b^3
 + 2*b^4)*sinh(d*x + c)^8 - 4*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^6 - 4*(3*a*b^
3 + 2*b^4 - 7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^3 - 3*(3*a*b^3 + 2*b^4
)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x
 + c)^4 + 2*(35*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^4 - 3*a^2*b^2 +
 7*a*b^3 + 6*b^4 - 30*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
3*a^2*b^2 + 5*a*b^3 + 2*b^4 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x +
 c)^5 - 10*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^3 - (3*a^2*b^2 - 7*a*b^3 - 6*b^4
)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^2 + 4*
(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^6 - 15*(3*a*b^3 + 2*b^4)*cos
```

h(d*x + c)^4 - 3*a*b^3 - 2*b^4 - 3*(3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^7 - 3*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^5 - (3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x + c)^3 - (3*a*b^3 + 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^8 + 8*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*sinh(d*x + c)^8 - 4*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^6 - 4*(a^3*b - 3*a*b^3 - 2*b^4 - 7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^3 - 3*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^4 - a^4 + 3*a^3*b + 3*a^2*b^2 - 7*a*b^3 - 6*b^4 - 30*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4 + 8*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^5 - 10*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^6 - 15*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^4 - a^3*b + 3*a*b^3 + 2*b^4 - 3*(a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^7 - 3*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^5 - (a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^3 - (a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^7 - 3*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^5 + 2*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^3 - (2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^8 - 4*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^5*b + 2*a^4*b^2 + a^3*b^3)*d)*sinh(d*x + c)^6 - 2*(a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 - 3*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 - 30*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^4 - 4*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 - 10*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 - (a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 - 15*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 - 3*(a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (a^5*b + 2*

```
a^4*b^2 + a^3*b^3)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 - 3*(a^5*
b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 - (a^6 - a^5*b - 5*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c)^3 - (a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c))*
sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth^3{\left(c+dx\right)}}{\left(a+b\tanh^2{\left(c+dx\right)}\right)^2}\,dx$$

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(118) = 236$.

Time = 0.22 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.24

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(3\,ab^2+2\,b^3)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^5+2\,a^4b+a^3b^2)d}+\frac{dx+c}{(a^2+2\,ab+b^2)d} - \frac{2\,((a^3+3\,a^2b+3\,ab^2+2\,b^3)e^{(-2\,dx-2\,c)}+2\,(a^3+a^2b-ab^2-2\,b^3)e^{(-4\,d}}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3-4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-2\,dx-2\,c)}-2\,(a^5-a^4b-5\,a^3b^2-3\,a^2b^3)e^{(-4\,dx-4\,c)}} + \frac{(a-2\,b)\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{(a-2\,b)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + 2*b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + (d*x + c)/((a^2 + 2*a*b + b^2
)*d) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + 2*(a^3 + a^2
*b - a*b^2 - 2*b^3)*e^(-4*d*x - 4*c) + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^
(-6*d*x - 6*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - 4*(a^4*b + 2*a^3*b^
2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(
-4*d*x - 4*c) - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-8*d*x - 8*c))*d) + (a - 2*b)*log(e^(-d*x
- c) + 1)/(a^3*d) + (a - 2*b)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(118) = 236.

Time = 0.39 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.60

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

$$= \frac{\frac{(3\,ab^2+2\,b^3)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^5+2\,a^4b+a^3b^2} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} + \frac{2\,(a-2\,b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^3} - \frac{4\left(\frac{(a^4+3\cdots}{\cdots}\right.}{\cdots}}{2\,d}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b^2 + 2*b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2
*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^5 + 2*a^4*b + a^3*b^2) - 2*(d
*x + c)/(a^2 + 2*a*b + b^2) + 2*(a - 2*b)*log(abs(e^(2*d*x + 2*c) - 1))/a^3
 - 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(6*d*x + 6*c)/(a + b) + 2*(a^
4 + a^3*b - a^2*b^2 - 2*a*b^3)*e^(4*d*x + 4*c)/(a + b) + (a^4 + 3*a^3*b + 3
*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d
*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)*a^3*
(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.189 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2}+\frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2d} - \frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d}-\frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2+1/2*b^(5/2)*(7*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)
/(a+b)^2/d-1/2*(2*a^2-2*a*b-5*b^2)*coth(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*co
th(d*x+c)^3/a^2/(a+b)/d+1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3751, 483, 597, 536, 212, 211}

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^2} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2d(a+b)}-\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3d(a+b)} + \frac{b\coth^3(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 + (b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(7/2)*(a + b)^2*d) - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(
a + b)*d) - ((2*a + 5*b)*Coth[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Coth[c + d
*x]^3)/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-5b+5bx^2}{x^4(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2a(a+b)d}$$

$$= -\frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{3(2a^2-2ab-5b^2)+3b(2a+5b)x^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{6a^2(a+b)d}$$

$$= -\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-3(2a^3-2a^2b+2ab^2+5b^3)-3b(2a^2-2ab-5b^2)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{6a^3(a+b)d}$$

$$= -\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} + \frac{(b^3(7a+5b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2a^3(a+b)^2d}$$

$$= \frac{x}{(a+b)^2} + \frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2d} - \frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{6(c+dx)}{(a+b)^2}+\frac{3b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^2}+\frac{4(-2a+3b)\coth(c+dx)}{a^3}-\frac{2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a^2}+\frac{3b^3\sinh(2(c+dx))}{a^3(a+b)(a-b+(a+b)\cosh(2(}}{6d}$$

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((6*(c + d*x))/(a + b)^2 + (3*b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^2) + (4*(-2*a + 3*b)*Coth[c + d*x])/a^3 -
(2*Coth[c + d*x]*Csch[c + d*x]^2)/a^2 + (3*b^3*Sinh[2*(c + d*x)])/(a^3*(a +
 b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(6*d)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{-\frac{-a+2b}{a^3\tanh(dx+c)}+\frac{1}{3a^2\tanh(dx+c)^3}-\frac{b^3\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(7a+5b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tan}{2}}{d}$
default	$-\frac{-\frac{-a+2b}{a^3\tanh(dx+c)}+\frac{1}{3a^2\tanh(dx+c)^3}-\frac{b^3\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(7a+5b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tan}{2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{12\,e^{8dx+8c}a^4+24\,e^{8dx+8c}a^3b-21a\,b^3e^{8dx+8c}-15b^4e^{8dx+8c}+12\,e^{6dx+6c}a^4-12b\,a^3e^{6dx+6c}-12a^2b^2e^{6dx}}{}$

```
[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-(-a+2*b)/a^3/tanh(d*x+c)+1/3/a^2/tanh(d*x+c)^3-b^3/(a+b)^2/a^3*((1/2
*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(7*a+5*b)/(a*b)^(1/2)*arctan(
b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/2/(a+b)^2*ln(ta
nh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4080 vs. $2(143) = 286$.

Time = 0.38 (sec) , antiderivative size = 8482, normalized size of antiderivative = 53.35

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. $2(143) = 286$.

Time = 0.68 (sec) , antiderivative size = 2345, normalized size of antiderivative = 14.75

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/4*(a^2*b - a*b^2 - b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x
 + 2*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + 1/4*(a^2*b - a*b^2 - b^3)*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 +
2*a^4*b + a^3*b^2)*d) + 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*arct
an(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b
^2)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b^2)*
sqrt(a*b)*d) + 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + 23*a*b^3 - 15*b^4 +
 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e^(8*d*x + 8*c) + 6*
(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)*e^(6*d*x + 6*c) - 2*(50
*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4)*e^(4*d*x + 4*c) - 2*(10
```

*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4)*e^(2*d*x + 2*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^(10*d*x + 10*c) + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(8*d*x + 8*c) + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(6*d*x + 6*c) - 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(4*d*x + 4*c) - (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(2*d*x + 2*c))*d) - 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + 23*a*b^3 - 15*b^4 - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 6*(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)*e^(-6*d*x - 6*c) + 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e^(-8*d*x - 8*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(-2*d*x - 2*c) - 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(-4*d*x - 4*c) + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(-6*d*x - 6*c) + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(-8*d*x - 8*c) - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^(-10*d*x - 10*c))*d) + 1/12*(8*a^3 + 7*a^2*b - 16*a*b^2 - 15*b^3 + 3*(8*a^3 + 11*a^2*b + 6*a*b^2 - 5*b^3)*e^(8*d*x + 8*c) + 6*(4*a^3 - 7*a^2*b - 13*a*b^2 + 10*b^3)*e^(6*d*x + 6*c) - 2*(8*a^3 - 44*a^2*b - 43*a*b^2 + 45*b^3)*e^(4*d*x + 4*c) - 2*(4*a^3 + 27*a^2*b + 5*a*b^2 - 30*b^3)*e^(2*d*x + 2*c))/((a^5 + 2*a^4*b + a^3*b^2 - (a^5 + 2*a^4*b + a^3*b^2)*e^(10*d*x + 10*c) + (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(8*d*x + 8*c) + 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^(6*d*x + 6*c) - 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^(4*d*x + 4*c) - (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(2*d*x + 2*c))*d) - 1/12*(8*a^3 + 7*a^2*b - 16*a*b^2 - 15*b^3 - 2*(4*a^3 + 27*a^2*b + 5*a*b^2 - 30*b^3)*e^(-2*d*x - 2*c) - 2*(8*a^3 - 44*a^2*b - 43*a*b^2 + 45*b^3)*e^(-4*d*x - 4*c) + 6*(4*a^3 - 7*a^2*b - 13*a*b^2 + 10*b^3)*e^(-6*d*x - 6*c) + 3*(8*a^3 + 11*a^2*b + 6*a*b^2 - 5*b^3)*e^(-8*d*x - 8*c))/((a^5 + 2*a^4*b + a^3*b^2 - (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(-2*d*x - 2*c) - 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^(-4*d*x - 4*c) + 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^(-6*d*x - 6*c) + (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(-8*d*x - 8*c) - (a^5 + 2*a^4*b + a^3*b^2)*e^(-10*d*x - 10*c))*d) + 1/8*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*b + 30*b^2)*e^(-2*d*x - 2*c) - 2*(10*a^2 - 2*a*b - 45*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + a*b + 10*b^2)*e^(-6*d*x - 6*c) + 3*(3*a*b + 5*b^2)*e^(-8*d*x - 8*c))/((a^4 + a^3*b - (a^4 + 5*a^3*b)*e^(-2*d*x - 2*c) - 2*(a^4 - 5*a^3*b)*e^(-4*d*x - 4*c) + 2*(a^4 - 5*a^3*b)*e^(-6*d*x - 6*c) + (a^4 + 5*a^3*b)*e^(-8*d*x - 8*c) - (a^4 + a^3*b)*e^(-10*d*x - 10*c))*d) + 1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(a^3*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^3*d) + 1/2*(a - b)*log(e^(2*d*x + 2*c) - 1)/(a^3*d) - b*log(e^(2*d*x + 2*c) - 1)/(a^3*d) - 1/2*(a - b)*log(e^(-2*d*x - 2*c) - 1)/(a^3*d) + b*log(e^(-2*d*x - 2*c) - 1)/(a^3*d) - 1/8*(3*a*b - 5*b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) - 3/16*(3*a*b + 5*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) + 1/8*(3*a*b - 5*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.77

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{3\,(7\,ab^3+5\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab}}+\frac{6\,(dx+c)}{a^2+2\,ab+b^2}-\frac{6\,\left(ab^3e^{(2\,dx+2\,c)}-b^4e^{(2\,dx+2\,c)}+ab^3+b^4\right)}{(a^5+2\,a^4b+a^3b^2)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2}}}{6\,d}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(7*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
+ a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b)) + 6*(d*x + c)/(a^
2 + 2*a*b + b^2) - 6*(a*b^3*e^(2*d*x + 2*c) - b^4*e^(2*d*x + 2*c) + a*b^3 +
 b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)) - 8*(3*a*e^(4*d*x + 4*c)
 - 3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 2*a -
3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth(c+d\,x)^4}{\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

```
[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

3.190 $\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} - \frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3d} + \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^3-1/8*(3*a^2+10*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(
1/2)/b^(5/2)/(a+b)^3/d+1/4*a*tanh(d*x+c)^3/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+
1/8*a*(3*a+7*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3751, 481, 592, 536, 212, 211}

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}d(a+b)^3} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} + \frac{a\tanh^3(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{x}{(a+b)^3}$$

```
[In] Int[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*b^(5/2)*(a + b)^3*d) + (a*Tanh[c + d*x]^3)/(4*b*(a + b)*d
*(a + b*Tanh[c + d*x]^2)^2) + (a*(3*a + 7*b)*Tanh[c + d*x])/(8*b^2*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(-3a-4b)x^2)}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4b(a+b)d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a(3a+7b)+(-3a^2-7ab-8b^2)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8b^2(a+b)^2d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d} - \frac{\left(a(3a^2+10ab+15b^2)\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8b^2(a+b)^3d}$$

$$= \frac{x}{(a+b)^3} - \frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3d} + \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$=\frac{8(c+dx)-\frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{5/2}}-\frac{4a^2(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{3a(a+b)(a+3b)\sinh(2(c+dx))}{b^2(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

```
[In] Integrate[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/b^(5/2) - (4*a^2*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a
+ b)*Cosh[2*(c + d*x)])^2) + (3*a*(a + b)*(a + 3*b)*Sinh[2*(c + d*x)])/(b^2
*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{a\left(\frac{-\frac{(5a^2+14ab+9b^2)\tanh(dx+c)^3}{8b}-\frac{a(3a^2+10ab+7b^2)\tanh(dx+c)}{8b^2}}{(a+b\tanh(dx+c)^2)^2}+\frac{(3a^2+10ab+15b^2)\arctan\left(\frac{b\tanh(dx+}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{a\left(\frac{-\frac{(5a^2+14ab+9b^2)\tanh(dx+c)^3}{8b}-\frac{a(3a^2+10ab+7b^2)\tanh(dx+c)}{8b^2}}{(a+b\tanh(dx+c)^2)^2}+\frac{(3a^2+10ab+15b^2)\arctan\left(\frac{b\tanh(dx+}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{a(3a^3e^{6dx+6c}+13a^2b\,e^{6dx+6c}+a\,b^2e^{6dx+6c}-9\,e^{6dx+6c}b^3+9a^3e^{4dx+4c}+21a^2b\,e^{4dx+4c}-9a\,b^2e^{4dx}}{4(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}}$

```
[In] int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-a/(a+b)^3*((-1/8*(5*a^2+14*a*b+9*b^2)/b
*tanh(d*x+c)^3-1/8*a*(3*a^2+10*a*b+7*b^2)/b^2*tanh(d*x+c))/(a+b*tanh(d*x+c)
^2)^2+1/8*(3*a^2+10*a*b+15*b^2)/b^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^
(1/2)))+1/2/(a+b)^3*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3603 vs. 2(130) = 260.

Time = 0.40 (sec) , antiderivative size = 7528, normalized size of antiderivative = 52.28

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3354 vs. 2(130) = 260.

Time = 1.24 (sec) , antiderivative size = 3354, normalized size of antiderivative = 23.29

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/512*(3*a^5 + 25*a^4*b + 150*a^3*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*ar
ctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 1/512*(3*a^5 + 25*a^4*b + 150*a^3*b^
2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) +
a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d)
 - 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b
^5 + 3*b^6 + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^5 - 3*b
^6)*e^(6*d*x + 6*c) + (9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^3 - 905*
a^2*b^4 + 66*a*b^5 + 9*b^6)*e^(4*d*x + 4*c) + (9*a^6 + 68*a^5*b - 659*a^4*b
^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(2*d*x + 2*c))/((a^7*b^2 + 5*a^6*b^3
 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7 + (a^7*b^2 + 5*a^6*b^3 + 1
```

0*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(8*d*x + 8*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(6*d*x + 6*c) + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 + 7*a^3*b^6 + 3*a^2*b^7) *e^(4*d*x + 4*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b ^6 - a^2*b^7)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b^5 + 3*b^6 + (9*a^6 + 68*a^5*b - 659*a^4*b ^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(-2*d*x - 2*c) + (9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^3 - 905*a^2*b^4 + 66*a*b^5 + 9*b^6)*e^(-4*d*x - 4 *c) + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^5 - 3*b^6)*e^(-6*d*x - 6*c))/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7 + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^ 2*b^7)*e^(-2*d*x - 2*c) + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 + 7*a^3*b^6 + 3*a^2*b^7)*e^(-4*d*x - 4*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5* b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(-6*d*x - 6*c) + (a^7*b^2 + 5*a^6* b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(-8*d*x - 8*c))*d) - 3/128*(3*a^5 + 17*a^4*b + 14*a^3*b^2 - 14*a^2*b^3 - 17*a*b^4 - 3*b^5 + (3* a^5 + 15*a^4*b - 98*a^3*b^2 - 98*a^2*b^3 + 15*a*b^4 + 3*b^5)*e^(6*d*x + 6*c) + (9*a^5 + 27*a^4*b - 110*a^3*b^2 + 110*a^2*b^3 - 27*a*b^4 - 9*b^5)*e^(4* d*x + 4*c) + (9*a^5 + 29*a^4*b - 86*a^3*b^2 - 86*a^2*b^3 + 29*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6 + (a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^6*b ^2 + 4*a^5*b^3 + 2*a^4*b^4 + 4*a^3*b^5 + 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^ 6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) + 3/128*(3*a^5 + 17*a^4*b + 14*a^3*b^2 - 14*a^2*b^3 - 17*a*b^4 - 3*b^5 + (9*a^5 + 29*a^4* b - 86*a^3*b^2 - 86*a^2*b^3 + 29*a*b^4 + 9*b^5)*e^(-2*d*x - 2*c) + (9*a^5 + 27*a^4*b - 110*a^3*b^2 + 110*a^2*b^3 - 27*a*b^4 - 9*b^5)*e^(-4*d*x - 4*c) + (3*a^5 + 15*a^4*b - 98*a^3*b^2 - 98*a^2*b^3 + 15*a*b^4 + 3*b^5)*e^(-6*d*x - 6*c))/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6 + 4*(a^6*b ^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^6*b^2 + 4*a ^5*b^3 + 2*a^4*b^4 + 4*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c))*d) - 15/256*(3*a^4 + 8* a^3*b + 10*a^2*b^2 + 8*a*b^3 + 3*b^4 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*e^ (6*d*x + 6*c) + (9*a^4 + 46*a^2*b^2 + 9*b^4)*e^(4*d*x + 4*c) + (9*a^4 + 2*a ^3*b - 2*a*b^3 - 9*b^4)*e^(2*d*x + 2*c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4 *(a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^(6*d*x + 6*c) + 2*(3*a^5*b^2 + a ^4*b^3 + a^3*b^4 + 3*a^2*b^5)*e^(4*d*x + 4*c) + 4*(a^5*b^2 + a^4*b^3 - a^3* b^4 - a^2*b^5)*e^(2*d*x + 2*c))*d) + 15/256*(3*a^4 + 8*a^3*b + 10*a^2*b^2 + 8*a*b^3 + 3*b^4 + (9*a^4 + 2*a^3*b - 2*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + (9*a^4 + 46*a^2*b^2 + 9*b^4)*e^(-4*d*x - 4*c) + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*e^(-6*d*x - 6*c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5 + 4*(a^ 5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5*b^2 + a^4* b^3 + a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b^2 + a^4*b^3 - a^3*b^

```
4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)
*e^(-8*d*x - 8*c))*d) + 5/64*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 -
13*a^2*b - 13*a*b^2 + 9*b^3)*e^(-2*d*x - 2*c) + 3*(3*a^3 - 5*a^2*b + 5*a*b^
2 - 3*b^3)*e^(-4*d*x - 4*c) + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^(-6*d*x - 6
*c))/((a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^(-2*d*x - 2*
c) + 2*(3*a^4*b^2 - 2*a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 -
a^2*b^4)*e^(-6*d*x - 6*c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c
))*d) + 1/4*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b
)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c)
+ (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 9
/256*(a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/(sqrt(a*b)*a^2*b^2*d) - 45/512*(a^2 - b^2)*arctan(1/2*((a + b)*e^(2*d
*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d) + 5/128*(3*a^2 - 2*a*b
+ 3*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b
)*a^2*b^2*d) + 9/256*(a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*
c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d) + 45/512*(a^2 - b^2)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(130) = 260$.

Time = 0.52 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.83

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{\left(3\,a^3+10\,a^2b+15\,ab^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,\left(3\,a^4e^{(6\,dx+6\,c)}+13\,a^3be^{(6\,dx+6\,c)}+a^2b^2e^{(6\,dx+6}}{}}{}$$

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^3 + 10*a^2*b + 15*a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*
d*x + 2*c) + a - b)/sqrt(a*b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*sqrt(
a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(3*a^4*e^(6*d*x + 6
*c) + 13*a^3*b*e^(6*d*x + 6*c) + a^2*b^2*e^(6*d*x + 6*c) - 9*a*b^3*e^(6*d*x
 + 6*c) + 9*a^4*e^(4*d*x + 4*c) + 21*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4
*d*x + 4*c) + 27*a*b^3*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*c) + 23*a^3*b*e
^(2*d*x + 2*c) - 13*a^2*b^2*e^(2*d*x + 2*c) - 27*a*b^3*e^(2*d*x + 2*c) + 3*
a^4 + 15*a^3*b + 21*a^2*b^2 + 9*a*b^3)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^
5)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*
d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 2669, normalized size of antiderivative = 18.53

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(tanh(c + d*x)^6/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh
(c + d*x)^3*(9*a*b + 5*a^2))/(8*b*(2*a*b + a^2 + b^2)) + (a*tanh(c + d*x)*(
7*a*b + 3*a^2))/(8*b^2*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4
 + 2*a*b*d*tanh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (ata
n((((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4
 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 +
 4*a^3*b^4*d^2 + a^4*b^3*d^2)) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 393
6*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 80
0*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3
 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) - (tanh(
c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^
11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5
*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d +
 a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*
d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d +
 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2)*1i)/(16*(
b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)) + ((-a*b^5)^(1/2)*((tanh(c +
d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))
/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2))
 - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d
^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2
)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5
*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) + (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*
b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1
280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 -
256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d
^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^
(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a
*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2)*1i)/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*
d + 3*a*b^7*d)))/((120*a*b^4 + 51*a^4*b + 9*a^5 + 185*a^2*b^3 + 139*a^3*b^2
)/(32*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5
*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) - ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*
a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7
*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) + (((224
*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280
*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^
```

9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)) + ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) + (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d))))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*1i)/(8*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d))

3.191 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} - \frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(a+2b)}{2b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4*a^2/b^2/(
a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*a*(a+2*b)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2
)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{a^2}{4b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(a+2b)}{2b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - a^2/(4*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(2*b^2
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^3} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d}\\
&\quad - \frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(a+2b)}{2b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{-4\log(\cosh(c+dx))-2\log\left(a+b\tanh^2(c+dx)\right)+\frac{a^2(a+b)^2}{b^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{2a(a+b)(a+2b)}{b^2\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)^3d}$$

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/4*(-4*Log[Cosh[c + d*x]] - 2*Log[a + b*Tanh[c + d*x]^2] + (a^2*(a + b)^2
)/(b^2*(a + b*Tanh[c + d*x]^2)^2) - (2*a*(a + b)*(a + 2*b))/(b^2*(a + b*Tan
h[c + d*x]^2)))/((a + b)^3*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\tanh(dx+c)^2\right)}+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\tanh(dx+c)^2\right)^2}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\tanh(dx+c)^2\right)}+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\tanh(dx+c)^2\right)^2}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)a\,e^{2dx+2c}}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^2d(a+b)^3}+$
parallelrisch	$-\frac{4\ln(1-\tanh(dx+c))a^2b^2+4\ln(1-\tanh(dx+c))\tanh(dx+c)^4b^4-2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^4b^4-4a\,b^3\tanh(d}{}$

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2/(a+b)^3*(-a*(a^2+3*a*b+2*b^2)/b^2/(
a+b*tanh(d*x+c)^2)+1/2*a^2*(a^2+2*a*b+b^2)/b^2/(a+b*tanh(d*x+c)^2)^2-ln(a+b
*tanh(d*x+c)^2))-1/2/(a+b)^3*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 2584, normalized size of antiderivative = 23.71

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^
2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x - a^2 - a*b)*sinh(d*x + c)^6 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x - a^2 - a*b
)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 +
 4*a*b)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (
3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)
^2 - 2*a^2 + 4*a*b)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^5 + 10*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 - 2*a
*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 +
2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x - a^2 - a*b)*c
osh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 +
 4*a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*s
inh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*
cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 -
 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x - a
^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
```

```
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(103) = 206$.

Time = 0.24 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.45

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$+ \frac{4}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2}$$
$$+ \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 4*((a^2 + a*b)*e^(-2*d*x -
2*c) + (a^2 - 2*a*b)*e^(-4*d*x - 4*c) + (a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d) + 1/2*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(103) = 206$.

Time = 0.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.25

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{2\,\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2-4}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2}\right.\right.}}{4\,d}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 - 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 12*b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) - 4*a + 12*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.82

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{a^4 + a^3\,b\,\left(2\tanh(c+d\,x)^2+4\right) - a\,b^3\,\left(-4\tanh(c+d\,x)^2 + \tanh(c+d\,x)^2\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+}{2\,a-a\tanh(c+d\,x)^2+b\tanh(}\right.\right.}{4\,d\,a^5\,b^2 + 8\,d\,a^4\,b^3\tanh\left(c+d\,x\right)^2 + 12\,d\,a^4\,b^3 + 4\,d\,a^3\,b^4\tanh\left(c+d\,x\right)^4 + 24\,d\,a^3\,b^4\tanh\left(c+d\,x\right)^2 + 12\,d\,a}$$

```
[In] int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (a^4 + a^3*b*(2*tanh(c + d*x)^2 + 4) - a*b^3*(tanh(c + d*x)^2*atan((a*tanh(
c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c +
 d*x)^2))*8i - 4*tanh(c + d*x)^2) + a^2*b^2*(6*tanh(c + d*x)^2 - atan((a*ta
nh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(
c + d*x)^2))*4i + 3) - b^4*tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i + b*t
anh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i)/(4*a^
2*b^5*d + 12*a^3*b^4*d + 12*a^4*b^3*d + 4*a^5*b^2*d + 4*b^7*d*tanh(c + d*x)
^4 + 24*a^2*b^5*d*tanh(c + d*x)^2 + 24*a^3*b^4*d*tanh(c + d*x)^2 + 8*a^4*b^
3*d*tanh(c + d*x)^2 + 12*a^2*b^5*d*tanh(c + d*x)^4 + 4*a^3*b^4*d*tanh(c + d
*x)^4 + 8*a*b^6*d*tanh(c + d*x)^2 + 12*a*b^6*d*tanh(c + d*x)^4)
```

3.192 $\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{x}{(a+b)^3} - \frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}(a+b)^3d} + \frac{a\tanh(c+dx)}{4b(a+b)d(a+b\tanh^2(c+dx))^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d(a+b\tanh^2(c+dx))}$$

```
[Out] x/(a+b)^3-1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)
/(a+b)^3/d/a^(1/2)+1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a
+5*b)*tanh(d*x+c)/b/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 481, 541, 536, 212, 211}

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = -\frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}d(a+b)^3} - \frac{(a+5b)\tanh(c+dx)}{8bd(a+b)^2(a+b\tanh^2(c+dx))} + \frac{a\tanh(c+dx)}{4bd(a+b)(a+b\tanh^2(c+dx))^2} + \frac{x}{(a+b)^3}$$

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*Sqrt[a]*b^(3/2)*(a + b)^3*d) + (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) - ((a + 5*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(-a-4b)x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4b(a+b)d}\\
&= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-a(a-3b)+a(a+5b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8ab(a+b)^2d}\\
&= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d}\\
&\quad - \frac{(a^2+6ab-3b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8b(a+b)^3d}\\
&= \frac{x}{(a+b)^3} - \frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}(a+b)^3d}\\
&\quad + \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$
\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx
$$

$$
= \frac{8(c+dx) - \frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{4a(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{(a-5b)(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}
$$

[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (8*(c + d*x) - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
]])/(Sqrt[a]*b^(3/2)) + (4*a*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Co
sh[2*(c + d*x)])^2 + ((a - 5*b)*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a +
 b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\tanh(dx+c)^3-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\right.}{8b\sqrt{ab}}}{(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\tanh(dx+c)^3-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\right.}{8b\sqrt{ab}}}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{a^3e^{6dx+6c}-9a^2be^{6dx+6c}-5ab^2e^{6dx+6c}+5e^{6dx+6c}b^3+3a^3e^{4dx+4c}-17a^2be^{4dx+4c}+13ab^2e^{4dx+}}{4b\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-\right.}$

[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2/(a+b)^3*ln(tanh(d*x+c)+1)-1/(a+b)^3
*(((1/8*a^2+3/4*a*b+5/8*b^2)*tanh(d*x+c)^3-1/8*a*(a^2-2*a*b-3*b^2)/b*tanh(d
*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(a^2+6*a*b-3*b^2)/b/(a*b)^(1/2)*arctan(b*t
anh(d*x+c)/(a*b)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3727 vs. 2(123) = 246.

Time = 0.40 (sec) , antiderivative size = 7757, normalized size of antiderivative = 56.62

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2432 vs. $2(123) = 246$.

Time = 0.95 (sec) , antiderivative size = 2432, normalized size of antiderivative = 17.75

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*arctan(1/2*((a + b)
*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*
b^4)*sqrt(a*b)*d) + 1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*
arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/64*(a^5 - 33*a^4*b - 54*a^3*b^2 -
 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154*a^2*b^3
- 19*a*b^4 - 3*b^5)*e^(6*d*x + 6*c) + (3*a^5 - 171*a^4*b + 310*a^3*b^2 - 25
4*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(4*d*x + 4*c) + (3*a^5 - 133*a^4*b + 86*a^3
*b^2 + 190*a^2*b^3 - 41*a*b^4 - 9*b^5)*e^(2*d*x + 2*c))/((a^7*b + 5*a^6*b^2
 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + (a^7*b + 5*a^6*b^2 + 10*
a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^7*b + 3*
a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*
(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*e^(4*
d*x + 4*c) + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2
*b^6)*e^(2*d*x + 2*c))*d) + 1/64*(a^5 - 33*a^4*b - 54*a^3*b^2 - 2*a^2*b^3 +
 21*a*b^4 + 3*b^5 + (3*a^5 - 133*a^4*b + 86*a^3*b^2 + 190*a^2*b^3 - 41*a*b^
4 - 9*b^5)*e^(-2*d*x - 2*c) + (3*a^5 - 171*a^4*b + 310*a^3*b^2 - 254*a^2*b^
3 + 39*a*b^4 + 9*b^5)*e^(-4*d*x - 4*c) + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154
*a^2*b^3 - 19*a*b^4 - 3*b^5)*e^(-6*d*x - 6*c))/((a^7*b + 5*a^6*b^2 + 10*a^5
*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3
- 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^7*b + 7*a^6*b^
2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^
7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(-6*d*x -
6*c) + (a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*
```

```
e^(-8*d*x - 8*c))*d) - 1/16*(a^4 - 4*a^3*b - 14*a^2*b^2 - 12*a*b^3 - 3*b^4
+ (a^4 - 26*a^3*b - 20*a^2*b^2 + 10*a*b^3 + 3*b^4)*e^(6*d*x + 6*c) + (3*a^4
 - 52*a^3*b + 6*a^2*b^2 - 12*a*b^3 - 9*b^4)*e^(4*d*x + 4*c) + (3*a^4 - 30*a
^3*b - 28*a^2*b^2 + 14*a*b^3 + 9*b^4)*e^(2*d*x + 2*c))/((a^6*b + 4*a^5*b^2
+ 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*
b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5
)*e^(6*d*x + 6*c) + 2*(3*a^6*b + 4*a^5*b^2 + 2*a^4*b^3 + 4*a^3*b^4 + 3*a^2*
b^5)*e^(4*d*x + 4*c) + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e^(2*d*x
 + 2*c))*d) + 1/16*(a^4 - 4*a^3*b - 14*a^2*b^2 - 12*a*b^3 - 3*b^4 + (3*a^4
- 30*a^3*b - 28*a^2*b^2 + 14*a*b^3 + 9*b^4)*e^(-2*d*x - 2*c) + (3*a^4 - 52*
a^3*b + 6*a^2*b^2 - 12*a*b^3 - 9*b^4)*e^(-4*d*x - 4*c) + (a^4 - 26*a^3*b -
20*a^2*b^2 + 10*a*b^3 + 3*b^4)*e^(-6*d*x - 6*c))/((a^6*b + 4*a^5*b^2 + 6*a^
4*b^3 + 4*a^3*b^4 + a^2*b^5 + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e
^(-2*d*x - 2*c) + 2*(3*a^6*b + 4*a^5*b^2 + 2*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^
5)*e^(-4*d*x - 4*c) + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e^(-6*d*x
 - 6*c) + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*e^(-8*d*x -
 8*c))*d) + 3/32*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b
^2 - 9*b^3)*e^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*
x - 4*c) + (a^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-2
*d*x - 2*c) + 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c)
+ 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 3*a^4
*b^2 + 3*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/4*log((a + b)*e^(4*d*x
 + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a +
 b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/32*(a + 3*b)*arctan(1/2*((a + b
)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d) + 1/32*(a + 3*b)*
arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d
) + 3/64*(a - 3*b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a^2*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(123) = 246$.

Time = 0.46 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.80

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{\left(a^2+6\,ab-3\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,\left(a^3e^{(6\,dx+6\,c)}-9\,a^2be^{(6\,dx+6\,c)}-5\,ab^2e^{(6\,dx+6\,c)}+5\,b^3e^{(6}\right.}{}}{}$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((a^2 + 6*a*b - 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*
c) + a - b)/sqrt(a*b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(a*b)) - 8*
(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*e^(6*d*x + 6*c) - 9*a^2*
b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 3*a^3
*e^(4*d*x + 4*c) - 17*a^2*b*e^(4*d*x + 4*c) + 13*a*b^2*e^(4*d*x + 4*c) - 15
*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 11*a^2*b*e^(2*d*x + 2*c) + a
*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b - 9*a*b^2 - 5
*b^3)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 2574, normalized size of antiderivative = 18.79

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - ((tanh
(c + d*x)^3*(a + 5*b))/(8*(2*a*b + a^2 + b^2)) - (a*tanh(c + d*x)*(a - 3*b)
)/(8*b*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4 + 2*a*b*d*tanh(
c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (atan(((((tanh(c + d
*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 + 4*a*b
^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) + (((96*b^9*d^2 + 544*
a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 96*a^5*
b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3 + a^6
*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3))
 - (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2 + 1280
*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 - 2304*
a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5*d + 3*a^
3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b
^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(3*a^2*
b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b + a^2 -
3*b^2)*1i)/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)) + (((tanh
(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 +
 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) - (((96*b^9*d^2
+ 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 9
6*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3
 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2
*d^3)) + (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2
+ 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 -
 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5*d
+ 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6
*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(
```

```
3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b +
a^2 - 3*b^2)*1i)/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))/((
27*a*b^2 + 11*a^2*b + a^3 - 15*b^3)/(32*(b^7*d^3 + 6*a*b^6*d^3 + a^6*b*d^3
+ 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3)) + (((t
anh(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^
2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) + (((96*b^9*d
^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2
+ 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*
d^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*
b^2*d^3)) - (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d
^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^
2 - 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5
*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2
+ 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(1
6*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b
 + a^2 - 3*b^2))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)) - (
((tanh(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5
*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) - (((96*b^
9*d^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d
^2 + 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b
^6*d^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a
^5*b^2*d^3)) + (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^1
0*d^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6
*d^2 - 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*
b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d
^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))
/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*
a*b + a^2 - 3*b^2))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))
)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*1i)/(8*(3*a^2*b^5*d + 3*a^3*b^4*d +
a^4*b^3*d + a*b^6*d))
```

3.193 $\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(a+b\tanh^2(c+dx))}{2(a+b)^3d} + \frac{a}{4b(a+b)d(a+b\tanh^2(c+dx))^2} - \frac{1}{2(a+b)^2d(a+b\tanh^2(c+dx))}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d+1/4*a/b/(a+b)
/d/(a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 78}

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{a}{4bd(a+b)(a+b\tanh^2(c+dx))^2} - \frac{1}{2d(a+b)^2(a+b\tanh^2(c+dx))} + \frac{\log(a+b\tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) + a/(4*b*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{a}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} \\
&\quad + \frac{a}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{4\log(\cosh(c+dx))+2\log\left(a+b\tanh^2(c+dx)\right)+\frac{a(a+b)^2}{b\left(a+b\tanh^2(c+dx)\right)^2}-\frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3 d}$$

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)^2)/(b*(a
+ b*Tanh[c + d*x]^2)^2) - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3
*d)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{-a-b}{a+b\tanh(dx+c)^2}}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{-a-b}{a+b\tanh(dx+c)^2}}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{2\left(a^2\mathrm{e}^{4dx+4c}-\mathrm{e}^{4dx+4c}b^2+2a^2\,\mathrm{e}^{2dx+2c}-2ab\,\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{2dx+2c}b^2\right.}{\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-2b\,\mathrm{e}^{2dx+2c}+a+b\right)^2}$
parallelrisch	$-\frac{4\ln(1-\tanh(dx+c))a^2b^2+4\ln(1-\tanh(dx+c))\tanh(dx+c)^4b^4-2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^4b^4+2a\,b^3\tanh(}{}$

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*(-ln(a+b*tanh(d*x+c)^2)-1/2*a*(a^2+2*a*b+b^2)/b/(a+b*tanh
(d*x+c)^2)^2-(-a-b)/(a+b*tanh(d*x+c)^2))-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2/
(a+b)^3*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(92) = 184$.

Time = 0.31 (sec) , antiderivative size = 2611, normalized size of antiderivative = 26.64

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x - a^2 + b^2)*sinh(d*x + c)^6
+ 8*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 - b^2)*d*x - a^
2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x -
2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 15*(2*(a^2 - b^2)*d*x - a^2 + b
^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 +
 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(
d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 4*(2*(a^2 - b^2)*d*x -
 a^2 + b^2)*cosh(d*x + c)^2 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6
 + 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 + 2*(a^2 - b^2)*d*x +
 6*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 -
a^2 + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*
b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2
- b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 -
 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x + c
)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 15*(a^2 - b^2)*cosh(d*x +
c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)
^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 -
b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*si
nh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2)) + 8*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*(2*(a^2
 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 + 2*((3*a^2 - 2*a*b + 3*b^2)*d*x -
 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (2*(a^2 - b^2)*d*x - a^2 + b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
```

```
 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3
*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^
5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^4 +
8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x +
 c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2
*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a^5 + 7*a^4*
b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(
d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c
osh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b
^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 +
3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(92) = 184$.

Time = 0.23 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.92

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}$$

$$+ \frac{2\left((\cdots\right.}{(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4-b^5)e^{(-2dx-2c)}+2(\cdots}$$

$$+ \frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((a^2 - b^2)*e^(-2*d*x -
2*c) + 2*(a^2 - a*b + b^2)*e^(-4*d*x - 4*c) + (a^2 - b^2)*e^(-6*d*x - 6*c))
/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4
*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7
*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
+ 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.50

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{2\log\left(\left|a\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+b\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+2a-2b\right|\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{3a\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+3b\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+4a(\cdots}{(a^2+2ab+b^2)\left(a\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)\cdots\right.}}{4d}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 + 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) - 4*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2))/
d
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.05

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{-a^3+b^3\left(2\tanh(c+d\,x)^2+\tanh(c+d\,x)^4\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+d\,x)^2\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^2+b\tanh(c+d\,x)^2}\right)\,4\mathrm{i}\right)+}{4\,d\,a^5\,b+8\,d\,a^4\,b^2\tanh\left(c+d\,x\right)^2+12\,d\,a^4\,b^2+4\,d\,a^3\,b^3\tanh\left(c+d\,x\right)^4+24\,d\,a^3\,b^3\tanh\left(c+d\,x\right)^2+12}$$

```
[In] int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] -(b^3*(tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(
2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i + 2*tanh(c + d*x)^2) - a^3
 + a*b^2*(tanh(c + d*x)^2*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i
)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*8i + 2*tanh(c + d*x)^2 + 1
) + a^2*b*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(
c + d*x)^2 + b*tanh(c + d*x)^2))*4i)/(4*a^2*b^4*d + 12*a^3*b^3*d + 12*a^4*b
^2*d + 4*b^6*d*tanh(c + d*x)^4 + 4*a^5*b*d + 24*a^2*b^4*d*tanh(c + d*x)^2 +
 24*a^3*b^3*d*tanh(c + d*x)^2 + 8*a^4*b^2*d*tanh(c + d*x)^2 + 12*a^2*b^4*d*
tanh(c + d*x)^4 + 4*a^3*b^3*d*tanh(c + d*x)^4 + 8*a*b^5*d*tanh(c + d*x)^2 +
 12*a*b^5*d*tanh(c + d*x)^4)
```

3.194 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} - \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a+b)^3d} - \frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^3-1/8*(3*a^2-6*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)
/(a+b)^3/d/b^(1/2)-1/4*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(3*a-b
)*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3751, 482, 541, 536, 212, 211}

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}d(a+b)^3} - \frac{(3a-b)\tanh(c+dx)}{8ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)} - \frac{\tanh(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{x}{(a+b)^3}$$

[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] x/(a + b)^3 - ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*Sqrt[b]*(a + b)^3*d) - Tanh[c + d*x]/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((3*a - b)*Tanh[c + d*x])/(8*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 482

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 536

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 541

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 3751

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],

```
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d} \\
&= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-5a-b+(-3a+b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a(a+b)^2d} \\
&= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d} \\
&\quad - \frac{(3a^2-6ab-b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a(a+b)^3d} \\
&= \frac{x}{(a+b)^3} - \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a+b)^3d} \\
&\quad - \frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{8(c+dx) + \frac{(-3a^2+6ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{4b(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} - \frac{(5a-b)(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) + ((-3*a^2 + 6*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(a^(3/2)*Sqrt[b]) - (4*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*C
osh[2*(c + d*x)])^2 - ((5*a - b)*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a
+ b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\frac{\frac{b(3a^2+2ab-b^2)\tanh(dx+c)^3}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(3a^2-6ab-b^2)\arctan\left(\right.}{8a\sqrt{ab}}}{(a+b)^3}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\frac{\frac{b(3a^2+2ab-b^2)\tanh(dx+c)^3}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(3a^2-6ab-b^2)\arctan\left(\right.}{8a\sqrt{ab}}}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}+\frac{5a^3e^{6dx+6c}-5a^2b\,e^{6dx+6c}-9a\,b^2e^{6dx+6c}+e^{6dx+6c}b^3+15a^3e^{4dx+4c}-13a^2b\,e^{4dx+4c}+17a\,b^2e^{4d}}{4a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a\right.}$

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)+1)-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/(a+b)^3*
((1/8*b*(3*a^2+2*a*b-b^2)/a*tanh(d*x+c)^3+(5/8*a^2+3/4*a*b+1/8*b^2)*tanh(d*
x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(3*a^2-6*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*ta
nh(d*x+c)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3744 vs. 2(123) = 246.

Time = 0.37 (sec) , antiderivative size = 7791, normalized size of antiderivative = 56.87

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(123) = 246$.

Time = 0.60 (sec) , antiderivative size = 1472, normalized size of antiderivative = 10.74

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x +
2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d
) + 1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*
x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*
b)*d) + 1/16*(5*a^4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (5*a^4 - 46*a^3*b - 4
0*a^2*b^2 + 14*a*b^3 + 3*b^4)*e^(6*d*x + 6*c) + (15*a^4 - 104*a^3*b + 58*a^
2*b^2 - 24*a*b^3 - 9*b^4)*e^(4*d*x + 4*c) + (15*a^4 - 58*a^3*b - 56*a^2*b^2
 + 26*a*b^3 + 9*b^4)*e^(2*d*x + 2*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4
*b^3 + 5*a^3*b^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^
3 - 3*a^3*b^4 - a^2*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 +
 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(4*d*x + 4*c) + 4*(a^7 + 3*a^6*b + 2*
a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(2*d*x + 2*c))*d) - 1/16*(5*a^
4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (15*a^4 - 58*a^3*b - 56*a^2*b^2 + 26*a*
b^3 + 9*b^4)*e^(-2*d*x - 2*c) + (15*a^4 - 104*a^3*b + 58*a^2*b^2 - 24*a*b^3
 - 9*b^4)*e^(-4*d*x - 4*c) + (5*a^4 - 46*a^3*b - 40*a^2*b^2 + 14*a*b^3 + 3*
b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^
4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^
5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^
4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3
 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*
a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) - 1/8*(5*a^3 + 13*a^2*b
 + 11*a*b^2 + 3*b^3 + (15*a^3 + 13*a^2*b - 11*a*b^2 - 9*b^3)*e^(-2*d*x - 2*
c) + (15*a^3 - a^2*b + 9*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) + (5*a^3 - a^2*b -
```

```
9*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4 + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-2*d*x - 2*c) + 2*(
3*a^6 + 4*a^5*b + 2*a^4*b^2 + 4*a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(
a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/4*log((a + b)*e^(4*
d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) +
a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 3/16*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(123) = 246$.

Time = 0.42 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.83

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{(3\,a^2-6\,ab-b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}} - \frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left(5\,a^3e^{(6\,dx+6\,c)}-5\,a^2be^{(6\,dx+6\,c)}-9\,ab^2e^{(6\,dx+6\,c)}+b^3e\cdots\right.}{}}{}$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^2 - 6*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*
c) + a - b)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) - 8*
(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(5*a^3*e^(6*d*x + 6*c) - 5*a^
2*b*e^(6*d*x + 6*c) - 9*a*b^2*e^(6*d*x + 6*c) + b^3*e^(6*d*x + 6*c) + 15*a^
3*e^(4*d*x + 4*c) - 13*a^2*b*e^(4*d*x + 4*c) + 17*a*b^2*e^(4*d*x + 4*c) - 3
*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*c) + a^2*b*e^(2*d*x + 2*c) - 11*
a*b^2*e^(2*d*x + 2*c) + 3*b^3*e^(2*d*x + 2*c) + 5*a^3 + 9*a^2*b + 3*a*b^2 -
 b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.86

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{a^2\,x}{(a+b)\,(a^2+2\,a\,b+b^2)} - \frac{\tanh(c+d\,x)\,(5\,a+b)}{8\,d\,(a^2+2\,a\,b+b^2)} + \frac{b^2\,x\,\tanh(c+d\,x)^4}{a^3+3\,a^2\,b+3\,a\,b^2+b^3} + \frac{2\,a\,b\,x\,\tanh(c+d\,x)^2}{a^3+3\,a^2\,b+3\,a\,b^2+b^3} - \frac{\tanh(c+d\,x)^3\,(3\,a\,b-b^2)}{8\,a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\,\tanh\left(c+d\,x\right)^2+b^2\,\tanh\left(c+d\,x\right)^4} + \frac{\mathrm{atan}\left(\frac{b\,\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(-3\,a^2+6\,a\,b+b^2\right)}{\sqrt{a\,b}\,\left(8\,a^4\,d+a\,b\,\left(24\,d\,a^2+24\,d\,a\,b+8\,d\,b^2\right)\right)}$$

```
[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] ((a^2*x)/((a + b)*(2*a*b + a^2 + b^2)) - (tanh(c + d*x)*(5*a + b))/(8*d*(2*
a*b + a^2 + b^2)) + (b^2*x*tanh(c + d*x)^4)/(3*a*b^2 + 3*a^2*b + a^3 + b^3)
 + (2*a*b*x*tanh(c + d*x)^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*
x)^3*(3*a*b - b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4
 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(6*a*b - 3
*a^2 + b^2))/((a*b)^(1/2)*(8*a^4*d + a*b*(24*a^2*d + 8*b^2*d + 24*a*b*d)))
```

3.195 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} - \frac{1}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4/(a+b)/d/(
a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 455, 46}

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} - \frac{1}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - 1/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*Ta
nh[c + d*x]^2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{b}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} \\
&\quad - \frac{1}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$=\frac{4\log(\cosh(c+dx))+2\log\left(a+b\tanh^2(c+dx)\right)-\frac{(a+b)^2}{\left(a+b\tanh^2(c+dx)\right)^2}-\frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3d}$$

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] - (a + b)^2/(a + b*Tan
h[c + d*x]^2)^2 - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2+2ab+b^2}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2+2ab+b^2}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-b\,e^{2dx+2c}+a+b\right)b\,e^{2dx+2c}}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^2d(a+b)^3}$
parallelrisch	$-\frac{4\ln(1-\tanh(dx+c))a^2b^2+4\ln(1-\tanh(dx+c))\tanh(dx+c)^4b^4-2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^4b^4+2a\,b^3\tanh(}{}$

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2*b/(a+b)^3*(1/b*ln(a+b*tanh(d*x+c)^2
)-1/2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2-(a+b)/b/(a+b*tanh(d*x+c)^2))-
1/2/(a+b)^3*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2554 vs. 2(88) = 176.

Time = 0.31 (sec) , antiderivative size = 2554, normalized size of antiderivative = 27.17

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^
2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x + a*b + b^2)*sinh(d*x + c)^6 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x + a*b + b^2
)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b -
 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (
3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)
^2 + 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^5 + 10*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a
*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 +
2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^2 + 8*(7*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x + a*b + b^2)*c
osh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b -
 2*b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*s
inh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*
cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^5 + ((3*a^2 -
 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x + a
*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
```

```
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.02

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

$$-\frac{4}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2\,(}$$

$$+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 4*((a*b + b^2)*e^(-2*d*x -
2*c) + (2*a*b - b^2)*e^(-4*d*x - 4*c) + (a*b + b^2)*e^(-6*d*x - 6*c))/((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d) + 1/2*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.61

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{2\,\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+12\,a}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right.\right.}}{4\,d}$$

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 + 12*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + 12*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.50

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} - \frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} - \frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} + \frac{\frac{\tanh(c+d\,x)^4\left(\frac{b^3}{4}+\frac{3\,a\,b^2}{4}\right)}{a^2\,d\,(a^2+2\,a\,b+b^2)}+\frac{\tanh(c+d\,x)^2\left(\frac{b^2}{2}+a\,b\right)}{a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4}$$

```
[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(a + b*tanh(c + d*x)^2)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - lo
g(1 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(tanh
(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c + d*x
)^4*((3*a*b^2)/4 + b^3/4))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*x)^2*(
a*b + b^2/2))/(a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4 + 2*a*b
*tanh(c + d*x)^2)
```

3.196 $\int \frac{1}{(a+b\tanh^2(c+dx))^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 142

$$\int \frac{1}{(a+b\tanh^2(c+dx))^3} dx = \frac{x}{(a+b)^3} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3d} + \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^3+1/8*(15*a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(
1/2)/a^(5/2)/(a+b)^3/d+1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/
8*b*(7*a+3*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3742, 425, 541, 536, 212, 211}

$$\int \frac{1}{(a+b\tanh^2(c+dx))^3} dx = \frac{b(7a+3b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^3} + \frac{b\tanh(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{x}{(a+b)^3}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a + b)^3 + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^3*d) + (b*Tanh[c + d*x])/(4*a*(a + b)*d*(
a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 3*b)*Tanh[c + d*x])/(8*a^2*(a + b)^2*
d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{b-4(a+b)+3bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+7ab+3b^2-b(7a+3b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^2d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d} + \frac{(b(15a^2+10ab+3b^2))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^3d}$$

$$= \frac{x}{(a+b)^3} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3d} + \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} - 4\log(1-\tanh(c+dx)) + 4\log(1+\tanh(c+dx)) + \frac{2b(a+b)^2\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)^2}}{8(a+b)^3d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

```
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]
)/a^(5/2) - 4*Log[1 - Tanh[c + d*x]] + 4*Log[1 + Tanh[c + d*x]] + (2*b*(a +
 b)^2*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^2) + (b*(a + b)*(7*a + 3*b)
*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)))/(8*(a + b)^3*d)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{\frac{b(7a^2+10ab+3b^2)\tanh(dx+c)^3}{8a^2}+\frac{(9a^2+14ab+5b^2)\tanh(dx+c)}{8a}}{(a+b\tanh(dx+c)^2)^2}+\frac{(15a^2+10ab+3b^2)\,a}{8a}\right.}{(a+b)^3}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{\frac{b(7a^2+10ab+3b^2)\tanh(dx+c)^3}{8a^2}+\frac{(9a^2+14ab+5b^2)\tanh(dx+c)}{8a}}{(a+b\tanh(dx+c)^2)^2}+\frac{(15a^2+10ab+3b^2)\,a}{8a}\right.}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{b(9a^3e^{6dx+6c}-a^2b\,e^{6dx+6c}-13a\,b^2e^{6dx+6c}-3\,e^{6dx+6c}b^3+27a^3e^{4dx+4c}-9a^2b\,e^{4dx+4c}+21a\,b^2e}{4(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2}}$

```
[In] int(1/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)+1)-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/(a+b)^3*
b*((1/8*b*(7*a^2+10*a*b+3*b^2)/a^2*tanh(d*x+c)^3+1/8*(9*a^2+14*a*b+5*b^2)/a
*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/
2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3587 vs. 2(128) = 256.

Time = 0.37 (sec) , antiderivative size = 7496, normalized size of antiderivative = 52.79

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(128) = 256$.

Time = 0.38 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.57

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\left(15\,a^2b+10\,ab^2+3\,b^3\right)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{ab}d} + \frac{9\,a^3b+21\,a^2b^2+15\,ab^3+3\,b^4+(27\,a^3b}{4\,(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5+4\,(a^7+3\,a^6b+2\,a^5b^2-2\,a^4b^3-3\,a^3b^4-a^2b^5)e^{(-2\,dx-}} + \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/4*
(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^
3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(
-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((
a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*
a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(
3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x
- 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^
(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2
*b^5)*e^(-8*d*x - 8*c))*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(128) = 256$.

Time = 0.33 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.88

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{(15\,a^2b+10\,ab^2+3\,b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab}}+\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,(9\,a^3be^{(6\,dx+6\,c)}-a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}}{}}{}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d
*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a
*b)) + 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x +
6*c) - a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 3*b^4*e^(6*d*x
+ 6*c) + 27*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4*d*x + 4*c) + 21*a*b^3*e^
(4*d*x + 4*c) + 9*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 13*a^2*b
^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 9*b^4*e^(2*d*x + 2*c) + 9*a
^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3
)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}\\ &\quad-\frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}\\ &\quad+\frac{\frac{\tanh(c+d\,x)^3\,\left(\frac{3\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)}{a^2\,d\,(a^2+2\,a\,b+b^2)}+\frac{\tanh(c+d\,x)\,(5\,b^2+9\,a\,b)}{8\,a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4}\\ &\quad+\frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,(15\,a^2\,b+10\,a\,b^2+3\,b^3)}{\sqrt{a\,b}\,\left(8\,a^5\,d+a\,b\,\left(24\,a^3\,d+a\,b\,\left(24\,a\,d+8\,b\,d\right)\right)\right)}\end{aligned}$$

```
[In] int(1/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(1
- tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c + d
*x)^3*((7*a*b^2)/8 + (3*b^3)/8))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*
```

```
x)*(9*a*b + 5*b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4
 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(10*a*b^2
+ 15*a^2*b + 3*b^3))/((a*b)^(1/2)*(8*a^5*d + a*b*(24*a^3*d + a*b*(24*a*d +
8*b*d))))
```

3.197 $\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log(\tanh(c+dx))}{a^3 d} - \frac{b(3a^2+3ab+b^2)\log(a+b\tanh^2(c+dx))}{2a^3(a+b)^3 d} + \frac{b}{4a(a+b)d(a+b\tanh^2(c+dx))^2} + \frac{b(2a+b)}{2a^2(a+b)^2 d(a+b\tanh^2(c+dx))}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+ln(tanh(d*x+c))/a^3/d-1/2*b*(3*a^2+3*a*b+b^2)*ln(
a+b*tanh(d*x+c)^2)/a^3/(a+b)^3/d+1/4*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*
b*(2*a+b)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 457, 84}

$$\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{\log(\tanh(c+dx))}{a^3 d} + \frac{b(2a+b)}{2a^2 d(a+b)^2(a+b\tanh^2(c+dx))} - \frac{b(3a^2+3ab+b^2)\log(a+b\tanh^2(c+dx))}{2a^3 d(a+b)^3} + \frac{b}{4ad(a+b)(a+b\tanh^2(c+dx))^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[Tanh[c + d*x]]/(a^3*d) - (b*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + b/(4*a*(a +
 b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(2*a + b))/(2*a^2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x} - \frac{b^2}{a(a+b)(a+bx)^3} - \frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)^2} - \frac{b^2(3a^2+3ab+b^2)}{a^3(a+b)^3(a+bx)}\right)dx, x, \tanh^2(c+dx\right.}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(\tanh(c+dx))}{a^3d} - \frac{b(3a^2+3ab+b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^3d}\\
&\quad + \frac{b}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(2a+b)}{2a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$=\frac{\frac{4\log(\cosh(c+dx))}{(a+b)^3}+\frac{4\log(\tanh(c+dx))+\frac{b\left(-2\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right)+\frac{a(a+b)\left(a(5a+3b)+2b(2a+b)\tanh^2(c+dx)\right)}{\left(a+b\tanh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{a^3}}{4d}$$

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((4*Log[Cosh[c + d*x]])/(a + b)^3 + (4*Log[Tanh[c + d*x]] + (b*(-2*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)*(a*(5*a + 3*b) + 2*b*
(2*a + b)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x]^2)^2))/(a + b)^3)/a^3)/(4*
d)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{-\frac{\ln(\tanh(dx+c))}{a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b^2\left(-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a\left(2a^2+3ab+b\right.}{b\left(a+b\tanh(dx+\right.}\right.}{2(a+b)^3a^3}}{d}$
default	$-\frac{-\frac{\ln(\tanh(dx+c))}{a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b^2\left(-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a\left(2a^2+3ab+b\right.}{b\left(a+b\tanh(dx+\right.}\right.}{2(a+b)^3a^3}}{d}$
parallelrisch	$-24\left(\frac{(a+b)^2\cosh(4dx+4c)}{4}+\left(a^2-b^2\right)\cosh(2dx+2c)+\frac{3a^2}{4}-\frac{ab}{2}+\frac{3b^2}{4}\right)\left(a^2+ab+\frac{1}{3}b^2\right)b\ln\left(a+b\tanh(dx+c)^2\right)-16\left(\frac{(a+b)^2}{\ldots}\right.$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6bc}{ad(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6b^2x}{a^2(a^3+3a^2b+3a\,b^2+b^3}$

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/a^3*ln(tanh(d*x+c))+1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2*b^2/(a+b)^3/
a^3*(-1/2*a^2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2+(3*a^2+3*a*b+b^2)/b*l
n(a+b*tanh(d*x+c)^2)-a*(2*a^2+3*a*b+b^2)/b/(a+b*tanh(d*x+c)^2))+1/2/(a+b)^3
*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4800 vs. $2(132) = 264$.

Time = 0.52 (sec) , antiderivative size = 4800, normalized size of antiderivative = 34.78

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 2*a^4*b +
 a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d
*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d
*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 14*(a^5 + 2*a^4*b
+ a^3*b^2)*d*x*cosh(d*x + c)^2 - 2*(a^5 - a^3*b^2)*d*x)*sinh(d*x + c)^6 + 8
*(14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 4*a^2*b
^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(6*a
^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x
+ c)^4 + 4*(35*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 +
2*a^2*b^3 + 2*a*b^4 + (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x - 15*(3*a^3*b^2 + 4
*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 16*(7*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 5*(3*a^3*b^2 + 4*a^
2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 - 2*a^2
*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^
4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 2*a^4*b + a^3*b^2
)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 15*(3*a^3*b^2 + 4*a
^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - a^3*b^2)
*d*x - 6*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d
*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 +
 5*a*b^4 + b^5)*cosh(d*x + c)^8 + 8*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4*b + 9*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*sinh(d*x + c)^8 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*cosh(d*x + c)^6 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b
^4 - b^5 + 7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5)*cosh(d*x + c)^3 + 3*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*
cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c
)^4 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(3*a^4*b +
9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^4 + 30*(3*a^4*b + 3*a
^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7
*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^5 + 10*(3
*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^3 + (9*a^4*b
+ 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 +
```

4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2 + 4*(7
*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^6 + 3*a^4
*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(3*a^4*b + 3*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^4 + 3*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3
+ 7*a*b^4 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4*b + 9*a^3*
b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^7 + 3*(3*a^4*b + 3*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^5 + (9*a^4*b + 3*a^3*b^2 + 6*a^2
*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + (3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)
^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^
2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^6 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^3 + 3*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x +
c)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2*(3*a^5
+ 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 2*(3
*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^4 + 30*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(
d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cos
h(d*x + c)^3 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*cosh(d*x + c)^2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4
+ 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^5 + 5*a^4*b + 10*a^3*b^2
+ 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b
^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3
*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^5 + 2*a^4*b + a^3*b^2)*
d*x*cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*
d*x)*cosh(d*x + c)^5 - 2*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*
b + 3*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a
^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b
^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^8 + 5*a^7*b
+ 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x
+ c)^7 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*

```
sinh(d*x + c)^8 + 4*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^
3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 +
5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5
*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 7*a^7*b + 6*a^6
*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 +
 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5
*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a
^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^8 + 7*a^7*b + 6*a^6*
b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 3*a^
7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7
*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x
 + c)^5 + 10*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^3 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 +
3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 5*a^7*b + 10*a^6*
b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^8 + 3*a^7
*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(3*
a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x +
 c)^2 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*si
nh(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*
b^5)*d + 8*((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)
*d*cosh(d*x + c)^7 + 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 -
 a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*
a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5
*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\coth\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(132) = 264$.

Time = 0.23 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.61

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= -\frac{(3\,a^2b+3\,ab^2+b^3)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)d}$$
$$+\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$+\frac{2\,\big((3\,a^2b}{(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5+4\,(a^7+3\,a^6b+2\,a^5b^2-2\,a^4b^3-3\,a^3b^4-a^2b^5)e^{(-2\,dx-}}$$
$$+\frac{\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(
-4*d*x - 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + (d*x + c
)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((3*a^2*b^2 + 4*a*b^3 + b^4)*e^(-
2*d*x - 2*c) + 2*(3*a^2*b^2 - a*b^3 - b^4)*e^(-4*d*x - 4*c) + (3*a^2*b^2 +
4*a*b^3 + b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3
+ 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^
4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3
+ 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 -
2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5
*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + log(e^(-d*x
 - c) + 1)/(a^3*d) + log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(132) = 264$.

Time = 0.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.14

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$
$$-\frac{\frac{(3\,a^2b+3\,ab^2+b^3)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{2\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^3}-}{2\,d}$$

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) +
 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^6 + 3*a^5*b + 3*a^4*
b^2 + a^3*b^3) + 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*log(abs(e^
(2*d*x + 2*c) - 1))/a^3 - 4*((3*a^2*b^2 + a*b^3)*e^(6*d*x + 6*c) + (3*a^2*b
^2 + a*b^3)*e^(2*d*x + 2*c) + 2*(3*a^3*b^2 - a^2*b^3 - a*b^4)*e^(4*d*x + 4*
c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^2*a^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\coth(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.198 $\int \frac{\coth^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{\coth^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} - \frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3d} - \frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\,(a+b\tanh^2(c+dx))^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\,(a+b\tanh^2(c+dx))}$$

```
[Out] x/(a+b)^3-1/8*b^(3/2)*(35*a^2+42*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/a^(7/2)/(a+b)^3/d-1/8*(8*a^2+27*a*b+15*b^2)*coth(d*x+c)/a^3/(a+b)^2/d
+1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(9*a+5*b)*coth(d*x
+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{3751, 483, 593, 597, 536, 212, 211\}$

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{b(9a+5b)\coth(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} - \frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^3} - \frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3d(a+b)^2} + \frac{b\coth(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{x}{(a+b)^3}$$

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(7/2)*(a + b)^3*d) - ((8*a^2 + 27*a*b + 15*b^2)*Coth[c
 + d*x])/(8*a^3*(a + b)^2*d) + (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) + (b*(9*a + 5*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-5b+5bx^2}{x^2(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+27ab+15b^2-3b(9a+5b)x^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^2d}$$

$$= -\frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8a^3+8a^2b+27ab^2+15b^3-b(8a^2+27ab+15b^2)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a^3(a+b)^2d}$$

$$= -\frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d} - \frac{(b^2(35a^2+42ab+15b^2))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^3(a+b)^3d}$$

$$= \frac{x}{(a+b)^3} - \frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3d} - \frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 6.80 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{-\frac{8(c+dx)}{(a+b)^3} + \frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^3} + \frac{8\coth(c+dx)}{a^3} + \frac{4b^3\sinh(2(c+dx))}{a^2(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{b^2(13a-}{a^3(a+b)^2(a-}}{8d}$$

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((-8*(c + d*x))/(a + b)^3 + (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^3) + (8*Coth[c + d*x])/
a^3 + (4*b^3*Sinh[2*(c + d*x)])/(a^2*(a + b)^2*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (b^2*(13*a + 7*b)*Sinh[2*(c + d*x)])/(a^3*(a + b)^2*(a - b + (
a + b)*Cosh[2*(c + d*x)])))/d
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{9}{4}a\,b^2+\frac{7}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(13a^2+22ab+9b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(35a^2+42ab+15b^2\right)\arctan\left(\right.}{8\sqrt{ab}}\right.}{(a+b)^3a^3}}{d}$
default	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{9}{4}a\,b^2+\frac{7}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(13a^2+22ab+9b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(35a^2+42ab+15b^2\right)\arctan\left(\right.}{8\sqrt{ab}}\right.}{(a+b)^3a^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{-60b^5e^{6dx+6c}+90b^5e^{4dx+4c}+8a^5+15b^5+40a^4b+93a^3b^2+113a^2b^3+67a\,b^4-60b^5e^{2dx+2c}+66a}{}$

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)+1)+b^2/(a+b)^3/a^3*(((11/8*a^2*b+9/4*a*b^
2+7/8*b^3)*tanh(d*x+c)^3+1/8*a*(13*a^2+22*a*b+9*b^2)*tanh(d*x+c))/(a+b*tanh
(d*x+c)^2)^2+1/8*(35*a^2+42*a*b+15*b^2)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a
*b)^(1/2)))+1/a^3/tanh(d*x+c)+1/2/(a+b)^3*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5772 vs. 2(162) = 324.

Time = 0.44 (sec) , antiderivative size = 11865, normalized size of antiderivative = 66.66

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth^2{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^3}\,dx$$

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. 2(162) = 324.

Time = 0.65 (sec) , antiderivative size = 1944, normalized size of antiderivative = 10.92

$$\int \frac{\coth^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(3*a^2*b + 3*a*b^2 + b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2
*d*x + 2*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/4*(3*a^2
*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*
c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/32*(15*a^3*b - 25
*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)*d) - 1/32*(15*a
^3*b - 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c)
 + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)*d) +
1/16*(8*a^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b^5 + (8*a
^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e^(8*d*x + 8*c) + 2*(16*a^5
 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^5)*e^(6*d*x + 6*c) +
 2*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45*b^5)*e^(4*d
*x + 4*c) + 2*(16*a^5 + 39*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 65*a*b^4 - 30*
b^5)*e^(2*d*x + 2*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
 + a^3*b^5 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
)*e^(10*d*x + 10*c) - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^
4 - 5*a^3*b^5)*e^(8*d*x + 8*c) - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 +
13*a^4*b^4 + 5*a^3*b^5)*e^(6*d*x + 6*c) + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a
^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^(4*d*x + 4*c) + (3*a^8 + 7*a^7*b - 2*a^6
*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(2*d*x + 2*c))*d) - 1/16*(8*a
^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b^5 + 2*(16*a^5 + 3
9*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 65*a*b^4 - 30*b^5)*e^(-2*d*x - 2*c) + 2
*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45*b^5)*e^(-4*d*
x - 4*c) + 2*(16*a^5 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^
5)*e^(-6*d*x - 6*c) + (8*a^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e
^(-8*d*x - 8*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^
3*b^5 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)
*e^(-2*d*x - 2*c) + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 +
5*a^3*b^5)*e^(-4*d*x - 4*c) - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*
a^4*b^4 + 5*a^3*b^5)*e^(-6*d*x - 6*c) - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a
^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(-8*d*x - 8*c) - (a^8 + 5*a^7*b + 10*a^6
*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-10*d*x - 10*c))*d) - 1/8*(8*a^
4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a^3*b - 55*a
*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b
^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*e
```

```
^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*e^(-8
*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (3*a^7 + 4
*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 2*
a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7 + 2*a^5*b^2 + 8*
a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a
^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/16*b*arctan(1/2*((a + b)*e^(-2*d*
x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) + 1/2*log(e^(2*d*x + 2*c) -
1)/(a^3*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(162) = 324$.

Time = 0.46 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.46

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{\left(35\,a^2b^2+42\,ab^3+15\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{ab}} - \frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\,\left(13\,a^3b^2e^{(6\,dx+6\,c)}+3\,a^2b^3e^{(6\,dx+6\,c)}-17\,ab^4e}{}}{}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((35*a^2*b^2 + 42*a*b^3 + 15*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^
(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e^(6
*d*x + 6*c) + 3*a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 7*b^5*
e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 5*a^2*b^3*e^(4*d*x + 4*c) +
25*a*b^4*e^(4*d*x + 4*c) + 21*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2
*c) + 25*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 21*b^5*e^(2*d
*x + 2*c) + 13*a^3*b^2 + 33*a^2*b^3 + 27*a*b^4 + 7*b^5)/((a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{{\mathrm{coth}\left(c+d\,x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.199 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$
\begin{aligned}
\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = &-\frac{\coth^2(c+dx)}{2a^3d}+\frac{\log(\cosh(c+dx))}{(a+b)^3d} \\
&+\frac{(a-3b)\log(\tanh(c+dx))}{a^4d} \\
&+\frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^4(a+b)^3d} \\
&-\frac{b^2}{4a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\
&-\frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

```
[Out] -1/2*coth(d*x+c)^2/a^3/d+ln(cosh(d*x+c))/(a+b)^3/d+(a-3*b)*ln(tanh(d*x+c))/
a^4/d+1/2*b^2*(6*a^2+8*a*b+3*b^2)*ln(a+b*tanh(d*x+c)^2)/a^4/(a+b)^3/d-1/4*b
^2/a^2/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/(a+b*t
anh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 90}

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(a-3b)\log(\tanh(c+dx))}{a^4d} - \frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} - \frac{\coth^2(c+dx)}{2a^3d} - \frac{b^2}{4a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^4d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*Coth[c + d*x]^2/(a^3*d) + Log[Cosh[c + d*x]]/((a + b)^3*d) + ((a - 3*b
)*Log[Tanh[c + d*x]])/(a^4*d) + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[a + b*Tanh
[c + d*x]^2])/(2*a^4*(a + b)^3*d) - b^2/(4*a^2*(a + b)*d*(a + b*Tanh[c + d*
x]^2)^2) - (b^2*(3*a + 2*b))/(2*a^3*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x^2} + \frac{a-3b}{a^4x} + \frac{b^3}{a^2(a+b)(a+bx)^3} + \frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)^2} + \frac{b^3(6a^2+8ab+3b^2)}{a^4(a+b)^3(a+bx)}\right)dx, x, \tan\right.}{2d}$$

$$= -\frac{\coth^2(c+dx)}{2a^3d} + \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{(a-3b)\log(\tanh(c+dx))}{a^4d} + \frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^4(a+b)^3d} - \frac{b^2}{4a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{\coth^2(c+dx)}{a^3} + \frac{b^4}{2a^4(a+b)\left(b+a\coth^2(c+dx)\right)^2} - \frac{b^3(4a+3b)}{a^4(a+b)^2\left(b+a\coth^2(c+dx)\right)} - \frac{b^2(6a^2+8ab+3b^2)\log\left(b+a\coth^2(c+dx)\right)}{a^4(a+b)^3} - \frac{2\log(\sinh(c}{(a+b)}}{2d}$$

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(Coth[c + d*x]^2/a^3 + b^4/(2*a^4*(a + b)*(b + a*Coth[c + d*x]^2)^2) -
 (b^3*(4*a + 3*b))/(a^4*(a + b)^2*(b + a*Coth[c + d*x]^2)) - (b^2*(6*a^2 +
8*a*b + 3*b^2)*Log[b + a*Coth[c + d*x]^2])/(a^4*(a + b)^3) - (2*Log[Sinh[c
+ d*x]])/(a + b)^3)/d
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{b^3\left(\frac{(6a^2+8ab+3b^2)\ln(a+b\tanh(dx+c)^2)}{b}-\frac{a^2(a^2+2ab+b^2)}{2b(a+b\tanh(dx+c)^2)^2}-\frac{a(3a^2+5ab+2b^2)}{b(a+b\tanh(dx+c)^2)}\right)}{2(a+b)^3a^4}+\frac{\ln(\tanh(dx}{2(a+b}}{d}$
default	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{b^3\left(\frac{(6a^2+8ab+3b^2)\ln(a+b\tanh(dx+c)^2)}{b}-\frac{a^2(a^2+2ab+b^2)}{2b(a+b\tanh(dx+c)^2)^2}-\frac{a(3a^2+5ab+2b^2)}{b(a+b\tanh(dx+c)^2)}\right)}{2(a+b)^3a^4}+\frac{\ln(\tanh(dx}{2(a+b}}{d}$
parallelrisch	$48(a^2+\frac{4}{3}ab+\frac{1}{2}b^2)\left(\left(\frac{(a+b)^2\cosh(4dx+4c)}{4}+(a^2-b^2)\cosh(2dx+2c)+\frac{3a^2}{4}-\frac{ab}{2}+\frac{3b^2}{4}\right)b^2\ln\left(a+b\tanh(dx+c)^2\right)-16\left(\frac{(a+b)^2}{}\right.\right.$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a^4}+\frac{6bc}{da^4}-\frac{12b^2x}{a^2(a^3+3a^2b+3ab^2+b^3)}-\frac{12b^2c}{a^2d(a^3+3a^2b+3ab^2+b^3)}-\frac{}{a^3(}$

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2*b^3/(a+b)^3/a^4*((6*a^2+8*a*b+3*b^2
)/b*ln(a+b*tanh(d*x+c)^2)-1/2*a^2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2-a
*(3*a^2+5*a*b+2*b^2)/b/(a+b*tanh(d*x+c)^2))+1/2/(a+b)^3*ln(tanh(d*x+c)+1)+(
-a+3*b)/a^4*ln(tanh(d*x+c))+1/2/a^3/tanh(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10720 vs. 2(163) = 326.

Time = 0.71 (sec) , antiderivative size = 10720, normalized size of antiderivative = 62.69

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(163) = 326$.

Time = 0.27 (sec) , antiderivative size = 770, normalized size of antiderivative = 4.50

$$\begin{aligned}&\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx\\&= \frac{(6\,a^2b^2+8\,ab^3+3\,b^4)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)d}\\&\quad+\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}\\&\quad-\frac{2\left((a^5+5\,a^4b+10\,a^3b^2+14\,a^2b^3+11\,ab^4+\right.}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5+2\,(a^8+a^7b-6\,a^6b^2-14\,a^5b^3-11\,a^4b^4-3\,a^3b^5)e^{(-2\,dx}}\\&\quad+\frac{(a-3\,b)\log\left(e^{(-dx-c)}+1\right)}{a^4d}+\frac{(a-3\,b)\log\left(e^{(-dx-c)}-1\right)}{a^4d}\end{aligned}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] 1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*
e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + (d*x
+ c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 +
 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 6*a^4*b + 4*a
^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 7*a^4*
b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 +
 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-8*d*x - 8*c) + (a^
5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-10*d*x - 10*c
))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 2*(a^8
 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-2*d*x - 2*c
) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b^4 - 15*a^3*b^5)*e^(-
4*d*x - 4*c) - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3
*b^5)*e^(-6*d*x - 6*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b
^4 - 15*a^3*b^5)*e^(-8*d*x - 8*c) + 2*(a^8 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3
 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-10*d*x - 10*c) + (a^8 + 5*a^7*b + 10*a^6*b^2
 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-12*d*x - 12*c))*d) + (a - 3*b)*log
(e^(-d*x - c) + 1)/(a^4*d) + (a - 3*b)*log(e^(-d*x - c) - 1)/(a^4*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(163) = 326$.

Time = 0.54 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.77

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\frac{(6a^2b^2+8ab^3+3b^4)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^7+3a^6b+3a^5b^2+a^4b^3} - \frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(a-3b)\log\left(\left|e^{(2dx+2c)}-1\right|\right.}{a^4}}{}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*((6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c
) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^7 + 3*a^6*b + 3*a
^5*b^2 + a^4*b^3) - 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a - 3*
b)*log(abs(e^(2*d*x + 2*c) - 1))/a^4 - 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 14*
a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(10*d*x + 10*c) + 2*(2*a^5 + 6*a^4*b + 4*a^3*
b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(8*d*x + 8*c) + 2*(3*a^5 + 7*a^4*b +
6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(6*d*x + 6*c) + 2*(2*a^5 + 6*a^
4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(4*d*x + 4*c) + (a^5 + 5*
a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/((a*e^
(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*
c) + a + b)^2*(a + b)^3*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

```
[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

3.200 $\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} + \frac{b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^3+1/8*b^(5/2)*(63*a^2+90*a*b+35*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/a^(9/2)/(a+b)^3/d-1/8*(8*a^3-8*a^2*b-55*a*b^2-35*b^3)*coth(d*x+c)/a^4
/(a+b)^2/d-1/24*(8*a^2+55*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a+b)^2/d+1/4*b*cot
h(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(11*a+7*b)*coth(d*x+c)^3/a
^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3751, 483, 593, 597, 536, 212, 211}

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{b(11a+7b)\coth^3(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)} + \frac{b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a+b)^3} - \frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3d(a+b)^2} - \frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4d(a+b)^2} + \frac{b\coth^3(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{x}{(a+b)^3}$$

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 + (b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(9/2)*(a + b)^3*d) - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35
*b^3)*Coth[c + d*x])/(8*a^4*(a + b)^2*d) - ((8*a^2 + 55*a*b + 35*b^2)*Coth[
c + d*x]^3)/(24*a^3*(a + b)^2*d) + (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2*d
*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-7b+7bx^2}{x^4(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4a(a+b)d}$$

$$= \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{8a^2+55ab+35b^2-5b(11a+7b)x^2}{x^4(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^2d}$$

$$= -\frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-3(8a^3-8a^2b-55ab^2-35b^3)-3b(8a^2+55ab+35b^2)x^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{24a^3(a+b)^2d}$$

$$= -\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{3(8a^4-8a^3b+8a^2b^2+55ab^3+35b^4)+3b(8a^3-8a^2b-55ab^2-35b^3)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{24a^4(a+b)^2d}$$

$$= -\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^3d} + \frac{(b^3(63a^2+90ab+35b^2))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{8a^4(a+b)^3d}$$

$$= \frac{x}{(a+b)^3} + \frac{b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{24(c+dx)}{(a+b)^3}+\frac{3b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{9/2}(a+b)^3}+\frac{8(-4a+9b)\coth(c+dx)}{a^4}-\frac{8\coth(c+dx)\operatorname{csch}^2(c+dx)}{a^3}+\frac{12b^4\sinh}{a^3(a+b)^2(a-b+(a}}{24d}$$

[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((24*(c + d*x))/(a + b)^3 + (3*b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(9/2)*(a + b)^3) + (8*(-4*a + 9*b)*Coth[c + d*x])/a^4 - (8*Coth[c + d*x]*Csch[c + d*x]^2)/a^3 + (12*b^4*Sinh[2*(c + d*x)])/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (3*b^3*(17*a + 11*b)*Sinh[2*(c + d*x)])/(a^4*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(24*d)

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{13}{4}ab^2+\frac{11}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(17a^2+30ab+13b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(63a^2+9}{}\right.}{(a+b)^3a^4}}{d}$
default	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{13}{4}ab^2+\frac{11}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(17a^2+30ab+13b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(63a^2+9}{}\right.}{(a+b)^3a^4}}{d}$
risch	Expression too large to display

[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] -1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2/(a+b)^3*ln(tanh(d*x+c)+1)-b^3/(a+b)^3/a^4*((((15/8*a^2*b+13/4*a*b^2+11/8*b^3)*tanh(d*x+c)^3+1/8*a*(17*a^2+30*a*b+13*b^2)*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(63*a^2+90*a*b+35*b^2)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))-(-a+3*b)/a^4/tanh(d*x+c)+1/3/a^3/tanh(d*x+c)^3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10858 vs. $2(210) = 420$.

Time = 0.55 (sec) , antiderivative size = 22038, normalized size of antiderivative = 96.66

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. $2(210) = 420$.

Time = 1.08 (sec) , antiderivative size = 4285, normalized size of antiderivative = 18.79

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
[Out] -1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log((a + b)*e^(4*d*x + 4*c) +
2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d
) + 1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2
*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*d) + 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*arctan(1/2*(
(a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*sqrt(a*b)*d) - 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5
)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*sqrt(a*b)*d) + 1/192*(176*a^6 + 781*a^5*b + 1571*a^4
*b^2 + 1538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 3*(96*a^6 + 465*a
^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 35*b^6)*e^(12*d
```

*x + 12*c) + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 - 728*a^3*b^3 - 1070*a^2*
b^4 - 240*a*b^5 + 105*b^6)*e^(10*d*x + 10*c) + (176*a^6 - 281*a^5*b + 3509*
a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*a*b^5 - 1575*b^6)*e^(8*d*x +
8*c) - 4*(184*a^6 + 48*a^5*b + 473*a^4*b^2 + 970*a^3*b^3 + 3684*a^2*b^4 + 1
070*a*b^5 - 525*b^6)*e^(6*d*x + 6*c) - (384*a^6 + 1127*a^5*b - 861*a^4*b^2
- 7146*a^3*b^3 - 11386*a^2*b^4 - 1965*a*b^5 + 1575*b^6)*e^(4*d*x + 4*c) + 2
*(136*a^6 - 96*a^5*b - 1309*a^4*b^2 - 2996*a^3*b^3 - 2238*a^2*b^4 - 4*a*b^5
 + 315*b^6)*e^(2*d*x + 2*c))/((a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*
a^5*b^4 + a^4*b^5 - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 +
a^4*b^5)*e^(14*d*x + 14*c) - (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*
a^5*b^4 - 7*a^4*b^5)*e^(12*d*x + 12*c) + (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66
*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^(10*d*x + 10*c) + (3*a^9 - a^8*b + 14
*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(8*d*x + 8*c) - (3*a^9 -
 a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(6*d*x + 6*c)
 - (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^
(4*d*x + 4*c) + (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a
^4*b^5)*e^(2*d*x + 2*c))*d) - 1/192*(176*a^6 + 781*a^5*b + 1571*a^4*b^2 + 1
538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 2*(136*a^6 - 96*a^5*b - 1
309*a^4*b^2 - 2996*a^3*b^3 - 2238*a^2*b^4 - 4*a*b^5 + 315*b^6)*e^(-2*d*x -
2*c) - (384*a^6 + 1127*a^5*b - 861*a^4*b^2 - 7146*a^3*b^3 - 11386*a^2*b^4 -
 1965*a*b^5 + 1575*b^6)*e^(-4*d*x - 4*c) - 4*(184*a^6 + 48*a^5*b + 473*a^4*
b^2 + 970*a^3*b^3 + 3684*a^2*b^4 + 1070*a*b^5 - 525*b^6)*e^(-6*d*x - 6*c) +
 (176*a^6 - 281*a^5*b + 3509*a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*
a*b^5 - 1575*b^6)*e^(-8*d*x - 8*c) + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 -
 728*a^3*b^3 - 1070*a^2*b^4 - 240*a*b^5 + 105*b^6)*e^(-10*d*x - 10*c) + 3*(
96*a^6 + 465*a^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 3
5*b^6)*e^(-12*d*x - 12*c))/((a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^
5*b^4 + a^4*b^5 + (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7
*a^4*b^5)*e^(-2*d*x - 2*c) - (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 6
5*a^5*b^4 - 21*a^4*b^5)*e^(-4*d*x - 4*c) - (3*a^9 - a^8*b + 14*a^7*b^2 + 78
*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(-6*d*x - 6*c) + (3*a^9 - a^8*b + 14*
a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(-8*d*x - 8*c) + (3*a^9 +
 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^(-10*d*x -
10*c) - (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*
e^(-12*d*x - 12*c) - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 +
 a^4*b^5)*e^(-14*d*x - 14*c))*d) + 1/48*(32*a^5 + 83*a^4*b - 60*a^3*b^2 - 3
46*a^2*b^3 - 340*a*b^4 - 105*b^5 + 3*(32*a^5 + 95*a^4*b + 154*a^3*b^2 + 84*
a^2*b^3 - 42*a*b^4 - 35*b^5)*e^(12*d*x + 12*c) + 6*(48*a^5 + 40*a^4*b - 117
*a^3*b^2 - 201*a^2*b^3 + 45*a*b^4 + 105*b^5)*e^(10*d*x + 10*c) + (224*a^5 +
 281*a^4*b + 384*a^3*b^2 + 2318*a^2*b^3 - 160*a*b^4 - 1575*b^5)*e^(8*d*x +
8*c) - 4*(16*a^5 - 136*a^4*b - 9*a^3*b^2 + 697*a^2*b^3 - 115*a*b^4 - 525*b^
5)*e^(6*d*x + 6*c) - (96*a^5 + 137*a^4*b - 1262*a^3*b^2 - 1840*a^2*b^3 + 12
30*a*b^4 + 1575*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 - 136*a^4*b - 435*a^3*b^2
- 35*a^2*b^3 + 563*a*b^4 + 315*b^5)*e^(2*d*x + 2*c))/((a^8 + 4*a^7*b + 6*a^
6*b^2 + 4*a^5*b^3 + a^4*b^4 - (a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*

b^4)*e^(14*d*x + 14*c) - (a^8 - 4*a^7*b - 18*a^6*b^2 - 20*a^5*b^3 - 7*a^4*b ^4)*e^(12*d*x + 12*c) + (3*a^8 + 4*a^7*b - 22*a^6*b^2 - 44*a^5*b^3 - 21*a^4 *b^4)*e^(10*d*x + 10*c) + (3*a^8 - 4*a^7*b + 18*a^6*b^2 + 60*a^5*b^3 + 35*a ^4*b^4)*e^(8*d*x + 8*c) - (3*a^8 - 4*a^7*b + 18*a^6*b^2 + 60*a^5*b^3 + 35*a ^4*b^4)*e^(6*d*x + 6*c) - (3*a^8 + 4*a^7*b - 22*a^6*b^2 - 44*a^5*b^3 - 21*a ^4*b^4)*e^(4*d*x + 4*c) + (a^8 - 4*a^7*b - 18*a^6*b^2 - 20*a^5*b^3 - 7*a^4* b^4)*e^(2*d*x + 2*c))*d) - 1/48*(32*a^5 + 83*a^4*b - 60*a^3*b^2 - 346*a^2*b ^3 - 340*a*b^4 - 105*b^5 + 2*(16*a^5 - 136*a^4*b - 435*a^3*b^2 - 35*a^2*b^3 + 563*a*b^4 + 315*b^5)*e^(-2*d*x - 2*c) - (96*a^5 + 137*a^4*b - 1262*a^3*b ^2 - 1840*a^2*b^3 + 1230*a*b^4 + 1575*b^5)*e^(-4*d*x - 4*c) - 4*(16*a^5 - 1 36*a^4*b - 9*a^3*b^2 + 697*a^2*b^3 - 115*a*b^4 - 525*b^5)*e^(-6*d*x - 6*c) + (224*a^5 + 281*a^4*b + 384*a^3*b^2 + 2318*a^2*b^3 - 160*a*b^4 - 1575*b^5) *e^(-8*d*x - 8*c) + 6*(48*a^5 + 40*a^4*b - 117*a^3*b^2 - 201*a^2*b^3 + 45*a *b^4 + 105*b^5)*e^(-10*d*x - 10*c) + 3*(32*a^5 + 95*a^4*b + 154*a^3*b^2 + 8 4*a^2*b^3 - 42*a*b^4 - 35*b^5)*e^(-12*d*x - 12*c))/((a^8 + 4*a^7*b + 6*a^6* b^2 + 4*a^5*b^3 + a^4*b^4 + (a^8 - 4*a^7*b - 18*a^6*b^2 - 20*a^5*b^3 - 7*a^ 4*b^4)*e^(-2*d*x - 2*c) - (3*a^8 + 4*a^7*b - 22*a^6*b^2 - 44*a^5*b^3 - 21*a ^4*b^4)*e^(-4*d*x - 4*c) - (3*a^8 - 4*a^7*b + 18*a^6*b^2 + 60*a^5*b^3 + 35* a^4*b^4)*e^(-6*d*x - 6*c) + (3*a^8 - 4*a^7*b + 18*a^6*b^2 + 60*a^5*b^3 + 35 *a^4*b^4)*e^(-8*d*x - 8*c) + (3*a^8 + 4*a^7*b - 22*a^6*b^2 - 44*a^5*b^3 - 2 1*a^4*b^4)*e^(-10*d*x - 10*c) - (a^8 - 4*a^7*b - 18*a^6*b^2 - 20*a^5*b^3 - 7*a^4*b^4)*e^(-12*d*x - 12*c) - (a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^ 4*b^4)*e^(-14*d*x - 14*c))*d) + 1/32*(16*a^4 + 147*a^3*b + 351*a^2*b^2 + 32 5*a*b^3 + 105*b^4 + 2*(8*a^4 + 32*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*e^(-2*d*x - 2*c) - (96*a^4 + 313*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b ^4)*e^(-4*d*x - 4*c) - 4*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525* b^4)*e^(-6*d*x - 6*c) - (176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 15 75*b^4)*e^(-8*d*x - 8*c) - 6*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*e^(-10*d*x - 10*c) + 15*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*e^(-12*d*x - 12*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-2*d*x - 2*c) - (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)* e^(-4*d*x - 4*c) - (3*a^7 - 7*a^6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-6*d*x - 6*c) + (3*a^7 - 7*a^6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-8*d*x - 8*c) + (3*a^ 7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-10*d*x - 10*c) - (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-12*d*x - 12*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a ^4*b^3)*e^(-14*d*x - 14*c))*d) + 3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(a^4*d) - 3/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^4*d) + 1/4*(2*a - 3*b)*log(e^(2*d*x + 2*c) - 1)/(a^4*d) - 3/2*b*log(e^(2*d*x + 2*c) - 1)/(a^4*d) - 1/4*(2*a - 3*b)*log(e^(-2*d*x - 2*c) - 1)/(a^4*d) + 3/2*b*log(e^(-2*d*x - 2*c) - 1)/(a ^4*d) - 5/32*(3*a*b - 7*b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s qrt(a*b))/(sqrt(a*b)*a^4*d) - 15/64*(3*a*b + 7*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^4*d) + 5/32*(3*a*b - 7*b^2)* arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^4*d)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(210) = 420.

Time = 0.56 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.16

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(63\,a^2b^3+90\,ab^4+35\,b^5)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{ab}}+\frac{24\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{6\,(17\,a^3b^3e^{(6\,dx+6\,c)}+7\,a^2b^4e^{(6\,dx+6\,c)}-21\,ab^5e^{(6\,d}}{}}{}$$

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(63*a^2*b^3 + 90*a*b^4 + 35*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*
sqrt(a*b)) + 24*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(17*a^3*b^3*e
^(6*d*x + 6*c) + 7*a^2*b^4*e^(6*d*x + 6*c) - 21*a*b^5*e^(6*d*x + 6*c) - 11*
b^6*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) - a^2*b^4*e^(4*d*x + 4*c)
+ 29*a*b^5*e^(4*d*x + 4*c) + 33*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2*d*x +
 2*c) + 37*a^2*b^4*e^(2*d*x + 2*c) - 47*a*b^5*e^(2*d*x + 2*c) - 33*b^6*e^(2
*d*x + 2*c) + 17*a^3*b^3 + 45*a^2*b^4 + 39*a*b^5 + 11*b^6)/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 16*(6*a*e^(4*d*x + 4*c) - 9*b*
e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + 4*a - 9*b)/(
a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx=\int \frac{\coth(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

```
[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

3.201 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 201

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \frac{x}{(a+b)^4} + \frac{\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d} + \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^4+1/16*(35*a^3+35*a^2*b+21*a*b^2+5*b^3)*arctan(b^(1/2)*tanh(d*x+c)/
a^(1/2))*b^(1/2)/a^(7/2)/(a+b)^4/d+1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*
x+c)^2)^3+1/24*b*(11*a+5*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2
+1/16*b*(19*a^2+16*a*b+5*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3742, 425, 541, 536, 212, 211}

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \frac{b(11a+5b)\tanh(c+dx)}{24a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3d(a+b)^3\left(a+b\tanh^2(c+dx)\right)} + \frac{\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^4} + \frac{b\tanh(c+dx)}{6ad(a+b)\left(a+b\tanh^2(c+dx)\right)^3} + \frac{x}{(a+b)^4}$$

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

```
[Out] x/(a + b)^4 + (Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a]])/(16*a^(7/2)*(a + b)^4*d) + (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^3) + (b*(11*a + 5*b)*Tanh[c + d*x])/(2
4*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(19*a^2 + 16*a*b + 5*b^2)
*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^4}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} - \frac{\text{Subst}\left(\int \frac{b-6(a+b)+5bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{6a(a+b)d}\\
&= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3(8a^2+11ab+5b^2)-3b(11a+5b)x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{24a^2(a+b)^2d}\\
&= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2}\\
&\quad + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-3(16a^3+19a^2b+16ab^2+5b^3)+3b(19a^2+16ab+5b^2)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{48a^3(a+b)^3d}
\end{aligned}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3(a+b)^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^4d} + \frac{(b(35a^3+35a^2b+21ab^2+5b^3))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{16a^3(a+b)^4d}$$

$$= \frac{x}{(a+b)^4} + \frac{\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d} + \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \frac{\frac{3\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}} - 24\log(1-\tanh(c+dx)) + 24\log(1+\tanh(c+dx)) + \frac{8b(a+b)^3 \cdots}{a(a+b\tan \cdots}}{48(a+b)^4d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

```
[Out] ((3*Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/a^(7/2) - 24*Log[1 - Tanh[c + d*x]] + 24*Log[1 + Tanh[c +
d*x]] + (8*b*(a + b)^3*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^3) + (2*b*
(a + b)^2*(11*a + 5*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)^2) + (3*
b*(a + b)*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(a^3*(a + b*Tanh[c + d*x
]^2)))/(48*(a + b)^4*d)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^4}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^5}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)}{6a^2}}{\left(a+b\tanh(dx+c)^2\right)^3}\right.}{}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^4}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^5}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)}{6a^2}}{\left(a+b\tanh(dx+c)^2\right)^3}\right.}{}}{d}$
risch	Expression too large to display

[In] int(1/(a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/(a+b)^4*ln(tanh(d*x+c)+1)-1/2/(a+b)^4*ln(tanh(d*x+c)-1)+1/(a+b)^4*b*((1/16*b^2*(19*a^3+35*a^2*b+21*a*b^2+5*b^3)/a^3*tanh(d*x+c)^5+1/6*b*(17*a^3+33*a^2*b+21*a*b^2+5*b^3)/a^2*tanh(d*x+c)^3+1/16*(29*a^3+61*a^2*b+43*a*b^2+11*b^3)/a*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^3+1/16*(35*a^3+35*a^2*b+21*a*b^2+5*b^3)/a^3/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9849 vs. $2(185) = 370$.

Time = 0.51 (sec) , antiderivative size = 20020, normalized size of antiderivative = 99.60

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Timed out}$$

[In] integrate(1/(a+b*tanh(d*x+c)**2)**4,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(185) = 370.

Time = 0.50 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.60

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$$
$$= -\frac{(35\,a^3b+35\,a^2b^2+21\,ab^3+5\,b^4)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{16\,(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{ab}d}$$
$$+\frac{87\,a^5b+319\,a^4b^2+450\,a}{24\,(a^{10}+7\,a^9b+21\,a^8b^2+35\,a^7b^3+35\,a^6b^4+21\,a^5b^5+7\,a^4b^6+a^3b^7+6\,(a^{10}+5\,a^9b+9\,a^8b^2+5\,a^7b^3}$$
$$+\frac{dx+c}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*((a + b)*e^(-2*
d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^
3*b^4)*sqrt(a*b)*d) + 1/24*(87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a^2*
b^4 + 103*a*b^5 + 15*b^6 + 3*(145*a^5*b + 267*a^4*b^2 + 34*a^3*b^3 - 178*a^
2*b^4 - 115*a*b^5 - 25*b^6)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 93*a^4*b^2 -
6*a^3*b^3 + 106*a^2*b^4 + 85*a*b^5 + 25*b^6)*e^(-4*d*x - 4*c) + 2*(435*a^5*
b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^5 - 75*b^6)*e^(-6*d*x
- 6*c) + 3*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105*a*b^5 +
 25*b^6)*e^(-8*d*x - 8*c) + 3*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*
b^4 - 31*a*b^5 - 5*b^6)*e^(-10*d*x - 10*c))/((a^10 + 7*a^9*b + 21*a^8*b^2 +
 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7 + 6*(a^10 + 5*a
^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^6 - a^3*b^7)
*e^(-2*d*x - 2*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7*b^3 + 15*a^6
*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-4*d*x - 4*c) + 4*(5*a^10 +
17*a^9*b + 21*a^8*b^2 + 9*a^7*b^3 - 9*a^6*b^4 - 21*a^5*b^5 - 17*a^4*b^6 - 5
*a^3*b^7)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7*b^3
 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-8*d*x - 8*c) + 6*(
a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^6
- a^3*b^7)*e^(-10*d*x - 10*c) + (a^10 + 7*a^9*b + 21*a^8*b^2 + 35*a^7*b^3 +
 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7)*e^(-12*d*x - 12*c))*d) + (d
*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(185) = 370$.

Time = 0.36 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.73

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$$
$$= \frac{\frac{3\,(35\,a^3b+35\,a^2b^2+21\,ab^3+5\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{ab}}+\frac{48\,(dx+c)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\,(87\,a^5be^{(10\,dx+10\,c)}+69\,a^4b^2e^{(10}}{}}{}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x +
 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*sqrt(a*b)) + 48*(d*x + c)/(a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4) - 2*(87*a^5*b*e^(10*d*x + 10*c) + 69*a^4*b^2*e^(10*d*x + 1
0*c) - 186*a^3*b^3*e^(10*d*x + 10*c) - 246*a^2*b^4*e^(10*d*x + 10*c) - 93*a
*b^5*e^(10*d*x + 10*c) - 15*b^6*e^(10*d*x + 10*c) + 435*a^5*b*e^(8*d*x + 8*
c) + 51*a^4*b^2*e^(8*d*x + 8*c) - 174*a^3*b^3*e^(8*d*x + 8*c) + 450*a^2*b^4
*e^(8*d*x + 8*c) + 315*a*b^5*e^(8*d*x + 8*c) + 75*b^6*e^(8*d*x + 8*c) + 870
*a^5*b*e^(6*d*x + 6*c) + 58*a^4*b^2*e^(6*d*x + 6*c) + 324*a^3*b^3*e^(6*d*x
+ 6*c) - 612*a^2*b^4*e^(6*d*x + 6*c) - 490*a*b^5*e^(6*d*x + 6*c) - 150*b^6*
e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 558*a^4*b^2*e^(4*d*x + 4*c) -
 36*a^3*b^3*e^(4*d*x + 4*c) + 636*a^2*b^4*e^(4*d*x + 4*c) + 510*a*b^5*e^(4*
d*x + 4*c) + 150*b^6*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*c) + 801*a^4*
b^2*e^(2*d*x + 2*c) + 102*a^3*b^3*e^(2*d*x + 2*c) - 534*a^2*b^4*e^(2*d*x +
2*c) - 345*a*b^5*e^(2*d*x + 2*c) - 75*b^6*e^(2*d*x + 2*c) + 87*a^5*b + 319*
a^4*b^2 + 450*a^3*b^3 + 306*a^2*b^4 + 103*a*b^5 + 15*b^6)/((a^7 + 4*a^6*b +
 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) +
2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 3685, normalized size of antiderivative = 18.33

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b*tanh(c + d*x)^2)^4,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d + 8*a*b^3*d + 8*a^
3*b*d) + ((tanh(c + d*x)^3*(16*a*b^3 + 5*b^4 + 17*a^2*b^2))/(6*a^2*(3*a*b^2
 + 3*a^2*b + a^3 + b^3)) + (tanh(c + d*x)*(32*a*b^2 + 29*a^2*b + 11*b^3))/(
```

16*a*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (b^2*tanh(c + d*x)^5*(16*a*b^2 + 19
*a^2*b + 5*b^3))/(16*a^2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(a^3*d + b^3
*d*tanh(c + d*x)^6 + 3*a^2*b*d*tanh(c + d*x)^2 + 3*a*b^2*d*tanh(c + d*x)^4)
 - log(tanh(c + d*x) - 1)/(2*d*(a + b)^4) - (atan((((-a^7*b)^(1/2)*((tanh(c
 + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2
450*a^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 +
6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) + ((((5
*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^
2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10
*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4
+ 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 +
36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*
a^12*b^3*d^3 + 36*a^13*b^2*d^3) - (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 +
 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*
a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 -
 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^
2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)
*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 +
20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 3
5*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^1
0*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(32*(a^11*d + a^7*b^4*d
 + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)) + ((-a^7*b)^(1/2)*((tanh(c + d*
x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a
^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7
*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) - ((((5*a^3*
b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^2 + (
953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10*b^6*
d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4 + 4*a
^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^
8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*
b^3*d^3 + 36*a^13*b^2*d^3) + (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a
^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*a^8*b
^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 - 2867
2*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^2*d^2
))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)*(a^1
2*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^
9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3
 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d
)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(32*(a^11*d + a^7*b^4*d + 4*
a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))/((((185*a*b^7)/128 + (25*b^8)/128 +
(303*a^2*b^6)/64 + (567*a^3*b^5)/64 + (1225*a^4*b^4)/128 + (665*a^5*b^3)/12
8)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3
+ 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*b^3*d^3 +
36*a^13*b^2*d^3) + ((-a^7*b)^(1/2)*((tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 79
1*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3))/(12

8*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) + ((((5*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4 + 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*b^3*d^3 + 36*a^13*b^2*d^3) - (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 - 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)) - ((-a^7*b)^(1/2)*((tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) - ((((5*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4 + 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*b^3*d^3 + 36*a^13*b^2*d^3) + (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 - 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d))))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(16*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d))

3.202 $\int \sqrt{1-\tanh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \sqrt{1-\tanh^2(x)}\,dx = \arcsin(\tanh(x))$$

```
[Out] arcsin(tanh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 222}

$$\int \sqrt{1-\tanh^2(x)}\,dx = \arcsin(\tanh(x))$$

```
[In] Int[Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] ArcSin[Tanh[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{\operatorname{sech}^2(x)}\,dx \\ &= \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(x)\right) \\ &= \arcsin(\tanh(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = \arctan(\sinh(x))\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

```
[In] Integrate[Sqrt[1 - Tanh[x]^2],x]
[Out] ArcTan[Sinh[x]]*Cosh[x]*Sqrt[Sech[x]^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arcsin(\tanh(x))$	4
default	$\arcsin(\tanh(x))$	4
risch	$i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\, e^{-x}(1+e^{2x})\ln(e^x+i) - i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\, e^{-x}(1+e^{2x})\ln(e^x-i)$	70

```
[In] int((1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2\arctan(\cosh(x) + \sinh(x))$$

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sqrt{1-\tanh^2(x)}\,dx = \int \sqrt{1-\tanh^2(x)}\,dx$$

```
[In] integrate((1-tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(1 - tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2\arctan(e^x)$$

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2\arctan(e^x)$$

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sqrt{1 - \tanh^2(x)}\, dx = \operatorname{asin}(\tanh(x))$$

```
[In] int((1 - tanh(x)^2)^(1/2),x)
[Out] asin(tanh(x))
```

3.203 $\int \sqrt{-1+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

```
[Out] -arctanh(tanh(x)/(-sech(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3738, 4207, 223, 212}

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

```
[In] Int[Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] -ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{-\operatorname{sech}^2(x)}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, \tanh(x)\right) \\
&= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) \\
&= -\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$
\int \sqrt{-1+\tanh^2(x)}\,dx = \arctan(\sinh(x))\cosh(x)\sqrt{-\operatorname{sech}^2(x)}
$$

```
[In] Integrate[Sqrt[-1 + Tanh[x]^2],x]
[Out] ArcTan[Sinh[x]]*Cosh[x]*Sqrt[-Sech[x]^2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\ln\left(\tanh\left(x\right)+\sqrt{\tanh\left(x\right)^{2}-1}\right)$	15
default	$-\ln\left(\tanh\left(x\right)+\sqrt{\tanh\left(x\right)^{2}-1}\right)$	15
risch	$i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^{2}}}\,e^{-x}(1+e^{2x})\ln\left(e^{x}+i\right)-i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^{2}}}\,e^{-x}(1+e^{2x})\ln\left(e^{x}-i\right)$	72

```
[In] int((tanh(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(tanh(x)+(tanh(x)^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 6.00

$$\int\sqrt{-1+\tanh^{2}(x)}\,dx$$
$$=-\log\left(\left(\cosh\left(x\right)e^{x}+e^{x}\sinh\left(x\right)+\sqrt{-\frac{e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)\right)e^{(-x)}\right)$$
$$+\log\left(\left(\cosh\left(x\right)e^{x}+e^{x}\sinh\left(x\right)-\sqrt{-\frac{e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)\right)e^{(-x)}\right)$$

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] -log((cosh(x)*e^x + e^x*sinh(x) + sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))*
(e^(2*x) + 1))*e^(-x)) + log((cosh(x)*e^x + e^x*sinh(x) - sqrt(-e^(2*x)/(e^
(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1))*e^(-x))
```

Sympy [F]

$$\int\sqrt{-1+\tanh^{2}(x)}\,dx=\int\sqrt{\tanh^{2}\left(x\right)-1}\,dx$$

```
[In] integrate((-1+tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(tanh(x)**2 - 1), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.31

$$\int \sqrt{-1+\tanh^2(x)}\,dx = 2i\arctan\left(e^x\right)$$

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*I*arctan(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.06

$$\int \sqrt{-1+\tanh^2(x)}\,dx = 0$$

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 0
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\ln\left(\tanh(x)+\sqrt{\tanh\left(x\right)^2-1}\right)$$

```
[In] int((tanh(x)^2 - 1)^(1/2),x)
[Out] -log(tanh(x) + (tanh(x)^2 - 1)^(1/2))
```

3.204 $\int \left(1-\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{1}{2}\arcsin(\tanh(x)) + \frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\tanh(x)$$

```
[Out] 1/2*arcsin(tanh(x))+1/2*(sech(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3738, 4207, 201, 222}

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{1}{2}\arcsin(\tanh(x)) + \frac{1}{2}\tanh(x)\sqrt{\operatorname{sech}^2(x)}$$

```
[In] Int[(1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcSin[Tanh[x]]/2 + (Sqrt[Sech[x]^2]*Tanh[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \operatorname{sech}^2(x)^{3/2}\,dx \\
&= \operatorname{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\tanh(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\arcsin(\tanh(x)) + \frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\tanh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$
\int \left(1-\tanh^2(x)\right)^{3/2}\,dx = \frac{\operatorname{sech}(x)(\arctan(\sinh(x)) + \operatorname{sech}(x)\tanh(x))}{2\sqrt{\operatorname{sech}^2(x)}}
$$

```
[In] Integrate[(1 - Tanh[x]^2)^(3/2),x]
[Out] (Sech[x]*(ArcTan[Sinh[x]] + Sech[x]*Tanh[x]))/(2*Sqrt[Sech[x]^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
default	$\frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
risch	$\frac{\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,(e^{2x}-1)}{1+e^{2x}}+\frac{i e^{-x}(1+e^{2x})\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,\ln(e^x+i)}{2}-\frac{i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)}{2}$	100

```
[In] int((1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*tanh(x)*(1-tanh(x)^2)^(1/2)+1/2*arcsin(tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.36

$$\int (1-\tanh^2(x))^{3/2}\,dx=\frac{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x\right.}{\cosh(x)^4+4\cosh(x)\sinh(x)}$$

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(
x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)
^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*si
nh(x) - cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x
)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (1-\tanh^2(x))^{3/2}\,dx=\int \left(1-\tanh^2(x)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((1-tanh(x)**2)**(3/2),x)
[Out] Integral((1 - tanh(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{e^{(3x)} - e^x}{e^{(4x)} + 2e^{(2x)} + 1} + \arctan\left(e^x\right)$$

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] (e^(3*x) - e^x)/(e^(4*x) + 2*e^(2*x) + 1) + arctan(e^x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{1}{4}\pi - \frac{e^{(-x)} - e^x}{\left(e^{(-x)} - e^x\right)^2 + 4} + \frac{1}{2}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/4*pi - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 1/2*arctan(1/2*(e^(2*x) -
1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{\mathrm{asin}(\tanh(x))}{2} + \frac{\tanh(x)\,\sqrt{1-\tanh\left(x\right)^2}}{2}$$

```
[In] int((1 - tanh(x)^2)^(3/2),x)
[Out] asin(tanh(x))/2 + (tanh(x)*(1 - tanh(x)^2)^(1/2))/2
```

3.205 $\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx=\frac{1}{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)-\frac{1}{2}\sqrt{-\text{sech}^2(x)}\tanh(x)$$

```
[Out] 1/2*arctanh(tanh(x)/(-sech(x)^2)^(1/2))-1/2*(-sech(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3738, 4207, 201, 223, 212}

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx=\frac{1}{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{-\text{sech}^2(x)}$$

```
[In] Int[(-1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]/2 - (Sqrt[-Sech[x]^2]*Tanh[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\operatorname{sech}^2(x)\right)^{3/2} \, dx \\
&= -\text{Subst}\left(\int \sqrt{-1+x^2} \, dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}} \, dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) \\
&= \frac{1}{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) - \frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \left(-1+\tanh^2(x)\right)^{3/2} \, dx = -\frac{1}{2}\sqrt{-\text{sech}^2(x)}(\arctan(\sinh(x))\cosh(x)+\tanh(x))$$

```
[In] Integrate[(-1 + Tanh[x]^2)^(3/2),x]
[Out] -1/2*(Sqrt[-Sech[x]^2]*(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\tanh(x)\sqrt{\tanh(x)^2-1}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)}{2}$	28
default	$-\frac{\tanh(x)\sqrt{\tanh(x)^2-1}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)}{2}$	28
risch	$-\frac{\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,(e^{2x}-1)}{1+e^{2x}}-\frac{i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)}{2}+\frac{i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)}{2}$	104

```
[In] int((tanh(x)^2-1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*tanh(x)*(tanh(x)^2-1)^(1/2)+1/2*ln(tanh(x)+(tanh(x)^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 11.00

$$\int \left(-1+\tanh^2(x)\right)^{3/2} \, dx = \frac{\left(4\cosh(x)e^x\sinh(x)^3+e^x\sinh(x)^4+2\left(3\cosh(x)^2+1\right)e^x\sinh(x)^2+4\left(\cosh(x)^3+c\right.\right.}{}$$

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*((4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sin
h(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1
)*e^x)*log((cosh(x)*e^x + e^x*sinh(x) + sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x)
+ 1))*(e^(2*x) + 1))*e^(-x)) - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2
```

```
*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (c
osh(x)^4 + 2*cosh(x)^2 + 1)*e^x)*log((cosh(x)*e^x + e^x*sinh(x) - sqrt(-e^(
2*x)/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1))*e^(-x)) - 2*((e^(2*x) + 1)*s
inh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^2 + (cosh(x)^3
 - cosh(x))*e^(2*x) + (3*cosh(x)^2 + (3*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)
 - cosh(x))*sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1)))/(4*cosh(x)*e^x*sinh(x
)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + co
sh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \int \left(\tanh^2(x)-1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((-1+tanh(x)**2)**(3/2),x)
[Out] Integral((tanh(x)**2 - 1)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \frac{-i\,e^{(3\,x)} + i\,e^{x}}{e^{(4\,x)} + 2\,e^{(2\,x)} + 1} - i\,\arctan\left(e^{x}\right)$$

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] (-I*e^(3*x) + I*e^x)/(e^(4*x) + 2*e^(2*x) + 1) - I*arctan(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \frac{\sqrt{-e^{(2\,x)}} + \frac{1}{\sqrt{-e^{(2\,x)}}}}{\left(\sqrt{-e^{(2\,x)}} + \frac{1}{\sqrt{-e^{(2\,x)}}}\right)^2 - 4}$$

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] (sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))/((sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))^2
 - 4)
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx=\frac{\ln\left(\tanh(x)+\sqrt{\tanh\left(x\right)^2-1}\right)}{2}-\frac{\tanh(x)\,\sqrt{\tanh\left(x\right)^2-1}}{2}$$

```
[In] int((tanh(x)^2 - 1)^(3/2),x)
[Out] log(tanh(x) + (tanh(x)^2 - 1)^(1/2))/2 - (tanh(x)*(tanh(x)^2 - 1)^(1/2))/2
```

3.206 $\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

```
[Out] tanh(x)/(sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3738, 4207, 197}

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

```
[In] Int[1/Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[Sech[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{\operatorname{sech}^2(x)}}\,dx\\ &= \text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

```
[In] Integrate[1/Sqrt[1 - Tanh[x]^2],x]
[Out] Tanh[x]/Sqrt[Sech[x]^2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
default	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
parallelrisch	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
risch	$\frac{e^{2x}}{2\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{1}{2(1+e^{2x})\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}}$	56

```
[In] int(1/(1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(1-tanh(x)^2)^(1/2)*tanh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \sinh(x)$$

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] sinh(x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$$

```
[In] integrate(1/(1-tanh(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(1 - tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*e^(-x) + 1/2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \sinh(x)$$

```
[In] int(1/(1 - tanh(x)^2)^(1/2),x)
[Out] sinh(x)
```

3.207 $\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

```
[Out] tanh(x)/(-sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3738, 4207, 197}

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

```
[In] Int[1/Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-\operatorname{sech}^2(x)}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{(-1+x^2)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

```
[In] Integrate[1/Sqrt[-1 + Tanh[x]^2],x]
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{\tanh(x)^2-1}}$	12
default	$\frac{\tanh(x)}{\sqrt{\tanh(x)^2-1}}$	12
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{1}{2(1+e^{2x})\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}}$	58

```
[In] int(1/(tanh(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] tanh(x)/(tanh(x)^2-1)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\sqrt{-\frac{e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^{(-x)}\sinh(x)$$

[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^(-x)*sinh(x)

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

[In] integrate(1/(-1+tanh(x)**2)**(1/2),x)

[Out] tanh(x)/sqrt(tanh(x)**2 - 1)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{e^{(-2x)}}{2\sqrt{-e^{(-2x)}}}+\frac{1}{2\sqrt{-e^{(-2x)}}}$$

[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{1}{2}\sqrt{-e^{(2x)}} - \frac{1}{2\sqrt{-e^{(2x)}}}$$

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-e^(2*x)) - 1/2/sqrt(-e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{\sinh(2x)\sqrt{-\frac{1}{\cosh(x)^2}}}{2}$$

```
[In] int(1/(tanh(x)^2 - 1)^(1/2),x)
[Out] -(sinh(2*x)*(-1/cosh(x)^2)^(1/2))/2
```

3.208 $\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)+1/3*(a-b)*(a+b*tanh(x)^2)^(3/2)/b^2-1/5*(a+b*tanh(x)^2)^(5/2)/b^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 90, 52, 65, 214}

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] + ((a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (a + b*Tanh[x]^2)^(5/2)/(
5*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(a-b)\sqrt{a+bx}}{b}+\frac{\sqrt{a+bx}}{1-x}-\frac{(a+bx)^{3/2}}{b}\right)\,dx, x, \tanh^2(x)\right)\\
&= \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}+\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\sqrt{a+b\tanh^2(x)}+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}\\
&\quad+\frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\sqrt{a+b\tanh^2(x)}+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}\\
&\quad+\frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}\\
&\quad+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx\\
&= \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)\\
&\quad+\frac{\sqrt{a+b\tanh^2(x)}\left(2a^2-5ab-15b^2-b(a+5b)\tanh^2(x)-3b^2\tanh^4(x)\right)}{15b^2}
\end{aligned}$$

```
[In] Integrate[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] + (Sqrt[a + b*Tanh[x
]^2]*(2*a^2 - 5*a*b - 15*b^2 - b*(a + 5*b)*Tanh[x]^2 - 3*b^2*Tanh[x]^4))/(1
5*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(71) = 142$.

Time = 0.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.31

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{15b^2}-\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+}}{2}$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{15b^2}-\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+}}{2}$

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a+b*tanh(x)^2)^(3/2)/b-1/5*tanh(x)^2*(a+b*tanh(x)^2)^(3/2)/b+2/15*a/b
^2*(a+b*tanh(x)^2)^(3/2)-1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/
2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+ta
nh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-
1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(71) = 142$.

Time = 0.49 (sec) , antiderivative size = 4529, normalized size of antiderivative = 52.06

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b
^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*
(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*c
osh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2
*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh
(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cos
h(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*
```

b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^
2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5
+ 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*co
sh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(
2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*s
inh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(
x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x
)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3
+ 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15
*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a
*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*s
inh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh
(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^
2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*
sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a
^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh
(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)
^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*c
osh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5
*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3
 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*
sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*
b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*c
osh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*c
osh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 +
28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b
^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^
3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*c
osh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sin
h(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((2*
a^2 - 6*a*b - 23*b^2)*cosh(x)^8 + 8*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)*sinh(x
)^7 + (2*a^2 - 6*a*b - 23*b^2)*sinh(x)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(
x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*si
nh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b
^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^4 + 2*(35*(2*a
^2 - 6*a*b - 23*b^2)*cosh(x)^4 + 30*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 6*
a^2 - 14*a*b - 49*b^2)*sinh(x)^4 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^5

```
+ 10*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^3 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)
)*sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b -
23*b^2)*cosh(x)^6 + 15*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^4 + 3*(6*a^2 - 14*a
*b - 49*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^2 + 2*a^2 - 6*a*b
- 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^
2)*cosh(x)^5 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^
2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(
x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)
*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^
7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(
x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 +
 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^
4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)
^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^
2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 +
 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)
), -1/30*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 +
5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 +
40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^
2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*
b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*c
osh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*
cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*
(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 +
 b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)
^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt((
(a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 +
 (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cos
h(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b
)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 15*(b^2*cosh(x)^10 +
 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cos
h(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh
(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 +
 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x)
)*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 +
b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 +
5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh
(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^
2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*
```

```
(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(
x))*sinh(x) + a + b)) - 2*sqrt(2)*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^8 + 8*(
2*a^2 - 6*a*b - 23*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 - 6*a*b - 23*b^2)*sinh(x
)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*
cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^
2)*cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 - 1
4*a*b - 49*b^2)*cosh(x)^4 + 2*(35*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^4 + 30*(
2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 6*a^2 - 14*a*b - 49*b^2)*sinh(x)^4 + 8*
(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^5 + 10*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)
^3 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b
^2)*cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^6 + 15*(2*a^2 - 5*a*b
 - 12*b^2)*cosh(x)^4 + 3*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^2 + 2*a^2 - 5*a*
b - 12*b^2)*sinh(x)^2 + 2*a^2 - 6*a*b - 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2
)*cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^5 + (6*a^2 - 14*a*b - 49*b
^2)*cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2
*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cos
h(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*c
osh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x
)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(
x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^
2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)
*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 +
4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^4(x)}{4}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**5,x)
```

```
[Out] -Piecewise((2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a + b
*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)/1
```

```
0 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3, Ne(b, 0)), (sqrt(a
)*(log(tanh(x)**2 - 1)/2 + tanh(x)**4/4 + tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\tanh{(x)}^5\ dx$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(71) = 142$.

Time = 1.13 (sec) , antiderivative size = 980, normalized size of antiderivative = 11.26

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4/15*(15*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^9*(2*a + 3*b) + 15*(sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))^8*(10*a + 9*b)*sqrt(a + b) + 20*(18*a^2 + 23*a*b + b^2)*(sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^7 + 20*(30*a^2 - 7*a*b - 65*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + 2*(330*a^
3 - 705*a^2*b - 1480*a*b^2 + 19*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 10*(18*a^3 - 279*a^2*
b + 68*a*b^2 + 349*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) - 20*(30*a^4 + 81*a^3*b
- 149*a^2*b^2 - 245*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(42*a^4 - 33*a^3*b
- 139*a^2*b^2 + 69*a*b^3 + 325*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 5*(90*a^5
- 121*a^4*b - 184*a^3*b^2 + 658*a^2*b^3 + 166*a*b^4 - 1233*b^5)*(sqrt(a + b
```

```
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
) - (90*a^5 - 215*a^4*b + 240*a^3*b^2 + 638*a^2*b^3 - 2034*a*b^4 + 1713*b^5
)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^5
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b^2} - 2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\tanh\left(x\right)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\,\sqrt{-\frac{a}{4}-\frac{b}{4}} - \sqrt{b\tanh\left(x\right)^2+a}\,\left(\left(a+b\right)\,\left(\frac{a+b}{b^2}-\frac{2\,a}{b^2}\right)+\frac{a^2}{b^2}\right) - \left(\frac{a+b}{3\,b^2}-\frac{2\,a}{3\,b^2}\right)\,\left(b\tanh(x)^2+a\right)^{3/2}$$

```
[In] int(tanh(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(5/2)/(5*b^2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/
4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2) - (a + b*tanh(x)^2)^(1/2)*((a
+ b)*((a + b)/b^2 - (2*a)/b^2) + a^2/b^2) - ((a + b)/(3*b^2) - (2*a)/(3*b^2
))*(a + b*tanh(x)^2)^(3/2)
```

3.209 $\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{(a^2-4ab-8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] 1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2
)+arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/8*(a+4*b
)*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

$= \{3751, 489, 596, 537, 223, 212, 385\}$

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{(a^2-4ab-8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(8
*b^(3/2)) + Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]
] - ((a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a + b
*Tanh[x]^2])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2(3a+(a+4b)x^2)}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{a(a+4b)+(-a^2+4ab+8b^2)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{8b}\\
&= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}\\
&\quad + (a+b)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&\quad + \frac{(a^2-4ab-8b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{8b}
\end{aligned}$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a+b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \frac{(a^2-4ab-8b^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b}$$

$$= \frac{(a^2-4ab-8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.21 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.79

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$= \frac{-\frac{b(a^2-4b^2)\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\sqrt{-\frac{a\coth^2(x)}{b}}\sqrt{-\frac{a(1+\cosh(2x))\text{csch}^2(x)}{b}}\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{csch}(2x)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a-}{}}\right.\right.}{a(a-b+(a+b)\cosh(2x))}}{} + \sqrt{\frac{a-b+a\cosh(2x)+b\cosh(2x)}{1+\cosh(2x)}}\left(\frac{\text{sech}(x)(-a\sinh(x)-6b\sinh(x))}{8b} + \frac{1}{4}\text{sech}^2(x)\tanh(x)\right)$$

```
[In] Integrate[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] (-((b*(a^2 - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*Sqrt[
-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b +
 (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b +
 (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a - b + (a +
b)*Cosh[2*x]))) - ((4*I)*b*(4*a*b + 4*b^2)*Sqrt[1 + Cosh[2*x]]*Sqrt[(a - b
+ (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*(((-1/4*I)*Sqrt[-((a*Coth[x]^2)/b)]*S
```

```
qrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*C
sch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*C
sch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + Cosh[2*x]]*Sqrt[a - b + (a
 + b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x
])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]
*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/
b]/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + Cosh[2*x]]*Sqrt[a - b + (a + b
)*Cosh[2*x]])))/Sqrt[a - b + (a + b)*Cosh[2*x]])/(4*b) + Sqrt[(a - b + a*Co
sh[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x]*(-(a*Sinh[x]) - 6*b*Sinh[
x]))/(8*b) + (Sech[x]^2*Tanh[x])/4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(99) = 198$.

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.79

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^{2}}\,\tanh(x)}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{2\sqrt{b}}-\frac{\tanh(x)\left(a+b\tanh(x)^{2}\right)^{\frac{3}{2}}}{4b}+\frac{a\tanh(x)\sqrt{a+b\,\mathrm{t}}}{8b}$
default	$-\frac{\sqrt{a+b\tanh(x)^{2}}\,\tanh(x)}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{2\sqrt{b}}-\frac{\tanh(x)\left(a+b\tanh(x)^{2}\right)^{\frac{3}{2}}}{4b}+\frac{a\tanh(x)\sqrt{a+b\,\mathrm{t}}}{8b}$

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*ta
nh(x)^2)^(1/2))-1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)/b+1/8*a/b*tanh(x)*(a+b*ta
nh(x)^2)^(1/2)+1/8*a^2/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-1/
2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)
+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln
((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2))/(tanh(x)-1))+1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2*
b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(99) = 198$.

Time = 0.58 (sec) , antiderivative size = 9360, normalized size of antiderivative = 77.36

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\tanh^4{(x)}\,dx$$

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**4,x)
[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**4, x)
```

Maxima [F]

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\tanh{(x)}^4\,dx$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(99) = 198$.

Time = 0.99 (sec) , antiderivative size = 938, normalized size of antiderivative = 7.75

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
```

```
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 1/4*(a^2 - 4*a*b - 8*b^2)*arctan(-1/2*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) - 1/2*((a^2 + 12*a*b + 16*b^2)*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^7 + (7*a^2 + 52*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + (21
*a^3 + 109*a^2*b + 28*a*b^2 - 48*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (35*a^3 + 115*a^2*b
- 156*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (35*a^4 + 130*a^3*b - 3
17*a^2*b^2 - 156*a*b^3 + 304*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 + (21*a^4 + 94*a^3*b - 379
*a^2*b^2 + 476*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (7*a^5 + 53*a^4
*b - 135*a^3*b^2 + 271*a^2*b^3 - 140*a*b^4 - 272*b^5)*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^5 +
 11*a^4*b - 17*a^3*b^2 + 65*a^2*b^3 - 116*a*b^4 + 112*b^5)*sqrt(a + b))/(((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^4*b)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx=\int \tanh(x)^4\,\sqrt{b\tanh\left(x\right)^2+a}\,dx$$

```
[In] int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2),x)
[Out] int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

3.210 $\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)-1/3*(a+b*tanh(x)^2)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 81, 52, 65, 214}

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} - \sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] - (a + b*Tanh[x]^2)^(3/2)/(3*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= -\sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$
$$= -\sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}$$
$$= \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(a+3b+b\tanh^2(x)\right)}{3b}$$

```
[In] Integrate[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[x
]^2]*(a + 3*b + b*Tanh[x]^2))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(51) = 102$.

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.02

method	result
derivativedivides	$-\frac{(a+b\tanh(x)^2)^{\frac{3}{2}}}{3b} - \frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} + \frac{\sqrt{b}\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}\right)}{2}$
default	$-\frac{(a+b\tanh(x)^2)^{\frac{3}{2}}}{3b} - \frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} + \frac{\sqrt{b}\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}\right)}{2}$

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a+b*tanh(x)^2)^(3/2)/b-1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2
)+1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))
+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(
1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tan
h(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh
(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(51) = 102$.

Time = 0.34 (sec) , antiderivative size = 2329, normalized size of antiderivative = 36.97

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4
 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x
)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(
b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(((a^3
 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*co
sh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a
^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x
)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)
^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*co
```

```
sh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a
^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2
 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2
+ 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2
*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*si
nh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + 3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3
*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3
+ 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*co
sh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(-((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*co
sh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)) - 4*sqrt(2)*((a + 4*b)*cosh(x)^4 + 4*(a + 4*b)*cosh(x)*sinh(x)^3 +
(a + 4*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 4*b)*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*((a + 4*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a
+ 4*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sin
h(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3
 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^
2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)
, -1/6*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^
4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(
x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*
(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2
+ a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*(b*cosh(
x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)
^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)
^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
```

```
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + 4*b)*cosh(x)^
4 + 4*(a + 4*b)*cosh(x)*sinh(x)^3 + (a + 4*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*(a + 4*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 4*b)*cosh(x)
^3 + (a + 2*b)*cosh(x))*sinh(x) + a + 4*b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(
x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)
^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)
^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**3,x)
```

```
[Out] -Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a + b
*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)/6
)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^3\,dx$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(51) = 102$.

Time = 0.77 (sec) , antiderivative size = 630, normalized size of antiderivative = 10.00

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx =$$

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{4\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5(a+2b)+3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(\ldots}}\right.\right.}{}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(a + 2*b) + 3*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^4*(3*a + 2*b)*sqrt(a + b) + 2*(3*a^2 - 3*a*b - 10*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
- 6*(a^2 + 3*a*b + 6*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 3*(3*a^3 + 4*a^2*b -
 9*a*b^2 - 26*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a^3 - 17*a*b^2 + 34*b^3)*sqrt(a + b))/
((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b\tanh(x)^2+a}-\frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3\,b}-2\,\mathrm{atan}\left(\frac{2\sqrt{b\tanh(x)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

```
[In] int(tanh(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/(3*b) - 2*atan((2*(a +
b*tanh(x)^2)^(1/2)*(- a/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

3.211 $\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -1/2*(a+2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctanh
((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/2*(a+b*tanh(x)^2)
^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 489, 537, 223, 212, 385}

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] +
 Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (Tanh[x
]*Sqrt[a + b*Tanh[x]^2])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a+(a+2b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(-a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) \\
&\quad + (a+b)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)} \\
&\quad + \frac{1}{2}(-a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) \\
&\quad + (a+b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) \\
&= -\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}} \\
&\quad + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.74 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.27

$$
\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}a\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right), 1\right) - 2\sqrt{2}a\sqrt{\frac{(a-b+(a+b}{}}\right.}{2\sqrt{2}\sqrt{(a-b}}
$$

```
[In] Integrate[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSi
n[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*
a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), Arc
Sin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a - b +
 (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*Sqrt[(a - b + (a + b)*Co
sh[2*x])*Sech[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(67) = 134$.

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.25

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}+\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2}-\frac{\sqrt{b}}{}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}+\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2}-\frac{\sqrt{b}}{}$

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*ta
nh(x)^2)^(1/2))+1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2*b^(1/2)
*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-
1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-
1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b
)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(67) = 134$.

Time = 0.40 (sec) , antiderivative size = 4825, normalized size of antiderivative = 56.76

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*
sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)
^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b
```

```
^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3
- 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*
(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)
^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 +
2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^
2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2
)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^
4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x
))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2
*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^
2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) + ((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x)
 + a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*
cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)
^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)
*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh
(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)
^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), 1/4*(2*((a + 2*b)*cosh(x)^4
+ 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)
^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3
 + (a + 2*b)*cosh(x))*sinh(x) + a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
```

```
b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x
)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*
cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)
^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2
 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x
)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 +
 a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*
(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b
^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 +
6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4
 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*co
sh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(
x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*
a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a
*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^
2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6
 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 +
15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (b*cosh(x)^4 +
 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b
)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(((a
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh
(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b
)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)
^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*c
osh(x))*sinh(x) + b), -1/4*(2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh
(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh
(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*
b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^
4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b -
2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 -
a*b - 2*b^2)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
```

```
b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x
)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*c
osh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a +
2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a + 2*b)*co
sh(x))*sinh(x) + a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*c
osh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a +
2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^
3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 +
cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*s
inh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
+ b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh
(x)^3 + b*cosh(x))*sinh(x) + b), -1/2*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cos
h(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)
*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2
 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3
+ (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(
x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + 2*b)*cosh(x)^4 + 4*(a +
 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(
3*(a + 2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a +
2*b)*cosh(x))*sinh(x) + a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh
(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a + b)) + sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x
) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
```

```
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sin
h(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*
(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\tanh^2(x)\,dx$$

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**2,x)
[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**2, x)
```

Maxima [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^2\,dx$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(67) = 134$.

Time = 0.74 (sec) , antiderivative size = 554, normalized size of antiderivative = 6.52

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= -\frac{(a+2b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(a+2b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+\cdots}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}\cdots}\right.\right.}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] -(a + 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*
sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqr
t(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqr
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) - sqrt(a + b))) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(3*a
 - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^2 - a*b + 2
*b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^
2
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx=\int \tanh(x)^2\,\sqrt{b\tanh\left(x\right)^2+a}\,dx$$

```
[In] int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

3.212 $\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 52, 65, 214}

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\sqrt{a+b\tanh^2(x)}+\frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\sqrt{a+b\tanh^2(x)}+\frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}
\end{aligned}
$$

$$= \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(36) = 72.

Time = 0.07 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.41

method	result
derivativedivides	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2} - \frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2} + \frac{\sqrt{a\cdots}}{\cdots}$
default	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2} - \frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2} + \frac{\sqrt{a\cdots}}{\cdots}$

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
[Out] -1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)
-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)
*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)
+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1
/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+
b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(36) = 72$.

Time = 0.30 (sec) , antiderivative size = 1543, normalized size of antiderivative = 35.07

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(((a^3
 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*co
sh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a
^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x
)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)
^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*co
sh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a
^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2
 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2
+ 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2
*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*si
nh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(-((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 4*sqrt(2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sq
rt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
+ a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^
2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*co
sh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2
```

```
*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh
(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \frac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2}+\frac{b(a+b)\,\mathrm{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\log\left(2\tanh^2(x)-2\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x),x)
[Out] -Piecewise((2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh(
x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b, Ne(b, 0)), (sqrt(a)*log(2*tanh(x)
**2 - 2)/2, True))
```

Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)\ dx$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(36) = 72.

Time = 0.56 (sec) , antiderivative size = 349, normalized size of antiderivative = 7.93

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx =$$

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)b-\cdots\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\cdots}\right.}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*b - sqrt(a + b)*b)/((sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b\tanh(x)^2+a} - 2\,\mathrm{atan}\left(\frac{2\sqrt{b\tanh(x)^2+a}\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

```
[In] int(tanh(x)*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(1/2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/4 - b/4)
^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

3.213 $\int \sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3742, 399, 223, 212, 385}

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

```
[In] Int[Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + Sqrt[a + b]*A
rcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\left((-a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\right)\\
&\quad - b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)
\end{aligned}$$

$$= -\left((-a-b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)$$
$$-b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$= -\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \sqrt{-a-b}\arctan\left(\frac{\sqrt{b}\text{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right) + \sqrt{b}\log\left(-\sqrt{b}\tanh(x)+\sqrt{a+b\tanh^2(x)}\right)$$

```
[In] Integrate[Sqrt[a + b*Tanh[x]^2],x]
[Out] Sqrt[-a - b]*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^2])/Sqr
t[-a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a + b*Tanh[x]^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(48) = 96$.

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.97

method	result
derivativedivides	$\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} - \frac{\sqrt{b}\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}\right)}{2} - \frac{\sqrt{a+b}}{}$
default	$\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} - \frac{\sqrt{b}\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}\right)}{2} - \frac{\sqrt{a+b}}{}$

```
[In] int((a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tanh(x
))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-1/2*(a+b)^(1/2)*
ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+
a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/
```

```
2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh
(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(48) = 96$.

Time = 0.36 (sec) , antiderivative size = 3443, normalized size of antiderivative = 57.38

$$\int \sqrt{a + b\tanh^2(x)}\, dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*si
nh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
+ 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(
x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^
3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
- 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 -
 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)
^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3
*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 1
8*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3
*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^
2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3
*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a
+ 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*
(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*
cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*si
nh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh
(x)^3 + cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
```

/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*co
sh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), sqrt(
-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/4*
sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)
^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b
^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3
- 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*
(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)
^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 +
2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^
2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2
)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^
4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x
))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2
*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^
2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) + 1/4*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*s
inh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), -1/2*sqrt(-a
- b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a -
b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^
2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^
2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b +
b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x)))
- 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*c
osh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*c
osh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(
x) + a + b)) + 1/2*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*

```
sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*co
sh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a
- 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x)
)*sinh(x) + 1)), -1/2*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b
+ b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4
+ (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*
b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*
b - 2*b^2)*cosh(x))*sinh(x))) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((
a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
 b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh
(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + sqrt(-b)*arctan(sqrt(2)*(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))]
```

Sympy [F]

$$\int \sqrt{a + b\tanh^2(x)}\,dx = \int \sqrt{a + b\tanh^2{(x)}}\,dx$$

```
[In] integrate((a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \sqrt{a + b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2 + a}\,dx$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(48) = 96$.

Time = 0.49 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.22

$$\int \sqrt{a+b\tanh^2(x)}\,dx = -\frac{2\,b\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt(a
 + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqrt(a +
b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\,dx$$

```
[In] int((a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int((a + b*tanh(x)^2)^(1/2), x)
```

3.214 $\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))*a^(1/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))*(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 457, 85, 65, 214}

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)$$

```
[In] Int[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\,(1-x^2)}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x}\,dx, x, \tanh^2(x)\right)\end{aligned}$$

$$= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}$$

$$= -\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

```
[In] Integrate[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Maple [F]

$$\int \coth(x)\sqrt{a+b\tanh(x)^2}dx$$

```
[In] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(44) = 88$.

Time = 0.36 (sec) , antiderivative size = 3467, normalized size of antiderivative = 61.91

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sin
h(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 +
 a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x
)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3
)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 +
 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sin
h(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^
2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 +
3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sq
rt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh
(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*co
sh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18
*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*
a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*s
qrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3
 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 +
3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(
3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*c
osh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*co
sh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), sqrt(
-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
```

)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/4*
sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*
b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(
2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2
*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2
(14(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2
b - b^3 + 3(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x)
)*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*
cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^
2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2
*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + 1/4*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*s
inh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), -1/2*sqrt(-a
- b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a +
b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*
b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^
2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b +
 b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)))
- 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*c
osh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*c
osh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(
x) + a + b)) + 1/2*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*
sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*co
sh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*

```
a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x)
)*sinh(x) + 1)), sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b
)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b
)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*si
nh(x) + a + b)) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2
+ a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4
+ (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b -
b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
a*b - b^2)*cosh(x))*sinh(x))) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((
a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
 b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh
(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))]
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth{(x)}\,dx$$

```
[In] integrate(coth(x)*(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}\,dx$$

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh\left(x\right)^2+a}\coth\left(x\right)\,dx$$

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)\,\sqrt{b\tanh\left(x\right)^2+a}\,dx$$

```
[In] int(coth(x)*(a + b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)*(a + b*tanh(x)^2)^(1/2), x)
```

3.215 $\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-coth(x)*(a+b
*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 486, 12, 385, 212}

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]*
Sqrt[a + b*Tanh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= -\coth(x)\sqrt{a+b\tanh^2(x)} + \text{Subst}\left(\int \frac{a+b}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\coth(x)\sqrt{a+b\tanh^2(x)} + (a+b)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)
\end{aligned}$$

$$= -\coth(x)\sqrt{a+b\tanh^2(x)}+(a+b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

$$= \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= -\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{(a+b)\tanh^2(x)}{a+b\tanh^2(x)}\right)\sqrt{a+b\tanh^2(x)}$$

```
[In] Integrate[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
[Out] -(Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, ((a + b)*Tanh[x]^2)/(a + b*Tanh[x
]^2)]*Sqrt[a + b*Tanh[x]^2])
```

Maple [F]

$$\int \coth(x)^2\sqrt{a+b\tanh(x)^2}dx$$

```
[In] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(40) = 80.

Time = 0.30 (sec) , antiderivative size = 1539, normalized size of antiderivative = 32.06

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(-((a*
b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)
```

*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*c
osh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2
+ b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x
)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*c
osh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*
b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^
2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 -
 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*
a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*s
inh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)
) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 4*sqrt(2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2
- a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*
b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*co
sh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh
(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]

Sympy [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth^2{(x)}\,dx$$

```
[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}^2\,dx$$

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(40) = 80$.

Time = 0.56 (sec) , antiderivative size = 348, normalized size of antiderivative = 7.25

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$+\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)a+\cdots\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\cdots}\right.}$$

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
```

```
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a + sqrt(a + b)*a)/((sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^2\sqrt{b\tanh(x)^2+a}\,dx$$

```
[In] int(coth(x)^2*(a + b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

3.216 $\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -1/2*(2*a+b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*ta
nh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 101, 162, 65, 214}

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^3(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x^2}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{\frac{1}{2}(2a+b)+\frac{bx}{2}}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&\quad + \frac{1}{4}(2a+b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&\quad + \frac{(2a+b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{2b}\\
&= -\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}\\
&\quad + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = &-\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}\\
&+ \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)\\
&- \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}
\end{aligned}
$$

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Maple [F]

$$\int \coth(x)^3 \sqrt{a + b\tanh(x)^2}dx$$

```
[In] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(65) = 130$.

Time = 0.41 (sec) , antiderivative size = 4891, normalized size of antiderivative = 58.93

$$\int \coth^3(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*
sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*
b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(
2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2
*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2
*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2
*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x)
)*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*
cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^
2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2
*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
```

```
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^
3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
- 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x)
+ 2*a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^
3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
+ 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*c
osh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2
*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x
) + 1)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^
2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) +
 a)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)
*sinh(x) + a*sinh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)
^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a), 1/4*(2*((2*a + b)*cosh(x)^4
+ 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)
^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3
 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x
)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*
cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^
8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3
+ a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 +
6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4
*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(
2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^
3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 +
 b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)
^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4
+ 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cos
```

h(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x
)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2
*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^
3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b
^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6
+ 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 1
5*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)
sinh(x)^2 + 4(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(a + b)*log(-((a
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh
(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b
)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) - 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 +
a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*c
osh(x))*sinh(x) + a), -1/4*(2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 -
a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh
(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^
2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^
4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b
- b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2
+ a*b - b^2)*cosh(x))*sinh(x))) + 2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x
)^3 - a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)
cosh(x)^4 + 4(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)) - ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a
+ b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*co
sh(x))*sinh(x) + 2*a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a
+ b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^
3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*s

```
inh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
+ a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh
(x)^3 - a*cosh(x))*sinh(x) + a), 1/2*(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*co
sh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a +
 b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cos
h(x))*sinh(x) + 2*a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 +
4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cos
h(x))*sinh(x) + a + b)) - (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^
4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*co
sh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 +
a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 +
(2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^
2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*
b - b^2)*cosh(x))*sinh(x))) - (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 -
a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(
x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a -
 b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x)
 + a*sinh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh
(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(
a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)]
```

Sympy [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\coth^3(x)\,dx$$

```
[In] integrate(coth(x)**3*(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**3, x)
```

Maxima [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)^3\,dx$$

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(65) = 130$.

Time = 0.72 (sec) , antiderivative size = 557, normalized size of antiderivative = 6.71

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{(2a+b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right. + \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right) - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right) + \frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(2a+b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2\right.\right.}$$

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] (2*a + b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*s
qrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*sqrt
(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs(-sqrt
(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b) - sqrt(a + b))) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b)*e^(2*x)
 - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(2*a
- 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*a^2 - a*b +
```

```
b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^2
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^3\,\sqrt{b\tanh\left(x\right)^2+a}\,dx$$

```
[In] int(coth(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a + b*tanh(x)^2)^(1/2), x)
```

3.217 $\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/3*(3*a+b)*
coth(x)*(a+b*tanh(x)^2)^(1/2)/a-1/3*coth(x)^3*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 486, 597, 12, 385, 212}

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a}$$

```
[In] Int[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a +
b)*Coth[x]*Sqrt[a + b*Tanh[x]^2])/(3*a) - (Coth[x]^3*Sqrt[a + b*Tanh[x]^2])
/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4(1-x^2)}\,dx, x, \tanh(x)\right) \\
&= -\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{3}\text{Subst}\left(\int \frac{3a+b+2bx^2}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) \\
&= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} \\
&\quad - \frac{\text{Subst}\left(\int -\frac{3a(a+b)}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3a} \\
&= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} \\
&\quad - (-a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) \\
&= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} \\
&\quad - (-a-b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) \\
&= \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) \\
&\quad - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.06

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= \frac{\cosh^4(x)\coth^3(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{\operatorname{sech}^4(x)\left(\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}+\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}\right)(a-2b\tanh^2}{\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}}\right.}{3\sqrt{a+b\tanh^2(x)}}$$

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] (Cosh[x]^4*Coth[x]^3*(1 + (b*Tanh[x]^2)/a)*(-((Sech[x]^4*(ArcSin[Sqrt[-(((a
 + b)*Sinh[x]^2)/a)]]*Sqrt[-(((a + b)*Sinh[x]^2)/a)] + Sqrt[Cosh[x]^2 + (b*
Sinh[x]^2)/a])*(a - 2*b*Tanh[x]^2))/Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]) - (4
*(a + b)*Hypergeometric2F1[2, 2, 3/2, -(((a + b)*Sinh[x]^2)/a)]*(a*Tanh[x]
+ b*Tanh[x]^3)^2)/a^2))/(3*Sqrt[a + b*Tanh[x]^2])
```

Maple [F]

$$\int \coth(x)^4\sqrt{a+b\tanh(x)^2}dx$$

```
[In] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(64) = 128$.

Time = 0.35 (sec) , antiderivative size = 2355, normalized size of antiderivative = 30.19

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4
 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x
)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(
a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(-((a*
b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)
```

*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*c
osh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2
+ b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x
)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*c
osh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*
b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^
2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 -
 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*
a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*s
inh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)
) + 3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 +
3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3
 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*c
osh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(((a + b)
cosh(x)^4 + 4(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*co
sh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)) - 4*sqrt(2)*((4*a + b)*cosh(x)^4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 +
(4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2*
a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 4*
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sin
h(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3
 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^
2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)
, -1/6*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^
4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(
x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*
(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(-a - b)*arctan(
sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*s
inh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b
+ b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 3*(a*cosh(

```
x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)
^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)
^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a
*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((4*a + b)*cosh(x)^
4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 + (4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(
x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x)
^3 - (2*a + b)*cosh(x))*sinh(x) + 4*a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(
x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)
^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)
^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a
*cosh(x)^3 + a*cosh(x))*sinh(x) - a)]
```

Sympy [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth^4{(x)}\,dx$$

```
[In] integrate(coth(x)**4*(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**4, x)
```

Maxima [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}^4\,dx$$

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(64) = 128.

Time = 0.83 (sec) , antiderivative size = 629, normalized size of antiderivative = 8.06

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$+\frac{4\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5(2a+b)-3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(}\right.\right.}{}$$

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(2*a + b) - 3*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^4*(2*a + 3*b)*sqrt(a + b) - 2*(10*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
+ 6*(6*a^2 + 3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + 3*(26*a^3 + 9*a^2*b
- 4*a*b^2 - 3*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b)) + (34*a^3 - 17*a^2*b + 3*b^3)*sqrt(a + b))/
((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^3
```

Mupad [F(-1)]

Timed out.

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx=\int \coth(x)^4\,\sqrt{b\tanh{(x)}^2+a}\,dx$$

```
[In] int(coth(x)^4*(a + b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

3.218 $\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(8a^2+4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -1/8*(8*a^2+4*a*b-b^2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arcta
nh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-1/8*(4*a+b)*coth(x)^2*(a+
b*tanh(x)^2)^(1/2)/a-1/4*coth(x)^4*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used

$= \{3751, 457, 101, 156, 162, 65, 214\}$

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(8a^2+4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} - \frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a}$$

```
[In] Int[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/8*((8*a^2 + 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/a^(3/2)
 + Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - ((4*a + b)*Coth
[x]^2*Sqrt[a + b*Tanh[x]^2])/(8*a) - (Coth[x]^4*Sqrt[a + b*Tanh[x]^2])/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^5\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x^3}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{\frac{1}{2}(4a+b)+\frac{3bx}{2}}{(1-x)x^2\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a}-\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(-8a^2-4ab+b^2\right)-\frac{1}{4}b(4a+b)x}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{4a}
\end{aligned}
$$

$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{(8a^2+4ab-b^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{16a}$$

$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{1}{8}\left(4+\frac{8a}{b}-\frac{b}{a}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)$$

$$= -\frac{(8a^2+4ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}} + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$= \frac{(-8a^2-4ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a}\left(8a\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \coth^2(x)\,(4a+b+2}{8a^{3/2}}$$

```
[In] Integrate[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((-8*a^2 - 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(8
*a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Coth[x]^2*(4*a
+ b + 2*a*Coth[x]^2)*Sqrt[a + b*Tanh[x]^2]))/(8*a^(3/2))
```

Maple [F]

$$\int \coth(x)^5 \sqrt{a + b \tanh(x)^2} dx$$

[In] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(99) = 198$.

Time = 0.57 (sec) , antiderivative size = 9642, normalized size of antiderivative = 79.69

$$\int \coth^5(x) \sqrt{a + b \tanh^2(x)}\, dx = \text{Too large to display}$$

[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \coth^5(x) \sqrt{a + b \tanh^2(x)}\, dx = \int \sqrt{a + b \tanh^2(x)} \coth^5(x)\, dx$$

[In] integrate(coth(x)**5*(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**5, x)

Maxima [F]

$$\int \coth^5(x) \sqrt{a + b \tanh^2(x)}\, dx = \int \sqrt{b \tanh(x)^2 + a} \coth(x)^5\, dx$$

[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^5, x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(99) = 198$.

Time = 1.01 (sec) , antiderivative size = 947, normalized size of antiderivative = 7.83

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 1/4*(8*a^2 + 4*a*b - b^2)*arctan(-1/2*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) + 1/2*((16*a^2 + 12*a*b + b^2)*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^7 - (16*a^2 + 52*a*b + 7*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) - (48
*a^3 - 28*a^2*b - 109*a*b^2 - 21*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (176*a^3 + 156*a^2*b
 - 115*a*b^2 - 35*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (304*a^4 - 156*a^3*b -
317*a^2*b^2 + 130*a*b^3 + 35*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - (48*a^4 + 476*a^3*b - 37
9*a^2*b^2 + 94*a*b^3 + 21*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - (272*a^5 + 140*
a^4*b - 271*a^3*b^2 + 135*a^2*b^3 - 53*a*b^4 - 7*b^5)*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (112*a
^5 - 116*a^4*b + 65*a^3*b^2 - 17*a^2*b^3 + 11*a*b^4 + b^5)*sqrt(a + b))/(((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^4*a)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^5\,\sqrt{b\tanh(x)^2+a}\,dx$$

```
[In] int(coth(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a + b*tanh(x)^2)^(1/2), x)
```

3.219 $\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^2
)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)-1/5*(a+b*tanh(x)^2)^(5/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 81, 52, 65, 214}

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-(a+b)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3 - (a + b*Tanh[x]^2)^(5/2)/(5*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^3(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b} + \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\left((a+b)\sqrt{a+b\tanh^2(x)}\right) - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a+b)^2\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= -\left((a+b)\sqrt{a+b\tanh^2(x)}\right) - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}$$

$$= (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(3a^2+20ab+15b^2+b(6a+5b)\tanh^2(x)+3b^2\tanh^4(x)\right)}{15b}$$

```
[In] Integrate[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh
[x]^2]*(3*a^2 + 20*a*b + 15*b^2 + b*(6*a + 5*b)*Tanh[x]^2 + 3*b^2*Tanh[x]^4
))/(15*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.95

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^{2}\right)^{\frac{5}{2}}}{5b}-\frac{\left(b(1+\tanh(x))^{2}-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}{6}+\frac{b\left(\frac{(2b(1+\tanh(x))-2b)\sqrt{b(1+\tanh(x))^{2}-2b(1+\tanh(}}{4b}$
default	$-\frac{\left(a+b\tanh(x)^{2}\right)^{\frac{5}{2}}}{5b}-\frac{\left(b(1+\tanh(x))^{2}-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}{6}+\frac{b\left(\frac{(2b(1+\tanh(x))-2b)\sqrt{b(1+\tanh(x))^{2}-2b(1+\tanh(}}{4b}$

```
[In] int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(a+b*tanh(x)^2)^(5/2)/b-1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2
)+1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))-1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+tanh
(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(
1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2
)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))-1/6*(b*(tanh(x
)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(
tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(a+
b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1)+b
)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+
2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/
2))/(tanh(x)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2188 vs. $2(66) = 132$.

Time = 0.49 (sec) , antiderivative size = 4941, normalized size of antiderivative = 60.26

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b
 + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 +
 a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*
sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a
*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 +
```

70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^
2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15
*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7
 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))
sinh(x)^3 + 5(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b
 + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a
*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)
cosh(x)^7 + 6(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2
)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2
*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)
^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3
+ a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*
b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b
- a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*
b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b
- b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(
x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)
sinh(x)^2 + sqrt(2)(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)
^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(
x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2
*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh
(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(
x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*si
nh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh
(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*((a*b + b^2)*cosh(x)^10 + 10
*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cos
h(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b
^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 +
 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(
x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b
^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh
(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*si
nh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b +
b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 +
 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cos
h(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*(
(a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 +
 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a +

b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a
+ b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2)) - 4*sqrt(2)*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^8 + 8*(3*a^2
+ 26*a*b + 23*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 26*a*b + 23*b^2)*sinh(x)^8
+ 4*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*co
sh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 +
54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^4 + 30
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49*b^2)*sinh(x)^4 +
 8*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^5 + 10*(3*a^2 + 20*a*b + 12*b^2)*co
sh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 + 20*a*b
+ 12*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^6 + 15*(3*a^2
+ 20*a*b + 12*b^2)*cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)*cosh(x)^2 + 3*a^
2 + 20*a*b + 12*b^2)*sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 + 8*((3*a^2 + 26*a
*b + 23*b^2)*cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^5 + (9*a^2 + 5
4*a*b + 49*b^2)*cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x))*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*sinh(x)^9 + b*sinh(x)^10
+ 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 + 40*(3*b*cosh(x)^3 + b*c
osh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(x)^4 + 14*b*cosh(x)^2 +
b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 + 15*b*cosh(x))*sinh(x)^5
 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(x)^4 + 15*b*cosh(x)^2 +
b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cosh(x)^3 + b*cosh(x
))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cosh(x)^6 + 30*b*cos
h(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 + 4*b*cosh(x)^7 +
6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/30*(15*((a*b +
b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10
 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x
)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*
b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^
2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cos
h(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*
(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(
x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*c
osh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*
b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6
+ 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^
2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*
b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)
)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x
)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4
*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2
)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2
 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*

```
sinh(x))) + 15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 +
 (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x
)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh
(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 +
14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)
^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b
 + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4
 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh
(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cos
h(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28
*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^
2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b +
 b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b
+ b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh
(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((3*a^2 + 26*a*b + 23*b^2)*cosh
(x)^8 + 8*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 26*a*b + 2
3*b^2)*sinh(x)^8 + 4*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 26
*a*b + 23*b^2)*cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^6 + 8*(7*(3*a^2
 + 26*a*b + 23*b^2)*cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x
)^5 + 2*(9*a^2 + 54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^4 + 30*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49
*b^2)*sinh(x)^4 + 8*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^5 + 10*(3*a^2 + 20
*a*b + 12*b^2)*cosh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x))*sinh(x)^3 + 4
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(
x)^6 + 15*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)
*cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 +
 8*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(
x)^5 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*cosh
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*sinh(x)^
9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 + 40*(3*
b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(x)^4 +
14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 + 15*b*c
osh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(x)^4 +
15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cos
h(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cos
h(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 +
 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

Sympy [A] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.72

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} \, dx =$$

$$-a\left(\begin{cases} \frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for}\ b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} \frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3} & \text{for}\ b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^4(x)}{4}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(tanh(x)**3*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -a*Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a +
 b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)
/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**2/2), True)
) - b*Piecewise((2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(
a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5
/2)/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3, Ne(b, 0)), (s
qrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**4/4 + tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} \, dx = \int \left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}\tanh\left(x\right)^3 \, dx$$

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(66) = 132$.

Time = 1.46 (sec) , antiderivative size = 1063, normalized size of antiderivative = 12.96

$$\int \tanh^3(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*l
og(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4/15*(15*(a^2 + 4*a*b
 + 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))^9 + 15*(7*a^2 + 20*a*b + 9*b^2)*(sqrt(a + b)*e^(2*x)
 - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^8*sqrt(
a + b) + 20*(15*a^3 + 39*a^2*b + 21*a*b^2 + b^3)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + 20*(21*a^
3 + 21*a^2*b - 57*a*b^2 - 65*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + 2*(105*a^4 -
 210*a^3*b - 1860*a^2*b^2 - 1590*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 - 10*(21*a^
4 + 126*a^3*b + 288*a^2*b^2 - 390*a*b^3 - 349*b^4)*(sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a +
b) - 20*(21*a^5 + 63*a^4*b - 18*a^3*b^2 - 378*a^2*b^3 - 235*a*b^4 + 19*b^5)
*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^3 - 20*(15*a^5 + 21*a^4*b - 126*a^3*b^2 - 90*a^2*b^3 + 367*a*
b^4 + 325*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 5*(21*a^6 + 24*a^5*b - 243*a^4*
b^2 + 280*a^3*b^3 + 815*a^2*b^4 - 944*a*b^5 - 1233*b^6)*(sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (15*
a^6 - 165*a^4*b^2 + 920*a^3*b^3 - 1147*a^2*b^4 - 504*a*b^5 + 1713*b^6)*sqrt
(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^5
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b} - \left(\frac{a+b}{3\,b}-\frac{a}{3\,b}\right)\left(b\tanh(x)^2+a\right)^{3/2} - (a+b)\left(\frac{a+b}{b}-\frac{a}{b}\right)\sqrt{b\tanh\left(x\right)^2+a} - \mathrm{atan}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh\left(x\right)^2+a}\,1\mathrm{i}}{a^2+2\,a\,b+b^2}\right)(a+b)^{3/2}\,1\mathrm{i}$$

```
[In] int(tanh(x)^3*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(5/2)/(5*b) - ((a + b)/(3*b) - a/(3*b))*(a + b*tanh(x)^
2)^(3/2) - atan(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2)*1i)/(2*a*b + a^2 + b
^2))*(a + b)^(3/2)*1i - (a + b)*((a + b)/b - a/b)*(a + b*tanh(x)^2)^(1/2)
```

3.220 $\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = -\frac{(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}} + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/8*(3*a^2+1
2*a*b+8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(5*
a+4*b)*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/4*b*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3751, 488, 596, 537, 223, 212, 385}

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = -\frac{(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}} + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/8*((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]
^2]])/Sqrt[b] + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh
[x]^2]] - ((5*a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/8 - (b*Tanh[x]^3*Sqrt
[a + b*Tanh[x]^2])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{4}\text{Subst}\left(\int \frac{x^2(-a(4a+3b)-b(5a+4b)x^2)}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-ab(5a+4b)-b(3a^2+12ab+8b^2)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{8b}\\
&= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}\\
&\quad +(a+b)^2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&\quad -\frac{1}{8}(3a^2+12ab+8b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}\\
&\quad +(a+b)^2\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&\quad -\frac{1}{8}(3a^2+12ab+8b^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)
\end{aligned}$$

$$= -\frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}} + (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.25 (sec) , antiderivative size = 584, normalized size of antiderivative = 4.75

$$\int \tanh^2(x)\left(a + b\tanh^2(x)\right)^{3/2} dx = \frac{1}{4}\left(-\frac{b(a^2-4ab-4b^2)\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\sqrt{-\frac{a\coth^2(x)}{b}}\sqrt{-\frac{a(1+\cosh(2x))\operatorname{csch}^2(x)}{b}}\sqrt{\frac{(a-b+(a+\cdots}{}}}{a(a-b+(a\cdots} \right.$$

$$-\frac{4ib(4a^2+8ab+4b^2)\sqrt{1+\cosh(2x)}\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\left(-\frac{i\sqrt{-\frac{a\coth^2(x)}{b}}\sqrt{-\frac{a(1+\cosh(2x))\operatorname{csch}^2(x)}{b}}\sqrt{\frac{(a-b+(a+b)\cosh(2x))\cdots}{b}}}{4a\sqrt{1+\cdots}}\right.}{\cdots}$$

$$+\sqrt{\frac{a-b+a\cosh(2x)+b\cosh(2x)}{1+\cosh(2x)}}\left(\frac{1}{8}\operatorname{sech}(x)(-5a\sinh(x)-6b\sinh(x)) + \frac{1}{4}b\operatorname{sech}^2(x)\tanh(x)\right)$$

```
[In] Integrate[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-((b*(a^2 - 4*a*b - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x]
)]*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[(
(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[(
(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a - b
 + (a + b)*Cosh[2*x]))) - ((4*I)*b*(4*a^2 + 8*a*b + 4*b^2)*Sqrt[1 + Cosh[2*
x]]*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*(((-1/4*I)*Sqrt[-((a*
Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a +
 b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b + (a +
 b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + Cosh[2*x]]
*Sqrt[a - b + (a + b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-(
(a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]
^2)/b]*Csch[2*x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2
*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + Cosh[2*x]]*Sqr
t[a - b + (a + b)*Cosh[2*x]])))/Sqrt[a - b + (a + b)*Cosh[2*x]])/4 + Sqrt[(
a - b + a*Cosh[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x]*(-5*a*Sinh[x]
 - 6*b*Sinh[x]))/8 + (b*Sech[x]^2*Tanh[x])/4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(101) = 202$.

Time = 0.08 (sec) , antiderivative size = 529, normalized size of antiderivative = 4.30

method	result
derivativedivides	$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4}-\frac{3a\left(\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}\right)}{4}-\frac{\left(b(\tanh(x)-1)^2+\right.}{}$
default	$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4}-\frac{3a\left(\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}\right)}{4}-\frac{\left(b(\tanh(x)-1)^2+\right.}{}$

```
[In] int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)-3/4*a*(1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)
+1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)))-1/6*(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(tanh(
x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(tan
h(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(a+b)*
((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1)+b)/b
^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b
+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))
/(tanh(x)-1)))+1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/2*b*(1/4*(
2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/8*(4*(
a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+
```

```
tanh(x))+a+b)^(1/2)))+1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2
)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b
)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x
))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(101) = 202$.

Time = 0.59 (sec) , antiderivative size = 10046, normalized size of antiderivative = 81.67

$$\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} \, dx = \int \left(a + b\tanh^2(x)\right)^{\frac{3}{2}} \tanh^2(x) \, dx$$

```
[In] integrate(tanh(x)**2*(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral((a + b*tanh(x)**2)**(3/2)*tanh(x)**2, x)
```

Maxima [F]

$$\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} \, dx = \int \left(b\tanh(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^2 \, dx$$

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(101) = 202$.

Time = 1.21 (sec) , antiderivative size = 949, normalized size of antiderivative = 7.72

$$\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/4*(3*a^2 + 12*a*b + 8*b^2)*arctan(-
1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)
*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*((
5*a^2 + 20*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + (35*a^2 + 76*a*b + 16*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^6*sqrt(a + b) + (105*a^3 + 153*a^2*b - 28*a*b^2 - 48*b^3)*(sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^5 + (175*a^3 - 25*a^2*b - 260*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b
) + (175*a^4 - 110*a^3*b - 417*a^2*b^2 + 60*a*b^3 + 304*b^4)*(sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
 + (105*a^4 - 210*a^3*b - 55*a^2*b^2 + 484*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*s
qrt(a + b) + (35*a^5 - 79*a^4*b + 53*a^3*b^2 + 195*a^2*b^3 - 308*a*b^4 - 27
2*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b)) + (5*a^5 - 17*a^4*b + 51*a^3*b^2 - 19*a^2*b^3 - 44*a*b^
4 + 112*b^5)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a
- 3*b)^4
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} \, dx = \int \tanh(x)^2 \left(b\tanh(x)^2 + a\right)^{3/2} dx$$

```
[In] int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2),x)
[Out] int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

3.221 $\int \tanh(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \tanh(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^2
)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 52, 65, 214}

$$\int \tanh(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

```
[In] Int[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\left((a+b)\sqrt{a+b\tanh^2(x)}\right) - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} + \frac{1}{2}(a+b)^2\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= -\left((a+b)\sqrt{a+b\tanh^2(x)}\right) - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}$$

$$= (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{3}\sqrt{a+b\tanh^2(x)}\left(4a+3b+b\tanh^2(x)\right)$$

```
[In] Integrate[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh
[x]^2]*(4*a + 3*b + b*Tanh[x]^2))/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(51) = 102.

Time = 0.06 (sec) , antiderivative size = 473, normalized size of antiderivative = 7.51

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6} - \frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b} + \frac{\left(4(a+b)b-4\right.}{}\right.}{2}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6} - \frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b} + \frac{\left(4(a+b)b-4\right.}{}\right.}{2}$

```
[In] int(tanh(x)*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1
)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/
b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((
b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)
^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2))/(tanh(x)-1)))-1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(3/2)+1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a
+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))-1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2
*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^
(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(51) = 102$.

Time = 0.37 (sec) , antiderivative size = 2385, normalized size of antiderivative = 37.86

$$\int \tanh(x) \left(a + b\tanh^2(x)\right)^{3/2} \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)
^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5
*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3
*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)
*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a
 + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a
^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14
*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*
a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4
 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 +
 a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b
^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*
cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x
)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
```

```
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*c
osh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b
^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^
5 + sinh(x)^6)) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a +
 b)*sinh(x)^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(
x)^4 + 4*(5*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*co
sh(x)^2 + 3*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
 6*((a + b)*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a
+ b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a + b)*cosh(x)^4 + 4*(
a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*(a
+ b)*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*(a + b)*cosh(x)^3 + (2*a + b)*co
sh(x))*sinh(x) + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^6 + 6*cosh(x)*sinh
(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh
(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2
+ 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/6*(3
*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(
a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)
*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a +
 b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)
^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*a
rctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqr
t(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cos
h(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6
*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*((
a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(a +
 b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)*co
sh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)
*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5
+ 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
 b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 8*sqrt(2
)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + (2
*a + b)*cosh(x)^2 + (6*(a + b)*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*(a + b
```

```
)*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(
cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4
 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6
*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh
(x))*sinh(x) + 1)]
```

Sympy [A] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = -a\left(\begin{cases}\frac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} & \text{for } b\neq 0\\ \frac{\sqrt{a}\log\left(2\tanh^2(x)-2\right)}{2} & \text{otherwise}\end{cases}\right) - b\left(\begin{cases}\frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -a*Piecewise((2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tan
h(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b, Ne(b, 0)), (sqrt(a)*log(2*tanh(
x)**2 - 2)/2, True)) - b*Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2
*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a
+ b*tanh(x)**2)**(3/2)/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2
+ tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \left(b\tanh(x)^2+a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(51) = 102.

Time = 1.02 (sec) , antiderivative size = 662, normalized size of antiderivative = 10.51

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+\cdots}\right|\right) - \frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right) - \frac{(a^2+2ab+b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a\cdots\right.\right.}{2\sqrt{a+b}} - \frac{8\left(3(ab+b^2)\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5+3(3ab+b^2)\left(\sqrt{a+b}e^{(2x)}\cdots\right.\right.}{\cdots}$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*l
og(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 8/3*(3*(a*b + b^2)*(s
qrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b))^5 + 3*(3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + 2*(3*a^2*b - 6*
a*b^2 - 5*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^3 - 6*(a^2*b + 4*a*b^2 + 3*b^3)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*
sqrt(a + b) - 3*(3*a^3*b + a^2*b^2 - 15*a*b^3 - 13*b^4)*(sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a
^3*b - 9*a^2*b^2 + 5*a*b^3 + 17*b^4)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=\operatorname{atanh}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh(x)^2+a}}{a^2+2\,a\,b+b^2}\right)(a+b)^{3/2}-(a+b)\sqrt{b\tanh(x)^2+a}-\frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3}$$

```
[In] int(tanh(x)*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2))/(2*a*b + a^2 + b^2))*(a + b)^
(3/2) - (a + b)*(a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/3
```

3.222 $\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=-\frac{1}{2}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/2*(3*a+2*b
)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)-1/2*b*(a+b*tanh(x)
^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3742, 427, 537, 223, 212, 385}

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])
 + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (b*
Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a(2a+b)-b(3a+2b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} + (a+b)^2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) - \frac{1}{2}(b(3a+2b))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} + (a+b)^2\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}(b(3a+2b))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$= -\frac{1}{2}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int (a+b\tanh^2(x))^{3/2}\,dx = \frac{1}{2}\left(-2(-a-b)^{3/2}\arctan\left(\frac{\sqrt{b}\text{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right) + \sqrt{b}(3a+2b)\log\right.$$

```
[In] Integrate[(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-2*(-a - b)^(3/2)*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^2
])/Sqrt[-a - b]] + Sqrt[b]*(3*a + 2*b)*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a + b*
Tanh[x]^2]] - b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 473, normalized size of antiderivative = 5.38

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{(4(a+b)b-4b^2}{}\right.}{2}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{(4(a+b)b-4b^2}{}\right.}{2}$

```
[In] int((a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1
)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/
b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((
b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)
^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2))/(tanh(x)-1)))+1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(3/2)-1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a
+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))+1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2
*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^
(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(70) = 140$.

Time = 0.43 (sec) , antiderivative size = 4841, normalized size of antiderivative = 55.01

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2
+ b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x
)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(
```

```
x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^
3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(
x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*
b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*
cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b
+ 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3*a + 2*b)*sinh
(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2 + 3*a + 2*b)*s
inh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*sinh(x) + 3*a +
2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a
 + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a -
2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*
sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x)
)*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*co
sh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b)*co
sh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)
^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(
x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1),
 1/4*(2*((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3*a + 2
*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2 + 3*a
+ 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*sinh(x)
+ 3*a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*co
```

sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*co
sh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x
) + a + b)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*si
nh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-
((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3
)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 +
b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^
3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*
b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*co
sh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^
3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6
+ 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh
(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a
^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2
+ 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2
*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a +
 b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 +
 (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x
))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*
cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^
4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
 1), -1/4*(2*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sin
h(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arcta
n(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a
 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)
*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*
b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(

```
2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 2*((a +
b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a + b)*
cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a + b)*cosh(x)^
3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) +
 (a + b)*sinh(x)^2 + a + b)) - ((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(
x)*sinh(x)^3 + (3*a + 2*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a
+ 2*b)*cosh(x)^2 + 3*a + 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a +
 2*b)*cosh(x))*sinh(x) + 3*a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(
a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 +
2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b
)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*
sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(co
sh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sin
h(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x
)^3 + cosh(x))*sinh(x) + 1), -1/2*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*s
inh(x)^3 + (a + b)*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2
 + a + b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a +
 b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 +
 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b
^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x)
)*sinh(x))) - ((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3
*a + 2*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2
+ 3*a + 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*si
nh(x) + 3*a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
 b)*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b
)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + sqrt(2)*(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x
```

```
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)]
```

Sympy [F]

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int \left(a+b\tanh^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*tanh(x)**2)**(3/2),x)
[Out] Integral((a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int \left(b\tanh\left(x\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(70) = 140$.

Time = 0.81 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.64

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=
-\frac{1}{2}\,(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)
+\frac{1}{2}\,(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)
-\frac{(3\,ab+2\,b^2)\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}}
-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-}{2\,\sqrt{a+b}}
-\frac{2\left((ab+2\,b^2)\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)^3+(3\,ab-2\,b^2)\left(\sqrt{a+b}e^{(2\,x)}\right.}{\left(\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2}\right.\right.}$$

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) - sqrt(a + b))) - (3*a*b + 2*b^2)*arctan(-1/2*(sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 2*((a*b + 2*b^2)*(s
qrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b))^3 + (3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (3*a^2*b - 3*a*
b^2 - 2*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b)) + (a^2*b - a*b^2 + 2*b^3)*sqrt(a + b))/((sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^2
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int {\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{3/2}\,dx$$

```
[In] int((a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int((a + b*tanh(x)^2)^(3/2), x)
```

3.223 $\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = -a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - b\sqrt{a+b\tanh^2(x)}$$

```
[Out] -a^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))+(a+b)^(3/2)*arctanh((a+b*ta
nh(x)^2)^(1/2)/(a+b)^(1/2))-b*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3751, 457, 86, 162, 65, 214}

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = a^{3/2}\left(-\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - b\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x\,(1-x^2)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(1-x)x}\,dx, x, \tanh^2(x)\right)$$

$$= -b\sqrt{a+b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a^2+(-2a-b)bx}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= -b\sqrt{a+b\tanh^2(x)} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{1}{2}(a+b)^2\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= -b\sqrt{a+b\tanh^2(x)} + \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}$$

$$= -a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - b\sqrt{a+b\tanh^2(x)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = -a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - b\sqrt{a+b\tanh^2(x)}$$

```
[In] Integrate[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Maple [F]

$$\int \coth(x)\left(a+b\tanh(x)^{2}\right)^{\frac{3}{2}}dx$$

[In] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(57) = 114$.

Time = 0.42 (sec) , antiderivative size = 4039, normalized size of antiderivative = 56.89

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3
 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh
(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b
^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3
 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5
+ 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*s
inh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x
)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
+ 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 +
sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*co
sh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*
cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 +
18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(
3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a
^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3
+ 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*co
sh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
h(x)^2 + a)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*
```

```
cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(
x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)
^2 + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)
^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x
)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*b*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/4*(4*(a*cosh(x)^
2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((a^3 + a^2*b)
*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 +
2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2
)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*si
nh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cos
h(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh
(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 +
 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b
- a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x
)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*s
inh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b -
 b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^
2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (
3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(c
osh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh
(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a +
b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(
a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh
(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*b*sqrt(((a + b)*cosh(x)^2 + (a + b)
```

```
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*c
osh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a
*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^2
 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
 b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - (a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(-((2*a + b)*cosh(
x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*co
sh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 +
4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)
^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*b*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/2*(2*(a*cosh(x)^2
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*c
osh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (
a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(
x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*
cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - ((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*
sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sqrt(2)*b*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
```

```
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [F]

$$\int \coth(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \int \left(a + b\tanh^2(x)\right)^{\frac{3}{2}} \coth(x)\, dx$$

```
[In] integrate(coth(x)*(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral((a + b*tanh(x)**2)**(3/2)*coth(x), x)
```

Maxima [F]

$$\int \coth(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \int \left(b\tanh(x)^2 + a\right)^{\frac{3}{2}} \coth(x)\, dx$$

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x), x)
```

Giac [F]

$$\int \coth(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \int \left(b\tanh(x)^2 + a\right)^{\frac{3}{2}} \coth(x)\, dx$$

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \int \coth(x) \left(b\tanh(x)^2 + a\right)^{3/2} dx$$

```
[In] int(coth(x)*(a + b*tanh(x)^2)^(3/2),x)
[Out] int(coth(x)*(a + b*tanh(x)^2)^(3/2), x)
```

3.224 $\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = -b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - a\coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -b^(3/2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))+(a+b)^(3/2)*arctanh
((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-a*coth(x)*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 485, 537, 223, 212, 385}

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = b^{3/2}\left(-\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - a\coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[In] Int[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + (a + b)^(3/2)
*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - a*Coth[x]*Sqrt[a +
b*Tanh[x]^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= -a\coth(x)\sqrt{a+b\tanh^2(x)} + \text{Subst}\left(\int \frac{a(a+2b)+b^2x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -a\coth(x)\sqrt{a+b\tanh^2(x)} - b^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&\quad + (a+b)^2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -a\coth(x)\sqrt{a+b\tanh^2(x)} - b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&\quad + (a+b)^2\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&= -b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&\quad + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - a\coth(x)\sqrt{a+b\tanh^2(x)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.56

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{a\left((a-b+(a+b)\cosh(2x))\text{csch}^2(x) - \sqrt{2}(a+2b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b}{}}}{}\right.\right.\right.}{\sqrt{2}\sqrt{(}}$$

```
[In] Integrate[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((a*((a - b + (a + b)*Cosh[2*x])*Csch[x]^2 - Sqrt[2]*(a + 2*b)*Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*(a + b)*Sqrt[((a - b + (a
```

```
 + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b +
(a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*Sqrt[(a -
b + (a + b)*Cosh[2*x])*Sech[x]^2]))
```

Maple [F]

$$\int \coth(x)^2 \left(a + b\tanh(x)^2\right)^{\frac{3}{2}} dx$$

```
[In] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(63) = 126$.

Time = 0.40 (sec) , antiderivative size = 3913, normalized size of antiderivative = 50.82

$$\int \coth^2(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*
sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^
2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cos
h(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*
b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^
3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
 sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2
*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*
(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*
b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
```

```
nh(x)^2 - b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x
)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^
2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)
*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh
(x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x
)^2 - a - b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)
^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x
)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), 1/4*(4*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a + b)*log(-((a*b^2 + b^3
)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 -
 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*s
inh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*co
sh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 +
 b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6
- 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(
x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*
sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b -
3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b
^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt
(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh
```

```
(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*c
osh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b
^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^2
 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arct
an(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a +
b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) - (b*cosh(x)^2 + 2*b*cosh(
x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*
b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(
a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh
(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x
)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^
3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*a*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)
*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arctan(sqrt(2)*(
b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
+ (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*co
sh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^
2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) - 2*(b*cosh(x)^2 + 2*
b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*
cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 +
 (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x
)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a
- b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 + a + b)) + 2*sqrt(2)*a*sqrt(((a + b)*cosh(x)^2 + (a +
```

```
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F]

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(a+b\tanh^2(x)\right)^{\frac{3}{2}} \coth^2(x)\, dx$$

```
[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral((a + b*tanh(x)**2)**(3/2)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(b\tanh(x)^2+a\right)^{\frac{3}{2}} \coth(x)^2\, dx$$

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(63) = 126.

Time = 0.96 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.58

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx =$$

$$-\frac{2\,b^2 \arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$

$$-\frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)$$

$$+\frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$

$$-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a\right.\right.}{2\sqrt{a+b}}$$

$$+\frac{4\left(\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)a^2+\sqrt{\right.}{\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,a\right.}$$

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a +
 b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*log(abs(-
sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*
e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*
(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 4*((sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a^2 + sqrt(a
+ b)*a^2)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2}\, dx = \int \coth(x)^2 \left(b\tanh(x)^2+a\right)^{3/2} dx$$

```
[In] int(coth(x)^2*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

3.225 $\int \sqrt{1+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \sqrt{1+\tanh^2(x)}\,dx = -\operatorname{arcsinh}(\tanh(x)) + \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

```
[Out] -arcsinh(tanh(x))+arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3742, 399, 221, 385, 212}

$$\int \sqrt{1+\tanh^2(x)}\,dx = \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right) - \operatorname{arcsinh}(\tanh(x))$$

```
[In] Int[Sqrt[1 + Tanh[x]^2],x]
[Out] -ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{1+x^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= 2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)\\
&= -\text{arcsinh}(\tanh(x)) + 2\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)\\
&= -\text{arcsinh}(\tanh(x)) + \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \sqrt{1+\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)-\operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{1+\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

```
[In] Integrate[Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh
[x]*Sqrt[1 + Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

method	result
derivativedivides	$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(1+\tan}}{}$
default	$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(1+\tan}}{}$

```
[In] int((1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((tanh(x)-1)^2+2*tanh(x))^(1/2)-arcsinh(tanh(x))+1/2*2^(1/2)*arctanh(1
/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/2*((1+tanh(x))^
2-2*tanh(x))^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x
))^2-2*tanh(x))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 679, normalized size of antiderivative = 21.90

$$\int \sqrt{1+\tanh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(
x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x
)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 +
30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 +
10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*
sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x
)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 +
 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqr
t(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cos
h(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x)
+ sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)
^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

Sympy [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2{(x)}+1}\,dx$$

```
[In] integrate((1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**2 + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh{(x)}^2+1}\,dx$$

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x)^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.35

$$\int \sqrt{1+\tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)\right.$$

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt
(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x) +
 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) +
 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \sqrt{1+\tanh^2(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{2} - \operatorname{asinh}(\tanh(x)) + \frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)}{2}$$

```
[In] int((tanh(x)^2 + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
1)))/2 - asinh(tanh(x)) + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(
1/2) + 1) - log(tanh(x) - 1)))/2
```

3.226 $\int \sqrt{-1-\tanh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$

```
[Out] arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))-arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^
(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3742, 399, 223, 209, 385}

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

```
[In] Int[Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sqr
t[-1 - Tanh[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{-1-x^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)\right) + \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tanh(x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\\
&= \arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh(x)\right)-\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{-1-\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

```
[In] Integrate[Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh
[x]*Sqrt[-1 - Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.16

method	result
derivativedivides	$\frac{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{2}+\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-$
default	$\frac{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{2}+\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-$

```
[In] int((-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(1+tanh(x))^2+2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(1+tanh(x))^2+2*t
anh(x))^(1/2))-1/2*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^
2+2*tanh(x))^(1/2))-1/2*(-(tanh(x)-1)^2-2*tanh(x))^(1/2)+1/2*arctan(tanh(x)
/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/2*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^
(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 5.02

$$\int \sqrt{-1-\tanh^2(x)}\,dx$$
$$= -\frac{1}{4}\sqrt{-2}\log\left(-\left(\sqrt{-2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right)$$
$$+\frac{1}{4}\sqrt{-2}\log\left(\left(\sqrt{-2}\sqrt{-2\,e^{(4\,x)}-2}-2\,e^{(2\,x)}-2\right)e^{(-2\,x)}\right)$$
$$+\frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2\,e^{(4\,x)}-2}(e^{(2\,x)}-2)+\sqrt{-2}e^{(4\,x)}-\sqrt{-2}e^{(2\,x)}+2\sqrt{-2}\right)e^{(-4\,x)}\right)$$
$$-\frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2\,e^{(4\,x)}-2}(e^{(2\,x)}-2)-\sqrt{-2}e^{(4\,x)}+\sqrt{-2}e^{(2\,x)}-2\sqrt{-2}\right)e^{(-4\,x)}\right)$$
$$+\frac{1}{2}i\log\left(-4\left(i\sqrt{-2\,e^{(4\,x)}-2}+e^{(2\,x)}-1\right)e^{(-2\,x)}\right)$$
$$-\frac{1}{2}i\log\left(-4\left(-i\sqrt{-2\,e^{(4\,x)}-2}+e^{(2\,x)}-1\right)e^{(-2\,x)}\right)$$

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-2)*log(-(sqrt(-2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)
) + 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-2*
x)) + 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sqrt(-2)*e^
(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) - 1/4*sqrt(-2)*log(-2*(sqr
t(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*s
qrt(-2))*e^(-4*x)) + 1/2*I*log(-4*(I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^
(-2*x)) - 1/2*I*log(-4*(-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh^2{(x)}-1}\,dx$$

```
[In] integrate((-1-tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-tanh(x)**2 - 1), x)
```

Maxima [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh(x)^2-1}\,dx$$

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-tanh(x)^2 - 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.31

$$\int \sqrt{-1-\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2}\right.\right.$$

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sq
rt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x)
 + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x)
 + 1))
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \sqrt{-1-\tanh^2(x)}\,dx = -\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right) - \ln\left(\tanh(x)-\sqrt{-\tanh(x)^2-1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int((- tanh(x)^2 - 1)^(1/2),x)
```

```
[Out] - log(tanh(x) - (- tanh(x)^2 - 1)^(1/2)*1i)*1i - 2^(1/2)*atan((2^(1/2)*tanh
(x))/(- tanh(x)^2 - 1)^(1/2))
```

3.227 $\int \left(1+\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = -\frac{5}{2}\text{arcsinh}(\tanh(x))+2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

```
[Out] -5/2*arcsinh(tanh(x))+2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2
)-1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3742, 427, 537, 221, 385, 212}

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = -\frac{5}{2}\text{arcsinh}(\tanh(x))+2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

```
[In] Int[(1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] (-5*ArcSinh[Tanh[x]])/2 + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh
[x]^2]] - (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-3-5x^2}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right) + 4\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{5}{2}\text{arcsinh}(\tanh(x)) - \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} + 4\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

$$= -\frac{5}{2}\text{arcsinh}(\tanh(x)) + 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(-4\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x) + 5\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x) + \cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)(1+ }{2\cosh^{\frac{3}{2}}(2x)}$$

```
[In] Integrate[(1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sq
rt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(1 + Tanh[x]^2)
^(3/2))/Cosh[2*x]^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(38) = 76.

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.16

method	result
derivativedivides	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\operatorname{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh\left(x\right)-}$
default	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\operatorname{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh\left(x\right)-}$

```
[In] int((1+tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((tanh(x)-1)^2+2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)-1)^2+2*tanh(x))^
(1/2)-5/2*arcsinh(tanh(x))-((tanh(x)-1)^2+2*tanh(x))^(1/2)+2^(1/2)*arctanh(
1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/6*((1+tanh(x))
^2-2*tanh(x))^(3/2)-1/4*tanh(x)*((1+tanh(x))^2-2*tanh(x))^(1/2)+((1+tanh(x)
)^2-2*tanh(x))^(1/2)-2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))
^2-2*tanh(x))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 1027, normalized size of antiderivative = 20.54

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4
 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(s
qrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^
6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2
+ 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*
sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*
cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x
) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 +
3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sq
rt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18
*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt
(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2
))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqr
```

```
t(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*
(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*si
nh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2)) - 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
 1)*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + si
nh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = \int \left(\tanh^{2}\left(x\right)+1\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((1+tanh(x)**2)**(3/2),x)
[Out] Integral((tanh(x)**2 + 1)**(3/2), x)
```

Maxima [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = \int \left(\tanh\left(x\right)^{2}+1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((tanh(x)^2 + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.04

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = -\frac{1}{4}\sqrt{2}\left(5\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-\right.}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2\right.}\right.$$

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(5*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sq
rt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*
x))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/
((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2 +
 4*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + 4*log(sqrt(e^(4*x) + 1) - e^(2*x)
) - 4*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = \sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right) - \frac{5\,\mathrm{asinh}(\tanh(x))}{2} - \frac{\tanh(x)\sqrt{\tanh(x)^2+1}}{2} + \sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)$$

```
[In] int((tanh(x)^2 + 1)^(3/2),x)
```

```
[Out] 2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) + 1
)) - (5*asinh(tanh(x)))/2 - (tanh(x)*(tanh(x)^2 + 1)^(1/2))/2 + 2^(1/2)*(lo
g(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(tanh(x) - 1))
```

3.228 $\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx=-\frac{5}{2}\arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)+2\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)+\frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)}$$

```
[Out] -5/2*arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))+2*arctan(2^(1/2)*tanh(x)/(-1-tanh
(x)^2)^(1/2))*2^(1/2)+1/2*(-1-tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3742, 427, 537, 223, 209, 385}

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx=-\frac{5}{2}\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+2\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}$$

```
[In] Int[(-1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] (-5*ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]])/2 + 2*Sqrt[2]*ArcTan[(Sqrt[2]*Tan
h[x])/Sqrt[-1 - Tanh[x]^2]] + (Tanh[x]*Sqrt[-1 - Tanh[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(-1-x^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-3-5x^2}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tanh(x)\right) + 4\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 4\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$

$$= -\frac{5}{2}\arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(-4\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x) + 5\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x) + \cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)(-1$$

$$2\cosh^{\frac{3}{2}}(2x)$$

```
[In] Integrate[(-1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sq
rt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(-1 - Tanh[x]^2
)^(3/2))/Cosh[2*x]^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.15

method	result
derivativedivides	$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}-$
default	$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}-$

```
[In] int((-1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-(tanh(x)-1)^2-2*tanh(x))^(3/2)+1/4*tanh(x)*(-(tanh(x)-1)^2-2*tanh(x)
)^(1/2)-5/4*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+(-(tanh(x)-1)^
2-2*tanh(x))^(1/2)-2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^
2-2*tanh(x))^(1/2))+1/6*(-(1+tanh(x))^2+2*tanh(x))^(3/2)+1/4*tanh(x)*(-(1+t
anh(x))^2+2*tanh(x))^(1/2)-5/4*arctan(tanh(x)/(-(1+tanh(x))^2+2*tanh(x))^(1
/2))-(-(1+tanh(x))^2+2*tanh(x))^(1/2)+2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(
1/2)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 5.39

$$\int (-1-\tanh^2(x))^{3/2}\,dx = \frac{2\left(\sqrt{-2}e^{(4x)}+2\sqrt{-2}e^{(2x)}+\sqrt{-2}\right)\log\left(2\left(\sqrt{-2}\sqrt{-2e^{(4x)}-2}+2e^{(2x)}+2\right)e^{(-2x)}\right)-}{}$$

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(2*(sqrt(-2)*s
qrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*sq
rt(-2)*e^(2*x) + sqrt(-2))*log(-2*(sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x
) - 2)*e^(-2*x)) - 5*(I*e^(4*x) + 2*I*e^(2*x) + I)*log(-4*(I*sqrt(-2*e^(4*x
) - 2) + e^(2*x) - 1)*e^(-2*x)) - 5*(-I*e^(4*x) - 2*I*e^(2*x) - I)*log(-4*(
-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*
sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sq
rt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) + 2*(sqrt(-2)*e^(
4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x)
```

```
- 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x)) + 2*sqrt
(-2*e^(4*x) - 2)*(e^(2*x) - 1))/(e^(4*x) + 2*e^(2*x) + 1)
```

Sympy [F]

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx=\int \left(-\tanh^{2}\left(x\right)-1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-1-tanh(x)**2)**(3/2),x)
[Out] Integral((-tanh(x)**2 - 1)**(3/2), x)
```

Maxima [F]

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx=\int \left(-\tanh\left(x\right)^{2}-1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((-tanh(x)^2 - 1)^(3/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.04

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx=-\frac{1}{4}\sqrt{2}\left(-5i\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(-3i\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{3}+i\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}+\cdots\right)}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-2\sqrt{e^{(4x)}+1}+\cdots\right)^{2}}\cdots\right)$$

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(-5*I*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/
(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(-3*I*(sqrt(e^(4*x) + 1) -
 e^(2*x))^3 + I*(sqrt(e^(4*x) + 1) - e^(2*x))^2 + I*sqrt(e^(4*x) + 1) - I*e
^(2*x) + I)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2
*x) - 1)^2 - 4*I*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) - 4*I*log(sqrt(e^(4*x
) + 1) - e^(2*x)) + 4*I*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx=\int \left(-\tanh(x)^2-1\right)^{3/2}dx$$

```
[In] int((- tanh(x)^2 - 1)^(3/2),x)
[Out] int((- tanh(x)^2 - 1)^(3/2), x)
```

3.229 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+(a-b)*(a+b*tanh(x)^2
)^(1/2)/b^2-1/3*(a+b*tanh(x)^2)^(3/2)/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 90, 65, 214}

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} + \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}$$

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b)*Sqrt[a +
b*Tanh[x]^2])/b^2 - (a + b*Tanh[x]^2)^(3/2)/(3*b^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^5}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a-b}{b\sqrt{a+bx}}+\frac{1}{(1-x)\sqrt{a+bx}}-\frac{\sqrt{a+bx}}{b}\right)dx,x,\tanh^2(x)\right)$$

$$=\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{1}{2}\text{Subst}\left(\int\frac{1}{(1-x)\sqrt{a+bx}}dx,x,\tanh^2(x)\right)$$

$$=\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}dx,x,\sqrt{a+b\tanh^2(x)}\right)}{b}$$

$$=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int\frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b+(a-2b)\cosh(2x))\text{sech}^2(x)\sqrt{a+b\tanh^2(x)}}{3b^2}$$

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b + (a - 2*b
)*Cosh[2*x])*Sech[x]^2*Sqrt[a + b*Tanh[x]^2])/(3*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.34

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}-\frac{\tanh(x)^2\sqrt{a+b\tanh(x)^2}}{3b}+\frac{2a\sqrt{a+b\tanh(x)^2}}{3b^2}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)\cdots}}{\tanh(x)-1}\cdots\right.}{2\sqrt{a+b}}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}-\frac{\tanh(x)^2\sqrt{a+b\tanh(x)^2}}{3b}+\frac{2a\sqrt{a+b\tanh(x)^2}}{3b^2}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)\cdots}}{\tanh(x)-1}\cdots\right.}{2\sqrt{a+b}}$

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a+b*tanh(x)^2)^(1/2)/b-1/3*tanh(x)^2/b*(a+b*tanh(x)^2)^(1/2)+2/3*a/b^2*(a
+b*tanh(x)^2)^(1/2)+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/
2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)^(1/2
)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x)
)+a+b)^(1/2))/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(58) = 116$.

Time = 0.43 (sec) , antiderivative size = 2827, normalized size of antiderivative = 40.39

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*co
sh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^
2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^
4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*
(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*
a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)
^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4
```

```
+ 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^
2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
+ 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a
 + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b
*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8
*sqrt(2)*((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sin
h(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^
2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*(
(a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(((a + b)
*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)))/((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (
a*b^2 + b^3)*sinh(x)^6 + 3*(a*b^2 + b^3)*cosh(x)^4 + 3*(a*b^2 + b^3 + 5*(a*
b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + 3*(a*b^2 +
 b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2 + 3*(5*(
a*b^2 + b^3)*cosh(x)^4 + a*b^2 + b^3 + 6*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
 + 6*((a*b^2 + b^3)*cosh(x)^5 + 2*(a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*c
osh(x))*sinh(x)), -1/6*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cos
h(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4
 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x
)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*
cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh
(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*
a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh
(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)
^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5
*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5
+ 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 4*sqrt(2)*((a^2 - a*b -
2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2
*b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x
)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cos
h(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b^2 + b^
3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6
```

```
+ 3*(a*b^2 + b^3)*cosh(x)^4 + 3*(a*b^2 + b^3 + 5*(a*b^2 + b^3)*cosh(x)^2)*s
inh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + 3*(a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2 + 3*(5*(a*b^2 + b^3)*cosh(x)^4
+ a*b^2 + b^3 + 6*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a*b^2 + b^3)*cos
h(x)^5 + 2*(a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^5{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh{(x)}^5}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^5/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(58) = 116$.

Time = 0.67 (sec) , antiderivative size = 592, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{8\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)^5+3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(}\right.\right.}{}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b) - 8/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 3*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqr
t(a + b) + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b))^3*(3*a - 5*b) + 6*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b)*(a -
 3*b) - 3*(3*a^2 + 6*a*b - 13*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (9*a^2 - 22*a*b + 17*b^2)
*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3\,b^2} - \left(\frac{a+b}{b^2}-\frac{2\,a}{b^2}\right)\sqrt{b\tanh\left(x\right)^2+a}$$

```
[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x)
^2)^(3/2)/(3*b^2) - ((a + b)/b^2 - (2*a)/b^2)*(a + b*tanh(x)^2)^(1/2)
```

3.230 $\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

```
[Out] 1/2*(a-2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh(
(a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-1/2*(a+b*tanh(x)^2)^
(1/2)*tanh(x)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 490, 537, 223, 212, 385}

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

```
[In] Int[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*b^(3/2)) +
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Tanh[x]
*Sqrt[a + b*Tanh[x]^2])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{2b}$$
$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{2b} + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b} + \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$= \frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\left(\sqrt{2}a(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right), 1\right) - 2\sqrt{2}ab\sqrt{(\cdots}\right.}{2\sqrt{2}b(a+\cdots}$$

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipti
cF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*
Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a
```

```
 + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]
- (a + b)*(a - b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*b*(a +
 b)*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{\sqrt{b}}-\frac{\sqrt{a+b\tanh(x)^{2}}\,\tanh(x)}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\frac{2a+2}{}\right.}{}$
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{\sqrt{b}}-\frac{\sqrt{a+b\tanh(x)^{2}}\,\tanh(x)}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\frac{2a+2}{}\right.}{}$

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*(a+b*tanh(x)^2)^(1/2
)*tanh(x)/b+1/2*a/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-
1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(70) = 140$.

Time = 0.48 (sec) , antiderivative size = 5494, normalized size of antiderivative = 62.43

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^4{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(tanh(x)**4/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh{(x)}^4}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^4/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(70) = 140$.

Time = 0.57 (sec) , antiderivative size = 559, normalized size of antiderivative = 6.35

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(a-2b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(a+2b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}\cdots}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}\cdots}\right.\right.}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a - 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) - 1
/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
 - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) -
 sqrt(a + b)))/sqrt(a + b) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(
3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^2 - a*b
+ 2*b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3
*b)^2*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\tanh(x)^4}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2), x)
```

3.231 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 81, 65, 214}

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^3}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)}}{b} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)}}{b} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}
\end{aligned}
$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b\tanh^2(x)}}{b}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b\tanh^2(x)}}{b}$$

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(39) = 78.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.74

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(}{}\right.}{}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(}{}\right.}{}$

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a+b*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b
)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)
^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(39) = 78$.

Time = 0.38 (sec) , antiderivative size = 1625, normalized size of antiderivative = 34.57

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*log
(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*
b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2
*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a
^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*c
osh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2
*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6
*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6
 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cos
h(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (
3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a
^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^
2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5
+ (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(
x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20
*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6)) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*
log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 -
 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^
2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b
+ b^2), -1/2*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a
 - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a +
 b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a
```

```
*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)
^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)))
 + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a - b)*arcta
n(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x
)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)
*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cos
h(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b + b^2)]
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(39) = 78$.

Time = 0.49 (sec) , antiderivative size = 345, normalized size of antiderivative = 7.34

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{\right.}}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\right.}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b) - 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/((sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2
 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{b\tanh(x)^2+a}}{b}$$

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x)
^2)^(1/2)/b
```

3.232 $\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 494, 223, 212, 385}

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

```
[In] Int[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)
\end{aligned}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$

$$-\frac{a\coth(x)\operatorname{EllipticPi}\left(\frac{b}{a+b}, \arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right), 1\right)\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x}}{b(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}$$

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((a*Coth[x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])
*Csch[x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])/(b
*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(48) = 96.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.28

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}} + \frac{\ln(}{}$
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}} + \frac{\ln(}{}$

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2/(a+b)^(1/2)*ln((2*a+
2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/
2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*
(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(48) = 96$.

Time = 0.44 (sec) , antiderivative size = 3361, normalized size of antiderivative = 56.02

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)
*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b
^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*co
sh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6
*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b
^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^
5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))
*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh
(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3
- 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^
2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4
- 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2
*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x
))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a
*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3
- 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^
4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(
x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + sqrt(a + b)*b*log(((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(a*b + b^2), 1/4*(4*(a + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
```

```
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)
^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2
+ 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b
)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8
+ 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 +
2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6
 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^
3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b +
4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^
3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2
+ 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 +
4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(
x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^
2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b
 - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s
inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^
2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 +
 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 +
6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*
cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*b*log
(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b + b^2), -1/2*(sqrt(-a - b)*b*arctan(sqrt(2)*(b*cosh(
x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b
+ b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2
 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh
(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + sqrt(-a - b)*b*arctan(sqrt
(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) - (a + b)*sqrt(b)*log(-((a + 2*b)
*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2
*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x
```

```
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a*b + b^2), -1/2*(sqrt(-
a - b)*b*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b
+ b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh
(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a
*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x
))) - 2*(a + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)
*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sin
h(x) + a + b)) + sqrt(-a - b)*b*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a + b)))/(a*b + b^2)]
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(tanh(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(48) = 96$.

Time = 0.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.20

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*log(abs(
-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*log(abs(-sq
rt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b
)))/sqrt(a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

3.233 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3751, 455, 65, 214}

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.93

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(t
anh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(
x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 1361, normalized size of antiderivative = 46.93

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*si
nh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3
```

```
+ a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(
x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3
+ 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)
^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 +
 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cos
h(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*c
osh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 1
8*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3
*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^
3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 +
 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cos
h(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))
*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1
/2*(sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 +
 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b
^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x)
)*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(
a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x
))*sinh(x) + a + b)))/(a + b)]
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } \sqrt{a}=0 \\ \frac{\log\left(2\sqrt{a}\tanh^2(x)-2\sqrt{a}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(1/2),x)
[Out] -Piecewise((atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/sqrt(-a - b), Ne(b, 0
)), (Piecewise((zoo*tanh(x)**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*tanh(x)**2
- 2*sqrt(a))/(2*sqrt(a)), True)), True))
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(23) = 46.

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.48

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2)
```

3.234 $\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3742, 385, 212}

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[In] Int[1/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\ &= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[In] Integrate[1/Sqrt[a + b*Tanh[x]^2],x]
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.68

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2}}{1+\tanh(x)}\right.}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2}}{1+\tanh(x)}\right.}{2\sqrt{a+b}}$

```
[In] int(1/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1
+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tan
h(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(25) = 50$.

Time = 0.30 (sec) , antiderivative size = 1287, normalized size of antiderivative = 41.52

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*s
inh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh
(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
- 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*s
inh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x
)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*co
sh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*
cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(
3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))
```

```
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b
^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*co
sh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cos
h(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))
*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1
/2*(sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 +
 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b
^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x)
)*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a +
 b)))/(a + b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{a+b\tanh^{2}{\left(x \right)}}}\,dx$$

```
[In] integrate(1/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{b\tanh\left(x\right)^{2}+a}}\,dx$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(25) = 50.

Time = 0.39 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.06

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\tanh(x)\sqrt{a+b}}{\sqrt{b\tanh(x)^2+a}}\right)}{\sqrt{a+b}}$$

```
[In] int(1/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((tanh(x)*(a + b)^(1/2))/(a + b*tanh(x)^2)^(1/2))/(a + b)^(1/2)
```

3.235 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 457, 88, 65, 214}

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}
\end{aligned}
$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[In] Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Maple [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh(x)^2}}dx$$

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(44) = 88.

Time = 0.38 (sec) , antiderivative size = 3527, normalized size of antiderivative = 62.98

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*
sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^
3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cos
h(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^
3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5
 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*
```

```
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(
x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 +
 sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*c
osh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2
*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 +
 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*
(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*
a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3
 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x
)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a +
 b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2
*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + sqrt(a + b)*a*log(-((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(a^2 + a*b), 1/4*(4*sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)
^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2
+ 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b
)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 +
 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a
^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6
+ 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6
*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a
^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(1
4*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a
^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 +
 a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5
+ a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4
*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x
)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2
```

```
 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*
b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
 a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
+ b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6
*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*c
osh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*a*log(
-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a^2 + a*b), -1/2*(a*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2
+ a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2
 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh
(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + a*sqrt(-a - b)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a +
 b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sin
h(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 +
4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - (a + b)*sqrt(a)
*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh
(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)
^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) +
2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)
*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)))/(a^2 + a*
b), 1/2*(2*sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) - a*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2
+ a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4
+ (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b -
b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
a*b - b^2)*cosh(x))*sinh(x))) - a*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
```

```
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b
)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x
)^3 + (a - b)*cosh(x))*sinh(x) + a + b)))/(a^2 + a*b)]
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth{(x)}}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(44) = 88$.

Time = 0.51 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.54

$$\begin{aligned}\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} \\ &- \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} \\ &+ \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\ &- \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}\end{aligned}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*log(abs(-
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/2*log(abs(-sqr
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)
))/sqrt(a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a + b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\tanh{(x)}^2 + a}}\,dx$$

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)/(a + b*tanh(x)^2)^(1/2), x)
```

3.236 $\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-coth(x)*(a+b
*tanh(x)^2)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 491, 12, 385, 212}

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

```
[In] Int[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x]
*Sqrt[a + b*Tanh[x]^2])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a} + \frac{\text{Subst}\left(\int \frac{a}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a}\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a} + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a} + \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)
\end{aligned}$$

$$=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$

$$-\frac{\cosh^2(x)\coth(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{4(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^2(x)(a+b\tanh^2(x))}{3a^2}+\frac{\arcsin\left(\sqrt{-\cdots}\right.}{a\sqrt{-\frac{(a+\cdots}{}}}\right.}{\sqrt{a+b\tanh^2(x)}}$$

```
[In] Integrate[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((Cosh[x]^2*Coth[x]*(1 + (b*Tanh[x]^2)/a)*((-4*(a + b)*Hypergeometric2F1[2
, 2, 5/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*(a + b*Tanh[x]^2))/(3*a^2) +
(ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + 2*b*Tanh[x]^2))/(a*Sqrt[-(((a
+ b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])))/Sqrt[a + b*Tanh[x]^2])
```

Maple [F]

$$\int \frac{\coth(x)^2}{\sqrt{a+b\tanh(x)^2}}dx$$

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 1565, normalized size of antiderivative = 30.69

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)*log
(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b
^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*
b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(
a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*
cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^
2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b
^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5
 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh
(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 2
0*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)
*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*
b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2
- a*b), -1/2*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a
 - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a -
 b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b
^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)
^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x)))
 + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a - b)*arcta
n(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + 2*sqrt(2)*(a + b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (
a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(coth(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(43) = 86.

Time = 0.48 (sec) , antiderivative size = 343, normalized size of antiderivative = 6.73

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{\cdots}\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\cdots}\right.}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b) + 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/((sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2
 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\mathrm{coth}(x)^2}{\sqrt{b\,\mathrm{tanh}\left(x\right)^2+a}}\,dx$$

```
[In] int(coth(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

3.237 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*ta
nh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/2)
/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 105, 162, 65, 214}

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

```
[In] Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/a^(3/2) + ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x^3(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) \\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2a+b)-\frac{bx}{2}}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a} \\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) \\
&\quad + \frac{(2a-b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{4a} \\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} \\
&\quad + \frac{(2a-b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{2ab} \\
&= -\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$
$$= \frac{(-2a^2-ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a}\left(2a\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\coth^2(x)\sqrt{a+\right.}{2a^{3/2}(a+b)}$$

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
[Out] ((-2*a^2 - a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(2*a
*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Coth[x]^2
*Sqrt[a + b*Tanh[x]^2]))/(2*a^(3/2)*(a + b))
```

Maple [F]

$$\int \frac{\coth(x)^3}{\sqrt{a+b\tanh(x)^2}}dx$$

```
[In] int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(70) = 140$.

Time = 0.49 (sec) , antiderivative size = 5711, normalized size of antiderivative = 64.90

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*tanh(x)**2)**(1/2),x)
[Out] Integral(coth(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(70) = 140$.

Time = 0.62 (sec) , antiderivative size = 565, normalized size of antiderivative = 6.42

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(2a-b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(2a+b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+\cdots}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}\cdots}\right.\right.}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a - b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) - 1
/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
 - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/2*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) -
 sqrt(a + b)))/sqrt(a + b) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(
2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*a^2 - a*
b + b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a +
 b)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\coth(x)^3}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

```
[In] int(coth(x)^3/(a + b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)^3/(a + b*tanh(x)^2)^(1/2), x)
```

3.238 $\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-a^2/b^2/(a+b)/(a+b*t
anh(x)^2)^(1/2)-(a+b*tanh(x)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 89, 65, 214}

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - a^2/(b^2*(a + b)
*Sqrt[a + b*Tanh[x]^2]) - Sqrt[a + b*Tanh[x]^2]/b^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^5}{(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{3/2}} - \frac{1}{b\sqrt{a+bx}} - \frac{1}{(a+b)(-1+x)\sqrt{a+bx}}\right) dx, x, \tanh^2(x)\right)$$

$$= -\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\, dx, x, \tanh^2(x)\right)}{2(a+b)}$$

$$= -\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\mathrm{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\, dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\, dx = \frac{-b^2\, \mathrm{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{a+b\tanh^2(x)}{a+b}\right) - (a+b)\left(2a-b+b\tanh^2(x)\right)}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
[Out] (-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) - (a + b
)*(2*a - b + b*Tanh[x]^2))/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(62) = 124.

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 4.47

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\tanh(x)^2}}-\frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}}-\frac{2a}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$
default	$\frac{1}{b\sqrt{a+b\tanh(x)^2}}-\frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}}-\frac{2a}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(a+b*tanh(x)^2)^(1/2)-tanh(x)^2/b/(a+b*tanh(x)^2)^(1/2)-2*a/b^2/(a+b*ta
nh(x)^2)^(1/2)-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b
)*(2*b*(tanh(x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+
a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)/(b*(1+tanh(x)
)^2-2*b*(1+tanh(x))+a+b)^(1/2)-b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b
^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b
-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))
/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(62) = 124$.

Time = 0.49 (sec) , antiderivative size = 3991, normalized size of antiderivative = 55.43

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2
 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3
)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2
 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b
^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a
^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(
x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^
3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*
b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^
2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*
```

b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cos
h(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(
x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cos
h(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a
^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*
a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cos
h(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*
sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b^2 + b^3)*cosh(x)^6 + 6*
(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)
*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*
(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3
+ (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 +
 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*
(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*l
og(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 -
2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^
4 + 4*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 4*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + 2*a^3 + 4*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 +
 2*a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3 + 3*(2
*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 4*a^2*b
+ 3*a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^
6 + 6*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^5 + (a^3*b^2 +
3*a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^6 + a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
+ (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + (3*a^3*b^2 + 5*a^2*b^3
+ a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*
a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4
- b^5)*cosh(x)^2 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a
^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 6*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^
5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*
b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)), -1/2*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a
*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*c
osh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a

```
*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 +
(3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6
*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3
*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh
(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*
(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a*b
^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sin
h(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*
sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cos
h(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b
^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x)
)*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) + 2*sqrt(2)*((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4
+ 4*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 4*a^2*b
+ 3*a*b^2 + b^3)*sinh(x)^4 + 2*a^3 + 4*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 2
*a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3 + 3*(2*a
^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 4*a^2*b +
3*a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^6
+ 6*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^5 + (a^3*b^2 + 3*
a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^6 + a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 +
(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + (3*a^3*b^2 + 5*a^2*b^3 +
a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^
2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^2 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 6*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^2)*sinh(x)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^5
+ 2*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*b^
3 + a*b^4 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^5}{(b\tanh(x)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(62) = 124.

Time = 0.57 (sec) , antiderivative size = 460, normalized size of antiderivative = 6.39

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = -\frac{\frac{(a^4b+a^3b^2)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5}+\frac{a^4b+a^3b^2}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} - \frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{\right.}}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\right.\right.}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^4*b + a^3*b^2)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) + (a^4*b + a^3*b^
2)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a
 - b)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/
2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2) - 4*(sqrt(a + b)
*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
- sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)*b)
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2\,a+2\,b)}{2\,(a+b)^{3/2}}\right)}{(a+b)^{3/2}} - \frac{\sqrt{b\tanh\left(x\right)^2+a}}{b^2} - \frac{a^2}{b^2\,\left(a+b\right)\,\sqrt{b\tanh\left(x\right)^2+a}}$$

```
[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(a + b)^(3/2
) - (a + b*tanh(x)^2)^(1/2)/b^2 - a^2/(b^2*(a + b)*(a + b*tanh(x)^2)^(1/2))
```

3.239 $\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+a*tanh(x)/b/(a+b)/(a+b*tanh(x)^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 481, 537, 223, 212, 385}

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(3/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (a*Tanh[x])/(b*(a +
 b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^4}{(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{a+(-a-b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b(a+b)}\\
&= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}\\
&= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}\\
&= -\frac{\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.97 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.24

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{a\left(-2a-2b+\sqrt{2}(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)+\right.}{\sqrt{2}b(a+b)^2\sqrt{(a-b+(a+}}$$

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((a*(-2*a - 2*b + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sq
rt[2]], 1] + Sqrt[2]*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Elli
pticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sq
rt[2]], 1])*Tanh[x])/(Sqrt[2]*b*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])*
Sech[x]^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.90

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{b^{\frac{3}{2}}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\text{ta}}}$
default	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{b^{\frac{3}{2}}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\text{ta}}}$

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+tanh(x)/b/(a+b*tanh(x)^2)^(1/2)-1/b^(3/2)*
ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1
+tanh(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+ta
nh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x
)))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tan
h(x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+
1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1434 vs. $2(70) = 140$.

Time = 0.54 (sec) , antiderivative size = 6973, normalized size of antiderivative = 83.01

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^4{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral(tanh(x)**4/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh{(x)}^4}{\left(b\tanh{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(70) = 140.

Time = 0.57 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.42

$$\begin{aligned}\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5}-\frac{a^3b^2+a^2b^3}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} \\
&- \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b} \\
&- \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} \\
&- \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} \\
&+ \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) - (a^3*b^2 + a^2
*b^3)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b) - 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(
4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-
b))/(sqrt(-b)*b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))
)/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) + 1
/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\tanh(x)^4}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

```
[In] int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2),x)
[Out] int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2), x)
```

3.240 $\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+a/b/(a+b)/(a+b*tanh(
x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 79, 65, 214}

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^3}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right) \\
&= \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}
\end{aligned}$$

$$= \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(44) = 88.

Time = 0.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.52

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} - \frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+\text{ta}}}$
default	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} - \frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+\text{ta}}}$

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(
1/2)-b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh(x))^2-2*b*(
1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(
1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*
(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*
```

```
ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+
a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. 2(44) = 88.

Time = 0.35 (sec) , antiderivative size = 2525, normalized size of antiderivative = 48.56

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2
)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b -
b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*co
sh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^
2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^
4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*
(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*
a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*c
osh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(
a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*co
sh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 -
```

```
b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a
*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*cosh(x)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)*sinh
(x)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sinh(x)^4 + a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4 + 2*(a^3*b + a^2*b^2 - a*b^3 - b^4)*cosh(x)^2 + 2*(a^3*b + a
^2*b^2 - a*b^3 - b^4 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^2)*sin
h(x)^2 + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^3 + (a^3*b + a^2*b^
2 - a*b^3 - b^4)*cosh(x))*sinh(x)), -1/2*(((a*b + b^2)*cosh(x)^4 + 4*(a*b +
 b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 +
 2*(3*(a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b +
b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*c
osh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a
*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a*b + b^2)*cosh(
x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b
^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b
^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sq
rt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*
sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*cosh(x)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)*sinh(x)^
3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sinh(x)^4 + a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4 + 2*(a^3*b + a^2*b^2 - a*b^3 - b^4)*cosh(x)^2 + 2*(a^3*b + a^2*b
^2 - a*b^3 - b^4 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^3 + (a^3*b + a^2*b^2 -
a*b^3 - b^4)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^3}{(b\tanh(x)^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(44) = 88.

Time = 0.44 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.52

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\frac{(a^3+a^2b)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^3+a^2b}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] ((a^3 + a^2*b)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^3 + a^2*b)/(a^3*b +
 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a +
```

```
b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) - 1/2*log(
abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{\left(b^2+a\,b\right)\,\sqrt{b\tanh\left(x\right)^2+a}}$$

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(3/2),x)
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) + a/((a*b + b^2)
*(a + b*tanh(x)^2)^(1/2))
```

3.241 $\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)-tanh(x)/(a+b
)/(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3751, 482, 385, 212}

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[x
]/((a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^2}{(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}$$

$$= -\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(53) = 106.

Time = 1.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\tanh(x)\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)(b+a\coth^2(x))\sqrt{\frac{(a+b)\tanh^2(x)}{a}}-(a+b)\sqrt{1}\right.}{(a+b)^2\sqrt{a+b\tanh^2(x)}\sqrt{1+\frac{b\tanh^2(x)}{a}}}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (Tanh[x]*(ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(b
 + a*Coth[x]^2)*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*Sqrt[1 + (b*Tanh[x]^2
)/a]))/((a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(45) = 90.

Time = 0.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 5.45

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}+\frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(}}$
default	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}+\frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(}}$

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))
+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh(x))^
2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*
(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(
a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2
*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)
^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(45) = 90$.

Time = 0.35 (sec) , antiderivative size = 2281, normalized size of antiderivative = 43.04

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2
+ b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x
)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(
x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^
3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(
x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*
b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*
cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b
+ 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
 - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*co
sh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
 - 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 - a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
```

```
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b +
b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 +
 a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x
)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\tanh^2{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)^2}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(45) = 90$.

Time = 0.46 (sec) , antiderivative size = 293, normalized size of antiderivative = 5.53

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] -((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a^2*b + a*b^2)/(a^
3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/
(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) + 1/2
*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\tanh(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

```
[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2),x)
[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

3.242 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-1/(a+b)/(a+b*tanh(x)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 53, 65, 214}

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - 1/((a + b)*Sqrt[
a + b*Tanh[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}
\end{aligned}$$

$$= -\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{a+b\tanh^2(x)}{a+b}\right)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*Sqrt[
a + b*Tanh[x]^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(41) = 82.

Time = 0.07 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.57

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} - \frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} + -$
default	$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} - \frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} + -$

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-b/(a+b)*(2*b*(1+tanh
(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b
*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(
x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x
```

```
)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-
1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(41) = 82.

Time = 0.36 (sec) , antiderivative size = 2277, normalized size of antiderivative = 46.47

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3 + a
^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)
^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x
))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b
)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
 (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x
)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a
^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*c
osh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a
^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*
a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^
3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

```
 - 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)
^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 +
 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x
)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]
```

Sympy [A] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{2(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}(a+b)}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } a^{\frac{3}{2}}=0 \\ \frac{\log\left(2a^{\frac{3}{2}}\tanh^2(x)-2a^{\frac{3}{2}}\right)}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(3/2),x)
[Out] -Piecewise((2*(b/(2*(a + b)*sqrt(a + b*tanh(x)**2)) + b*atan(sqrt(a + b*tan
h(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)*(a + b)))/b, Ne(b, 0)), (Piecewise((
zoo*tanh(x)**2, Eq(a**(3/2), 0)), (log(2*a**(3/2)*tanh(x)**2 - 2*a**(3/2))/
(2*a**(3/2)), True)), True))
```

Maxima [F]

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(41) = 82.

Time = 0.45 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.96

$$\begin{aligned}\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = &-\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} \\ &-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} \\ &+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} \\ &-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^2*b + a*b^2)/(a^
3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/
(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) - 1/2
*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\ \sqrt{b\tanh\left(x\right)^2+a}}$$

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) - 1/((a + b)*(a
+ b*tanh(x)^2)^(1/2))
```

3.243 $\int \frac{1}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*tanh(x)/a/
(a+b)/(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3742, 390, 385, 212}

$$\int \frac{1}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[(a + b*Tanh[x]^2)^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Tan
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}\\
&= \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}\\
&= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.98

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx =$$

$$-\frac{\sinh^2(x)\left(\frac{15}{4}a(3a-2b+(3a+2b)\cosh(2x))\text{csch}(x)\text{sech}(x)\left((a-b)\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)+(a+b)\right.\right.}{}$$

[In] Integrate[(a + b*Tanh[x]^2)^(-3/2),x]

[Out] -1/15*(Sinh[x]^2*((15*a*(3*a - 2*b + (3*a + 2*b)*Cosh[2*x])*Csch[x]*Sech[x]
*((a - b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (a + b)*ArcSin[Sqrt[-(((
a + b)*Sinh[x]^2)/a)]]*Cosh[2*x] - 2*a*Sqrt[-(((a + b)*(b + a*Coth[x]^2)*Si
nh[x]^4)/a^2)]))/4 + Sqrt[2]*a^2*(a + b)*Hypergeometric2F1[2, 2, 7/2, -(((a
 + b)*Sinh[x]^2)/a)]*(-(((a + b)*(a - b + (a + b)*Cosh[2*x])*Sinh[x]^2)/a^2
))^(3/2)*Tanh[x]))/(a^4*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2 + (
b*Sinh[x]^2)/a]*Sqrt[a + b*Tanh[x]^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(48) = 96.

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.86

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$
default	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$

[In] int(1/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x
)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b
(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)/(b(1+tanh(x))^2-2*b*(1+tan
h(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x
))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(48) = 96$.

Time = 0.36 (sec) , antiderivative size = 2509, normalized size of antiderivative = 44.80

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b
)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 -
a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh
(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(
a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)
*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2
+ 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 +
6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(
x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(
x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*c
osh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a
^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)
^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2
+ 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sin
h(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3
*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a
^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) +
15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 +
6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*
cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*
(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*c
osh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 +
a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b
^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + 3*a^3*b + 3*a^2*
```

```
b^2 + a*b^3)*cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh
(x)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a
^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sin
h(x)^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b -
a^2*b^2 - a*b^3)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 +
 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 +
a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*c
osh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b
^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a^2 + a*b)*cosh(
x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a
*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a
*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sq
rt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*
sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^
3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 2*(a^4 + a^3*b -
 a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*
b^2 - a*b^3)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\tanh^{2}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^2 + a)^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(48) = 96$.

Time = 0.44 (sec) , antiderivative size = 288, normalized size of antiderivative = 5.14

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{(ab^2+b^3)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{ab^2+b^3}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\,(a+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a+b)^{\frac{3}{2}}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a+b)^{\frac{3}{2}}}$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] ((a*b^2 + b^3)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a*b^2 + b^3)/(a^3*b +
 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a +
b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) + 1/2*log(
abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b*tanh(x)^2)^(3/2),x)
[Out] int(1/(a + b*tanh(x)^2)^(3/2), x)
```

3.244 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(3/2)+b/a/(a+b)/(a+b*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3751, 457, 87, 162, 65, 214}

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(3/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + b/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\,(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a-b+bx}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}\\
&= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}\\
&= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{ab}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}\\
&= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\,\text{Hypergeometric2F1}\left(-\right.}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
[Out] (-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tanh[x]^2)/a])/(a*(a + b)*Sqrt[a + b
*Tanh[x]^2])
```

Maple [F]

$$\int \frac{\coth(x)}{(a+b\tanh(x)^2)^{\frac{3}{2}}}dx$$

[In] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(64) = 128$.

Time = 0.54 (sec) , antiderivative size = 6955, normalized size of antiderivative = 89.17

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\coth(x)}{(a+b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

[In] integrate(coth(x)/(a+b*tanh(x)**2)**(3/2),x)

[Out] Integral(coth(x)/(a + b*tanh(x)**2)**(3/2), x)

Maxima [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\coth(x)}{(b\tanh(x)^2+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(3/2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(64) = 128$.

Time = 0.51 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.77

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3}+\frac{a^3b^2+a^2b^3}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} + \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) + (a^3*b^2 + a^2
*b^3)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b) + 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(
4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-
a))/(sqrt(-a)*a) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))
)/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) - 1
/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth(x)}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a + b*tanh(x)^2)^(3/2), x)
```

3.245 $\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*coth(x)/a/
(a+b)/(a+b*tanh(x)^2)^(1/2)-(a+2*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^2/(a+b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 483, 597, 12, 385, 212}

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = -\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Cot
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) - ((a + 2*b)*Coth[x]*Sqrt[a + b*Tan
h[x]^2])/(a^2*(a + b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a-2b+2bx^2}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a(a+b)}$$

$$= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst}\left(\int \frac{a^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a^2(a+b)}$$

$$= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}$$

$$= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.09

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx =$$

$$-\frac{\cosh^2(x)\coth(x)\left(\frac{8(a+b)\cosh^2(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)(ia\tanh(x)+ib\tanh^3(x))^2}{15a^3}-\frac{8(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},-\frac{(a\cdots}{}\right.}{}\right.}{}$$

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((Cosh[x]^2*Coth[x]*((8*(a + b)*Cosh[x]^2*HypergeometricPFQ[{2, 2, 2}, {1,
 7/2}, -(((a + b)*Sinh[x]^2)/a)]*(I*a*Tanh[x] + I*b*Tanh[x]^3)^2)/(15*a^3)
- (8*(a + b)*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]
^2*(2*a^2 + 5*a*b*Tanh[x]^2 + 3*b^2*Tanh[x]^4))/(15*a^3) - (Coth[x]^2*(3*a^
2 + 12*a*b*Tanh[x]^2 + 8*b^2*Tanh[x]^4)*(ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/
a)]]*(-a - b*Tanh[x]^2) + a*Sech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(
a + b*Tanh[x]^2))/a^2)]))/(a^2*(a + b)*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*
(a + b*Tanh[x]^2))/a^2)])))/(a*Sqrt[a + b*Tanh[x]^2]))
```

Maple [F]

$$\int \frac{\coth(x)^2}{(a+b\tanh(x)^2)^{\frac{3}{2}}}dx$$

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. 2(75) = 150.

Time = 0.49 (sec) , antiderivative size = 3929, normalized size of antiderivative = 46.22

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 +
 a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 +
a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b
)*cosh(x))*sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 +
 a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2
*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2
+ b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh
(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a
*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b +
 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 +
 b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6
*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2
 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*co
sh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh
(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*co
sh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*
b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2
*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*c
osh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*co
sh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*s
inh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + a^2*b)*cosh(x)^6 + 6
*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b
)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5
*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b
 - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b
+ 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2
*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*
log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^
4 + 4*(a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b
+ 4*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3 + 2*(a^3 + a
^2*b - 2*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3 + 3*(a
^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b +
4*a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
```

h(x)*sinh(x) + sinh(x)^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^
6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^5 + (a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^6 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3
+ (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^4 + (a^5 - a^4*b - 5*a^3*b^
2 - 3*a^2*b^3 + 15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b -
5*a^3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^
2*b^3)*cosh(x)^2 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3 - 15*(a^5 + 3*a^4*b
 + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 - 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^
5 + 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^3 - (a^5 - a^4*b - 5*a^
3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a
^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*c
osh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b -
(a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6
*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a
^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh
(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*
(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a^3
 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sin
h(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*
sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cos
h(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3
 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x)
)*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a +
 b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
 b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) + 2*sqrt(2)*((a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^4
+ 4*(a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b +
4*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3 + 2*(a^3 + a^2
*b - 2*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3 + 3*(a^3
 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 4*
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*cosh(x))*sinh(x)
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6
+ 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^5 + (a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^6 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 +

```
(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^4 + (a^5 - a^4*b - 5*a^3*b^2
- 3*a^2*b^3 + 15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b - 5*
a^3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*
b^3)*cosh(x)^2 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3 - 15*(a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 - 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*c
osh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5
+ 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^3 - (a^5 - a^4*b - 5*a^3*
b^2 - 3*a^2*b^3)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\coth^2{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(3/2),x)
[Out] Integral(coth(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\coth\left(x\right)^2}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(75) = 150$.

Time = 0.57 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.40

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{(a^2b^3+ab^4)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3} - \frac{a^2b^3+ab^4}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a}\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\right.\right.}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^3 + a*b^4)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) - (a^2*b^3 + a*b^
4)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a
 - b)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/
2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2) + 4*(sqrt(a + b)
*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
+ sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

```
[In] int(coth(x)^2/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

3.246 $\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+a*(a+2*b)*tanh(x)/b^2/(a+b)^2/(
a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)^3/b/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3751, 481, 592, 537, 223, 212, 385}

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tanh[x]^3)/(3*b*
(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b)*Tanh[x])/(b^2*(a + b)^2*Sqr
t[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
```

```
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^6}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2(3a-3(a+b)x^2)}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3b(a+b)}\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{3a(a+2b)-3(a+b)^2x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3b^2(a+b)^2}\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{(a+b)^2}\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^2}\\
&= -\frac{\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}\\
&\quad + \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.96

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{(a-b+(a+b)\cosh(2x))\mathrm{sech}^2(x)}\left(-\frac{3\sqrt{2}a\coth(x)\left((a^2+3ab+2b^2)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{(a\cdots}}{\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((-3*Sqrt[2]*a*Coth[x]*((a^2 +
 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^
2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*
x])*Csch[x]^2)/b]) + (a*(a + b)*(3*a^2 + 2*a*b - 7*b^2 + (3*a^2 + 10*a*b +
7*b^2)*Cosh[2*x])*Sinh[2*x])/(a - b + (a + b)*Cosh[2*x])^2))/(3*Sqrt[2]*b^2
*(a + b)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(100) = 200.

Time = 0.11 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.65

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)^3}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a\cdots}\right.}{b^{\frac{5}{2}}}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)^3}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a\cdots}\right.}{b^{\frac{5}{2}}}$

```
[In] int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+
1/3*tanh(x)^3/b/(a+b*tanh(x)^2)^(3/2)+1/b^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1
/b^(5/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))+1/3*tanh(x)/b/(a+b*tanh(
x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/6/(a+b)/(b*(tanh(x)-1)^
2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)
+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(
```

```
1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a
+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/
2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-
1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(3/2)+1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/
3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)
^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))
^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+
tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh
(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4472 vs. 2(100) = 200.

Time = 1.23 (sec) , antiderivative size = 19265, normalized size of antiderivative = 163.26

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)**6/(a+b*tanh(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**6/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^6}{(b\tanh(x)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^6/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(100) = 200$.

Time = 0.64 (sec) , antiderivative size = 805, normalized size of antiderivative = 6.82

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{(3a^9b^8+22a^8b^9+65a^7b^{10}+100a^6b^{11}+85a^5b^{12}+38a^4b^{13}+7a^3b^{14})e^{(2x)}}{a^8b^{10}+6a^7b^{11}+15a^6b^{12}+20a^5b^{13}+15a^4b^{14}+6a^3b^{15}+a^2b^{16}} + \frac{3(a^9b^8+2a^8b^9-9a^7b^{10}-36\cdots}{a^8b^{10}+6a^7b^{11}+15a^6b^{12}+\cdots}\right.\right.}{} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}} - \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b^2}$$

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((((3*a^9*b^8 + 22*a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12 +
 38*a^4*b^13 + 7*a^3*b^14)*e^(2*x)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 2
0*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16) + 3*(a^9*b^8 + 2*a^8*b^9
- 9*a^7*b^10 - 36*a^6*b^11 - 49*a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a^8*b
^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a
^2*b^16))*e^(2*x) - 3*(a^9*b^8 + 2*a^8*b^9 - 9*a^7*b^10 - 36*a^6*b^11 - 49*
a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 +
 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16))*e^(2*x) - (3*a^9*b^8 +
 22*a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12 + 38*a^4*b^13 + 7*a^
3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b^14 +
6*a^3*b^15 + a^2*b^16))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/
((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt
(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))
/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))
)/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) -
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a +
b))/sqrt(-b))/(sqrt(-b)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^6}{\left(b\tanh{(x)}^2+a\right)^{5/2}}\,dx$$

```
[In] int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2),x)
[Out] int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2), x)
```

3.247 $\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)+a*(a+2*b)/b^2/(a+b)^
2/(a+b*tanh(x)^2)^(1/2)-1/3*a^2/b^2/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 457, 89, 65, 214}

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}} dx = -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - a^2/(3*b^2*(a +
b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b))/(b^2*(a + b)^2*Sqrt[a + b*Tanh[
x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^5}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{5/2}} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^{3/2}} - \frac{1}{(a+b)^2(-1+x)\sqrt{a+bx}}\right) dx, x, \tanh^2(x)\right)$$

$$= -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)^2}$$

$$= -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{-b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right) + (a+b)\left(2a+b+3b\tanh^2(x\right.}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (-(b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a +
b)*(2*a + b + 3*b*Tanh[x]^2))/(3*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(72) = 144$.

Time = 0.10 (sec) , antiderivative size = 469, normalized size of antiderivative = 5.58

method	result
derivativedivides	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)^2}{b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{2a}{3b^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a-\right.}$
default	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)^2}{b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{2a}{3b^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a-\right.}$

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/(a+b*tanh(x)^2)^(3/2)+tanh(x)^2/b/(a+b*tanh(x)^2)^(3/2)+2/3*a/b^2/(a+
b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/
6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)
/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(t
anh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+
2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/6
/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/6*b/(a+b)/a/(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh
(x))+a+b)^(1/2)-1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b
*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+ta
nh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3234 vs. $2(72) = 144$.

Time = 0.67 (sec) , antiderivative size = 7033, normalized size of antiderivative = 83.73

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^5{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh{(x)}^5}{\left(b\tanh{(x)}^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(72) = 144$.

Time = 0.53 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{\left(a^9+8\,a^8b+25\,a^7b^2+40\,a^6b^3+35\,a^5b^4+16\,a^4b^5+3\,a^3b^6\right)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8} + \frac{3\left(a^9+6\,a^8b+13\,a^7b^2+12\,a^6b^3+3\,a^5 \cdots\right.}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a \cdots}\right.\right.\right.}{3\left(ae^{(4\,x)} \cdots\right.}$$
$$- \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)} - \sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$
$$+ \frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)} + \sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b} + \sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$
$$- \frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)} + \sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b} - \sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
[Out] 2/3*((((a^9 + 8*a^8*b + 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 +
 3*a^3*b^6)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4
*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^9 + 6*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 +
3*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5
```

```
*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^9 + 6*a^8*b + 13*a
^7*b^2 + 12*a^6*b^3 + 3*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a^7*b^3
 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + (
a^9 + 8*a^8*b + 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b
^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7
 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3
/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/((a^2 + 2*a
*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/((a^2 + 2*
a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/((a^2 + 2
*a*b + b^2)*sqrt(a + b))
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}{2\,{\left(a+b\right)}^{5/2}}\right)}{{\left(a+b\right)}^{5/2}}-\frac{\frac{a^2}{3\,\left(a+b\right)}-\frac{\left(a^2+2\,b\,a\right)\,\left(b\tanh(x)^2+a\right)}{{\left(a+b\right)}^2}}{b^2\,{\left(b\,{\tanh\left(x\right)}^2+a\right)}^{3/2}}$$

```
[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/
(a + b)^(5/2) - (a^2/(3*(a + b)) - ((2*a*b + a^2)*(a + b*tanh(x)^2))/(a + b
)^2)/(b^2*(a + b*tanh(x)^2)^(3/2))
```

3.248 $\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(a+4*b)*
tanh(x)/b/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)/b/(a+b)/(a+b*tanh(x)^
2)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 481, 541, 12, 385, 212}

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tan
h[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((a + 4*b)*Tanh[x])/(3*b*(a +
 b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+(-a-3b)x^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3b(a+b)}$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{3ab}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3ab(a+b)^2}$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{(a+b)^2}$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\tanh^3(x)\left(3\text{arctanh}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)(b+a\coth^2(x))^2\sqrt{\frac{(a+b)\tanh^2(x)}{a}}-(a+b)\left(\right.\right.}{3(a+b)^3(a+b\tanh^2(x))^{3/2}\sqrt{1+\frac{b\tanh^2(}{a}}}$$

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Tanh[x]^3*(3*ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]
]*(b + a*Coth[x]^2)^2*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*(a + 4*b + 3*a*
Coth[x]^2)*Sqrt[1 + (b*Tanh[x]^2)/a]))/(3*(a + b)^3*(a + b*Tanh[x]^2)^(3/2)
*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(76) = 152.

Time = 0.10 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.46

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}}+\frac{}{6(a+b)\left(b(1+\tanh\right.}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}}+\frac{}{6(a+b)\left(b(1+\tanh\right.}$

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+
1/3*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)+1
/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1+ta
nh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(x))^
2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)
*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-
1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/
(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(
```

```
5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x
)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2545 vs. $2(76) = 152$.

Time = 0.64 (sec) , antiderivative size = 5719, normalized size of antiderivative = 63.54

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^4{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**4/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh{(x)}^4}{\left(b\tanh{(x)}^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(76) = 152$.

Time = 0.53 (sec) , antiderivative size = 684, normalized size of antiderivative = 7.60

$$\int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{4\left(\left(\left(\frac{(a^7b^2+5a^6b^3+10a^5b^4+10a^4b^5+5a^3b^6+a^2b^7)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8} - \frac{3(a^6b^3+4a^5b^4+6a^4b^5+4a^3b^6+a^2b^7)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)} + \frac{3(\cdots}{a^8b^2+6a^7\cdots}\right.\right.}{3\left(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2\cdots}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a^2 + 2ab + b^2)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)\sqrt{a+b}}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*((((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^
7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*
a^3*b^7 + a^2*b^8) - 3*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6 + a^2*b
^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7
 + a^2*b^8))*e^(2*x) + 3*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6 + a^2
*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b
^7 + a^2*b^8))*e^(2*x) - (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5
*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4
*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(
2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a -
b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a +
 b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^4}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

```
[In] int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2),x)
[Out] int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2), x)
```

3.249 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh(
x)^2)^(1/2)+1/3*a/b/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 79, 53, 65, 214}

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + a/(3*b*(a + b)*(
a + b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^3}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{a}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}\\
&= \frac{a}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)^2}\\
&= \frac{a}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)^2}\\
&= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{a}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$
\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{a(a+b) - 3b\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{a+b\tanh^2(x)}{a+b}\right)\left(a+b\tanh^2(x)\right)}{3b(a+b)^2\,\left(a+b\tanh^2(x)\right)^{3/2}}
$$

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (a*(a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]
*(a + b*Tanh[x]^2))/(3*b*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

method	result
derivativedivides	$\frac{1}{3b\left(a+b\tanh(x)^{2}\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(1+\tanh(x))^{2}-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}-\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^{2}-2b(1+\tanh(x))-\right.}$
default	$\frac{1}{3b\left(a+b\tanh(x)^{2}\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(1+\tanh(x))^{2}-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}-\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^{2}-2b(1+\tanh(x))-\right.}$

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(3/2)-1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)-1/
3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)-1/2/(a+b)
^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^2/a/(b*(1+tanh(x))
^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+
tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh
(x)))-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(
b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh
(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*
b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3028 vs. $2(62) = 124$.

Time = 0.66 (sec) , antiderivative size = 6621, normalized size of antiderivative = 89.47

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^3}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(62) = 124$.

Time = 0.53 (sec) , antiderivative size = 725, normalized size of antiderivative = 9.80

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{\left(a^8b+2\,a^7b^2-5\,a^6b^3-20\,a^5b^4-25\,a^4b^5-14\,a^3b^6-3\,a^2b^7\right)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\left(a^8b+2\,a^7b^2-a^6b^3-4\,a^5b^4-a^4b^5+\cdots\right.}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4\cdots}\cdots}{3\left(ae^{(4\,x)}\cdots\right.}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/3*((((a^8*b + 2*a^7*b^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b^
6 - 3*a^2*b^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*
a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^8*b + 2*a^7*b^2 - a^6*b^3 - 4*a^5*b^4
 - a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^
```

```
5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^8*b + 2*a^7*b^2 -
 a^6*b^3 - 4*a^5*b^4 - a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3
+ 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + (a
^8*b + 2*a^7*b^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b^6 - 3*a^2
*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b
^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^
(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/((a^2 + 2
*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/((a^2 +
2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/((a^2 +
 2*a*b + b^2)*sqrt(a + b))
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}{2\,{(a+b)}^{5/2}}\right)}{{(a+b)}^{5/2}}+\frac{\frac{a}{3\,(a+b)}-\frac{b\,\left(b\tanh(x)^2+a\right)}{{(a+b)}^2}}{b\,{\left(b\tanh\left(x\right)^2+a\right)}^{3/2}}$$

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/
(a + b)^(5/2) + (a/(3*(a + b)) - (b*(a + b*tanh(x)^2))/(a + b)^2)/(b*(a + b
*tanh(x)^2)^(3/2))
```

3.250 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(2*a-b)*
tanh(x)/a/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*tanh(x)/(a+b)/(a+b*tanh(x)^2)^(
3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 482, 541, 12, 385, 212}

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) - Tanh[x
]/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((2*a - b)*Tanh[x])/(3*a*(a + b)^2*
Sqrt[a + b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1+2x^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3(a+b)}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{3a}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3a(a+b)^2}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{(a+b)^2}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.45

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\sinh^2(x)\tanh(x)\left(-\frac{4(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{9}{2},-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^2(x)\left(a+b\tanh^2(x)\right)}{35a^2} -\right.}{}$$

[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]

[Out] (Sinh[x]^2*Tanh[x]*((-4*(a + b)*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*(a + b*Tanh[x]^2))/(35*a^2) - (Coth[x]^4*(-5*a - 2*b*Tanh[x]^2)*(3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + b*Tanh[x]^2)^2 + a*Sech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]*(-4*b*Tanh[x]^2 + a*(-3 - Tanh[x]^2))))/(3*a*(a + b)^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])))/(3*a^2*Sqrt[a + b*Tanh[x]^2]*(1 + (b*Tanh[x]^2)/a))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(74) = 148.

Time = 0.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.16

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{6(a+b)a\left(b(\tan\right.}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{6(a+b)a\left(b(\tan\right.}$

[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2939 vs. $2(74) = 148$.

Time = 0.66 (sec) , antiderivative size = 6507, normalized size of antiderivative = 73.94

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^2}{(b\tanh(x)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(74) = 148$.

Time = 0.52 (sec) , antiderivative size = 728, normalized size of antiderivative = 8.27

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{\left(\left(\frac{(3a^7b^2+14a^6b^3+25a^5b^4+20a^4b^5+5a^3b^6-2a^2b^7-ab^8)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}+\frac{3(a^7b^2+2a^6b^3-a^5b^4-4a^4b^5-a^3b^6+2a^2b^7+ab^8)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)}-\frac{3(a^7b^2\cdots}{a^8b^2+6a\cdots}\right.}{3\left(ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2\cdots}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((((3*a^7*b^2 + 14*a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*a
^2*b^7 - a*b^8)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15
*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^7*b^2 + 2*a^6*b^3 - a^5*b^4 - 4*a^4*
b^5 - a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a
^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^7*b^2 + 2*a^6*b^
3 - a^5*b^4 - 4*a^4*b^5 - a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a^7*b^3
 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - (
3*a^7*b^2 + 14*a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*a^2*b^7 -
a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*
b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/((a^2 +
2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/((a^2 +
 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/((a^2
+ 2*a*b + b^2)*sqrt(a + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

```
[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2),x)
[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

3.251 $\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh(
x)^2)^(1/2)-1/3/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 53, 65, 214}

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - 1/(3*(a + b)*(a
+ b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}$$

$$= -\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)^2}$$

$$= -\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \frac{a+b\tanh^2(x)}{a+b}\right)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*(
a + b*Tanh[x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(58) = 116.

Time = 0.05 (sec) , antiderivative size = 420, normalized size of antiderivative = 6.00

method	result
derivativedivides	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)}$
default	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)}$

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(ta
nh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/
2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(
tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/6/(a+b)/(b*(1+tanh(
x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*(1+tanh
(x))+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+
b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/
2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)
^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. $2(58) = 116$.

Time = 0.64 (sec) , antiderivative size = 5779, normalized size of antiderivative = 82.56

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{6(a+b)\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}+\frac{b}{2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}(a+b)^2}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } a^{\frac{5}{2}}=0 \\ \frac{\log\left(2a^{\frac{5}{2}}\tanh^2(x)-2a^{\frac{5}{2}}\right)}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] -Piecewise((2*(b/(6*(a + b)*(a + b*tanh(x)**2)**(3/2)) + b/(2*(a + b)**2*sq
rt(a + b*tanh(x)**2)) + b*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt
(-a - b)*(a + b)**2))/b, Ne(b, 0)), (Piecewise((zoo*tanh(x)**2, Eq(a**(5/2)
, 0)), (log(2*a**(5/2)*tanh(x)**2 - 2*a**(5/2))/(2*a**(5/2)), True)), True)
)
```

Maxima [F]

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)}{(b\tanh(x)^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(58) = 116.

Time = 0.51 (sec) , antiderivative size = 683, normalized size of antiderivative = 9.76

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{4\left(\left(\left(\frac{\left(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7\right)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\left(a^7b^2+4\,a^6b^3+6\,a^5b^4+4\,a^4b^5+a^3b^6\right)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2\,x)}+\frac{3\,(}{a^8b^2+6\,a^7}\right.\right.}{3\left(ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*((((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^
7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*
a^3*b^7 + a^2*b^8) + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b
^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7
 + a^2*b^8))*e^(2*x) + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3
*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b
^7 + a^2*b^8))*e^(2*x) + (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5
*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4
*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(
2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a -
b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a +
 b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}{2\,(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{1}{3\,(a+b)}+\frac{b\tanh(x)^2+a}{(a+b)^2}}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}$$

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/
(a + b)^(5/2) - (1/(3*(a + b)) + (a + b*tanh(x)^2)/(a + b)^2)/(a + b*tanh(x
)^2)^(3/2)
```

3.252 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(5*a+2
*b)*tanh(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*b*tanh(x)/a/(a+b)/(a+b*ta
nh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3742, 425, 541, 12, 385, 212}

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[(a + b*Tanh[x]^2)^(-5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Tan
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(5*a + 2*b)*Tanh[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-3(a+b)+2bx^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{3a^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3a^2(a+b)^2}$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{(a+b)^2}$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.82 (sec) , antiderivative size = 976, normalized size of antiderivative = 10.49

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\cosh(x)\sinh(x)\left(1575\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right) + \frac{3150(a+b)\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right.}{a}\right.}{}$$

```
[In] Integrate[(a + b*Tanh[x]^2)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sinh[x]*(1575*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (3150*(a +
b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2)/a + (1575*(a + b)^2*Ar
cSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4)/a^2 + (2100*b*ArcSin[Sqrt[-
(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^2)/a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a
+ b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^2)/a^2 + (2100*b*(a + b)^2*ArcSin[Sq
rt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^2)/a^3 + (840*b^2*ArcSin[Sq
rt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^4)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqr
t[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^4)/a^3 + (840*b^2*(a + b)^2*
ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^4)/a^4 + 2100*(-((
(a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 96*Hyp
ergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/
a))^(7/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 24*HypergeometricPFQ[{2,
2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)
*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + (2800*b*(-(((a + b)*Sinh[x]^2)/a))
^(3/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (168*b*Hypergeo
metric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(
7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (48*b*Hypergeomet
ricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2
)/a))^(7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (1120*b^2*
(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2
))/a])/a^2 + (72*b^2*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]
*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^
2))/a])/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sin
h[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a +
 b*Tanh[x]^2))/a])/a^2 - 1575*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Ta
nh[x]^2))/a^2)] - (2100*b*Tanh[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a
+ b*Tanh[x]^2))/a^2)])/a - (840*b^2*Tanh[x]^4*Sqrt[-(((a + b)*Cosh[x]^2*Sin
h[x]^2*(a + b*Tanh[x]^2))/a^2)])/a^2))/(315*a^2*(-(((a + b)*Sinh[x]^2)/a))^
(5/2)*Sqrt[a + b*Tanh[x]^2]*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a]*(1 + (b*T
anh[x]^2)/a))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(79) = 158$.

Time = 0.10 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.52

method	result
derivativedivides	$\frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)a^2\sqrt{}}$
default	$\frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)a^2\sqrt{}}$

```
[In] int(1/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(x))
^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+t
anh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2
)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1
+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/(b*(tanh(x)-
1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)
-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2
/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^
(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(
x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3152 vs. $2(79) = 158$.

Time = 0.68 (sec) , antiderivative size = 6933, normalized size of antiderivative = 74.55

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\tanh^2\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^2 + a)^(-5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(79) = 158$.

Time = 0.52 (sec) , antiderivative size = 714, normalized size of antiderivative = 7.68

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{\left(3\,a^6b^3+16\,a^5b^4+35\,a^4b^5+40\,a^3b^6+25\,a^2b^7+8\,ab^8+b^9\right)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\left(a^6b^3+2\,a^5b^4-3\,a^4b^5-12\,a^3b^6-13\cdots\right.}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a\cdots}\right.\right.\right.}{3\left(ae^{(4\,x)}\cdots\right.}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
[Out] 2/3*((((3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a
*b^8 + b^9)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4
*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^3*b
^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5
*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^6*b^3 + 2*a^5*b^4
- 3*a^4*b^5 - 12*a^3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a^7*b^3
 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - (
3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a*b^8 + b
^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7
 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3
/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/((a^2 + 2*a
*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/((a^2 + 2*
a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/((a^2 + 2
*a*b + b^2)*sqrt(a + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b*tanh(x)^2)^(5/2),x)
[Out] int(1/(a + b*tanh(x)^2)^(5/2), x)
```

3.253 $\int \frac{\coth(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(5/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(5/2)+b*(2*a+b)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3
*b/a/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3751, 457, 87, 157, 162, 65, 214}

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(5/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + b/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2))
 + (b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{-a-b+bx}{(1-x)x(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}\\
&= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2+\frac{1}{2}b(2a+b)x}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{a^2(a+b)^2}\\
&= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a^2}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2(a+b)^2}\\
&= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{a^2b}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b(a+b)^2}
\end{aligned}
$$

$$= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right) + (a+b)\,\text{Hypergeometric2F1}}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
[Out] (-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)
*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tanh[x]^2)/a])/(3*a*(a + b)*(a + b
*Tanh[x]^2)^(3/2))
```

Maple [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}dx$$

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
[Out] int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4510 vs. 2(90) = 180.

Time = 1.28 (sec) , antiderivative size = 19305, normalized size of antiderivative = 178.75

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth{\left(x \right)}}{\left(a+b\tanh^{2}{\left(x \right)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(5/2),x)
[Out] Integral(coth(x)/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth\left(x\right)}{\left(b\tanh\left(x\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(90) = 180$.

Time = 0.65 (sec) , antiderivative size = 808, normalized size of antiderivative = 7.48

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\left(\left(\frac{(7\,a^{14}b^{3}+38\,a^{13}b^{4}+85\,a^{12}b^{5}+100\,a^{11}b^{6}+65\,a^{10}b^{7}+22\,a^{9}b^{8}+3\,a^{8}b^{9})e^{(2\,x)}}{a^{16}b^{2}+6\,a^{15}b^{3}+15\,a^{14}b^{4}+20\,a^{13}b^{5}+15\,a^{12}b^{6}+6\,a^{11}b^{7}+a^{10}b^{8}}+\frac{3\,(7\,a^{14}b^{3}+30\,a^{13}b^{4}+49\,a^{12}b\cdots}{a^{16}b^{2}+6\,a^{15}b^{3}+15\,a^{14}b^{4}+\cdots}\cdots\right.\right.}{\cdots}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\,(a^{2}+2\,ab+b^{2})\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^{2}+2\,ab+b^{2})\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^{2}+2\,ab+b^{2})\sqrt{a+b}}$$
$$+\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/3*((((7*a^14*b^3 + 38*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^7
 + 22*a^9*b^8 + 3*a^8*b^9)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 2
```

```
0*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8) + 3*(7*a^14*b^3 + 30*a^13
*b^4 + 49*a^12*b^5 + 36*a^11*b^6 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a^16*
b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a
^10*b^8))*e^(2*x) + 3*(7*a^14*b^3 + 30*a^13*b^4 + 49*a^12*b^5 + 36*a^11*b^6
 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 +
 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) + (7*a^14*b^3
+ 38*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^7 + 22*a^9*b^8 + 3*a
^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 +
6*a^11*b^7 + a^10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/
((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt
(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))
/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))
)/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) -
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a +
b))/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth(x)}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a + b*tanh(x)^2)^(5/2), x)
```

3.254 $\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 131

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(7*a+4
*b)*coth(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*(3*a+2*b)*(a+4*b)*coth(x)
*(a+b*tanh(x)^2)^(1/2)/a^3/(a+b)^2+1/3*b*coth(x)/a/(a+b)/(a+b*tanh(x)^2)^(3
/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3751, 483, 593, 597, 12, 385, 212}

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Cot
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(7*a + 4*b)*Coth[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2]) - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sqrt[a
+ b*Tanh[x]^2])/(3*a^3*(a + b)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-4b+4bx^2}{x^2(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{(3a+2b)(a+4b)-2b(7a+4b)x^2}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3a^2(a+b)^2}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2} - \frac{\text{Subst}\left(\int -\frac{3a^3}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{3a^3(a+b)^2}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{(a+b)^2}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^2}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.66 (sec) , antiderivative size = 1375, normalized size of antiderivative = 10.50

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = -\frac{\cosh^2(x)\coth(x)\left(-\frac{20a\operatorname{csch}^2(x)}{3(a+b)} - \frac{5a^2\operatorname{csch}^4(x)}{(a+b)^2} - \frac{40b\operatorname{sech}^2(x)}{a+b} - \frac{30ab\operatorname{csch}^2(x)\operatorname{sech}^2(x)}{(a+b)^2} - \frac{40b^2\operatorname{sech}^4(x)}{(a+b)^2} - \frac{92(a+b)\text{Hy}}{}\right.}{}$$

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -((Cosh[x]^2*Coth[x]*((-20*a*Csch[x]^2)/(3*(a + b)) - (5*a^2*Csch[x]^4)/(a
+ b)^2 - (40*b*Sech[x]^2)/(a + b) - (30*a*b*Csch[x]^2*Sech[x]^2)/(a + b)^2
- (40*b^2*Sech[x]^4)/(a + b)^2 - (92*(a + b)*Hypergeometric2F1[2, 2, 9/2, -
```

(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2)/(105*a) - (24*(a + b)*HypergeometricPFQ
[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2)/(35*a) - (16*(a
+ b)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]
*Sinh[x]^2)/(105*a) - (160*b^2*Sech[x]^2*Tanh[x]^2)/(3*a*(a + b)) - (124*b*
(a + b)*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Ta
nh[x]^2)/(35*a^2) - (16*b*(a + b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(
((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^2)/(7*a^2) - (16*b*(a + b)*Hyperg
eometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*
Tanh[x]^2)/(35*a^2) - (64*b^3*Sech[x]^2*Tanh[x]^4)/(3*a^2*(a + b)) - (152*b
^2*(a + b)*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2
*Tanh[x]^4)/(35*a^3) - (88*b^2*(a + b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2
}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^4)/(35*a^3) - (16*b^2*(a + b
)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Si
nh[x]^2*Tanh[x]^4)/(35*a^3) - (176*b^3*(a + b)*Hypergeometric2F1[2, 2, 9/2,
 -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^6)/(105*a^4) - (32*b^3*(a + b)
*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2
*Tanh[x]^6)/(35*a^4) - (16*b^3*(a + b)*HypergeometricPFQ[{2, 2, 2, 2}, {1,
1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^6)/(105*a^4) + (5*ArcS
in[Sqrt[-(((a + b)*Sinh[x]^2)/a)]])/((-(((a + b)*Sinh[x]^2)/a))^(5/2)*Sqrt[
(Cosh[x]^2*(a + b*Tanh[x]^2))/a]) + (30*b*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)
/a)]]*Tanh[x]^2)/(a*(-(((a + b)*Sinh[x]^2)/a))^(5/2)*Sqrt[(Cosh[x]^2*(a + b
*Tanh[x]^2))/a]) + (40*b^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^4
)/(a^2*(-(((a + b)*Sinh[x]^2)/a))^(5/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/
a]) + (16*b^3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^6)/(a^3*(-(((a
 + b)*Sinh[x]^2)/a))^(5/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a]) + (5*ArcS
in[Sqrt[-(((a + b)*Sinh[x]^2)/a)]])/Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a
+ b*Tanh[x]^2))/a^2)] + (10*a*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Csch[x
]^2)/((a + b)*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])
 + (60*b*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sech[x]^2)/((a + b)*Sqrt[-(
((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]) + (30*b*ArcSin[Sqrt[
-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^2)/(a*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^
2*(a + b*Tanh[x]^2))/a^2)]) + (80*b^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]
]*Sech[x]^2*Tanh[x]^2)/(a*(a + b)*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a +
b*Tanh[x]^2))/a^2)]) + (40*b^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[
x]^4)/(a^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]) +
(32*b^3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sech[x]^2*Tanh[x]^4)/(a^2*(a
 + b)*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]) + (16*b
^3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^6)/(a^3*Sqrt[-(((a + b)*C
osh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]) + (16*b^3*(I*Tanh[x] - I*Tanh[
x]^3)^2)/(a*(a + b)^2)))/(a^2*Sqrt[a + b*Tanh[x]^2]*(1 + (b*Tanh[x]^2)/a))

Maple [F]

$$\int \frac{\coth\left(x\right)^{2}}{\left(a+b\tanh\left(x\right)^{2}\right)^{\frac{5}{2}}}dx$$

[In] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)

[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5021 vs. $2(113) = 226$.

Time = 1.34 (sec) , antiderivative size = 10671, normalized size of antiderivative = 81.46

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\text{Too large to display}$$

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth^{2}\left(x\right)}{\left(a+b\tanh^{2}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(5/2),x)

[Out] Integral(coth(x)**2/(a + b*tanh(x)**2)**(5/2), x)

Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth\left(x\right)^{2}}{\left(b\tanh\left(x\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(5/2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(113) = 226$.

Time = 0.74 (sec) , antiderivative size = 898, normalized size of antiderivative = 6.85

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((((9*a^13*b^4 + 50*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b^
8 + 34*a^8*b^9 + 5*a^7*b^10)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 +
 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8) + 3*(3*a^13*b^4 + 6*a^1
2*b^5 - 11*a^11*b^6 - 44*a^10*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^10)/(
a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^
7 + a^10*b^8))*e^(2*x) - 3*(3*a^13*b^4 + 6*a^12*b^5 - 11*a^11*b^6 - 44*a^10
*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3 + 15*a^
14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) - (9*a
^13*b^4 + 50*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b^8 + 34*a^8*b
^9 + 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^
12*b^6 + 6*a^11*b^7 + a^10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(
a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*
x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt
(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2
*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqr
t(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 4*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))
/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

```
[In] int(coth(x)^2/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

3.255 $\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3742, 385, 212}

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

```
[In] Int[1/Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{1+\tanh^2(x)}}$$

```
[In] Integrate[1/Sqrt[1 + Tanh[x]^2],x]
[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[1 + Tanh[x
]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}$	62
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}$	62

```
[In] int(1/(1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*arctanh(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/
2))-1/4*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))
^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 21.72

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(
x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x
)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 +
30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 +
10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*
sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x
)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 +
 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqr
t(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cos
h(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x)
```

```
+ sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh^2(x)+1}}\,dx$$

```
[In] integrate(1/(1+tanh(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(tanh(x)**2 + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh(x)^2+1}}\,dx$$

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(tanh(x)^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = -\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
- e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}-\tanh(x)+1\right)\right)}{4} + \frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)}{4}$$

```
[In] int(1/(tanh(x)^2 + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
1)))/4 + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(t
anh(x) - 1)))/4
```

3.256 $\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3742, 385, 209}

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

```
[In] Int[1/Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\\ &= \frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{-1-\tanh^2(x)}}$$

```
[In] Integrate[1/Sqrt[-1 - Tanh[x]^2],x]
[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[-1 - Tanh[
x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\,\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{4}$	66
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\,\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{4}$	66

```
[In] int(1/(-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(
1/2))+1/4*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2+2*tanh(
x))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 6.48

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx &= \frac{1}{8}i\sqrt{2}\log\left(\frac{1}{2}\left(i\sqrt{2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right)\\ &\quad-\frac{1}{8}i\sqrt{2}\log\left(\frac{1}{2}\left(-i\sqrt{2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right)\\ &\quad-\frac{1}{8}i\sqrt{2}\log\left(\left(\sqrt{-2\,e^{(4\,x)}-2}\left(e^{(2\,x)}-2\right)+i\sqrt{2}e^{(4\,x)}-i\sqrt{2}e^{(2\,x)}+2i\sqrt{2}\right)e^{(-4\,x)}\right)\\ &\quad+\frac{1}{8}i\sqrt{2}\log\left(\left(\sqrt{-2\,e^{(4\,x)}-2}\left(e^{(2\,x)}-2\right)-i\sqrt{2}e^{(4\,x)}+i\sqrt{2}e^{(2\,x)}-2i\sqrt{2}\right)e^{(-4\,x)}\right)\end{aligned}$$

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*I*sqrt(2)*log(1/2*(I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-
2*x)) - 1/8*I*sqrt(2)*log(1/2*(-I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x)
+ 2)*e^(-2*x)) - 1/8*I*sqrt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + I*
sqrt(2)*e^(4*x) - I*sqrt(2)*e^(2*x) + 2*I*sqrt(2))*e^(-4*x)) + 1/8*I*sqrt(2
)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - I*sqrt(2)*e^(4*x) + I*sqrt(2)*e
^(2*x) - 2*I*sqrt(2))*e^(-4*x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh^2{(x)}-1}}\,dx$$

```
[In] integrate(1/(-1-tanh(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(-tanh(x)**2 - 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh{(x)}^2-1}}\,dx$$

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-tanh(x)^2 - 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$$
$$= \frac{1}{4}i\sqrt{2}\Big(\log\left(\sqrt{e^{(4\,x)}+1}-e^{(2\,x)}+1\right)+\log\left(\sqrt{e^{(4\,x)}+1}-e^{(2\,x)}\right)-\log\left(-\sqrt{e^{(4\,x)}+1}+e^{(2\,x)}+1\right)\Big)$$

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*I*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
 - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right)}{2}$$

```
[In] int(1/(- tanh(x)^2 - 1)^(1/2),x)
[Out] (2^(1/2)*atan((2^(1/2)*tanh(x))/(- tanh(x)^2 - 1)^(1/2)))/2
```

3.257 $\int \left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx=\left(a^2+b^2\right)x+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a^2+b^2)*x+2*a*b*ln(cosh(d*x+c))/d-b^2*tanh(d*x+c)/d-a*b*tanh(d*x+c)^2/d-1
/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3742, 1824, 647, 31}

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{ab\tanh^2(c+dx)}{d}-\frac{(a+b)^2\log(1-\tanh(c+dx))}{2d}+\frac{(a-b)^2\log(\tanh(c+dx)+1)}{2d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[In] Int[(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -1/2*((a + b)^2*Log[1 - Tanh[c + d*x]])/d + ((a - b)^2*Log[1 + Tanh[c + d*x
]])/(2*d) - (b^2*Tanh[c + d*x])/d - (a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c +
 d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^3)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b^2-2abx-b^2x^2-b^2x^4+\frac{a^2+b^2+2abx}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}\\
&\quad-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{\text{Subst}\left(\int \frac{a^2+b^2+2abx}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}\\
&\quad-\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tanh(c+dx)\right)}{2d}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh(c+dx)\right)}{2d}\\
&= -\frac{(a+b)^2\log(1-\tanh(c+dx))}{2d}+\frac{(a-b)^2\log(1+\tanh(c+dx))}{2d}\\
&\quad-\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{15((a+b)^2\log(1-\tanh(c+dx))-(a-b)^2\log(1+\tanh(c+dx)))+30b^2\tanh(c+dx)+30ab\tanh^2(\ldots}{30d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -1/30*(15*((a + b)^2*Log[1 - Tanh[c + d*x]] - (a - b)^2*Log[1 + Tanh[c + d*
x]]) + 30*b^2*Tanh[c + d*x] + 30*a*b*Tanh[c + d*x]^2 + 10*b^2*Tanh[c + d*x]
^3 + 6*b^2*Tanh[c + d*x]^5)/d
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{-3\tanh(dx+c)^5b^2-5b^2\tanh(dx+c)^3+15a^2dx-30abdx+15b^2dx-15\tanh(dx+c)^2ab-30\ln(1-\tanh(dx+c))ab-15b^2\tan\ldots}{15d}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^5b^2}{5}-\frac{b^2\tanh(dx+c)^3}{3}-\tanh(dx+c)^2ab-b^2\tanh(dx+c)-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}+\frac{(a^2-2ab+b^2)\ln(\tan\ldots}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^5b^2}{5}-\frac{b^2\tanh(dx+c)^3}{3}-\tanh(dx+c)^2ab-b^2\tanh(dx+c)-\frac{(a^2+2ab+b^2)\ln(\tanh(dx+c)-1)}{2}+\frac{(a^2-2ab+b^2)\ln(\tan\ldots}{2}}{d}$
parts	$a^2x+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2ab\left(-\frac{\tanh(dx+c)^2}{2}\ldots\right.}{}$
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}+\frac{2b(30a\,e^{8dx+8c}+45b\,e^{8dx+8c}+90a\,e^{6dx+6c}+90b\,e^{6dx+6c}+90a\,e^{4dx+4c}+140b\,e^{4dx+\ldots}}{15d\left(e^{2dx+2c}+1\right)^5}$

```
[In] int((a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-3*tanh(d*x+c)^5*b^2-5*b^2*tanh(d*x+c)^3+15*a^2*d*x-30*a*b*d*x+15*b^2
*d*x-15*tanh(d*x+c)^2*a*b-30*ln(1-tanh(d*x+c))*a*b-15*b^2*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 2074, normalized size of antiderivative = 23.30

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^10 + 150*(a^2 - 2*a*b + b^2)
*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 15*(a^2 - 2*a*b + b^2)*d*x*sinh(d*x +
c)^10 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^8 + 15
*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 5*(a^2 - 2*a*b + b^2)*d*x +
4*a*b + 6*b^2)*sinh(d*x + c)^8 + 120*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x +
 c)^3 + (5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^7 + 30*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^6 +
30*(105*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 5*(a^2 - 2*a*b + b^2)*d*x
 + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^2 + 6*a*b +
 6*b^2)*sinh(d*x + c)^6 + 60*(63*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^5 +
14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^3 + 3*(5*(a^2
- 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(15
*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^4 + 10*(315*(a^2
- 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 105*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b
 + 6*b^2)*cosh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 45*(5*(a^2 - 2*a*b
 + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^2 + 18*a*b + 28*b^2)*sinh(d*x +
c)^4 + 40*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 21*(5*(a^2 - 2*a*b
+ b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^5 + 15*(5*(a^2 - 2*a*b + b^2)*d*x
 + 6*a*b + 6*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b +
28*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 15*(a^2 - 2*a*b + b^2)*d*x + 5*(15
*(a^2 - 2*a*b + b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c)^2 + 5*(135*(a^2 -
 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 84*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b +
 6*b^2)*cosh(d*x + c)^6 + 90*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*co
sh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 12*(15*(a^2 - 2*a*b + b^2)*d*x
 + 18*a*b + 28*b^2)*cosh(d*x + c)^2 + 12*a*b + 28*b^2)*sinh(d*x + c)^2 + 46
*b^2 + 30*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*
b*sinh(d*x + c)^10 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*
b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh
(d*x + c)^2 + a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cos
h(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x
+ c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x
+ c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))
*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a
```

```
*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b +
10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4
*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c
)/(cosh(d*x + c) - sinh(d*x + c))) + 10*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*
x + c)^9 + 12*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^7 +
 18*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^5 + 4*(15*(a^
2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b
+ b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c
)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d
*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d
*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh
(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 +
10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d
*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cos
h(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x +
 c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx = \begin{cases} a^2x+2abx-\frac{2ab\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{ab\tanh^2\left(c+dx\right)}{d}+b^2x-\frac{b^2\tanh^5\left(c+dx\right)}{5d}-\frac{b^2\tanh^3\left(c+dx\right)}{3d}-\frac{b^2\tanh\left(c+dx\right)}{d} & \text{for} \\ x\left(a+b\tanh^3\left(c\right)\right)^2 & \text{oth} \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c +
 d*x)**2/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**2*tanh(c + d*x)**3/(
3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**3)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.18

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \, dx$$
$$= \frac{1}{15}\,b^2\left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right)$$
$$+ 2\,ab\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + a^2x$$

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x -
 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \, dx$$
$$= \frac{30\,ab\log\left(e^{(2\,dx+2\,c)} + 1\right) + 15\,(a^2 - 2\,ab + b^2)(dx + c) + \frac{2\,(23\,b^2+15\,(2\,ab+3\,b^2)e^{(8\,dx+8\,c)}+90\,(ab+b^2)e^{(6\,dx+6\,c)}+10\,(9\,ab+}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(30*a*b*log(e^(2*d*x + 2*c) + 1) + 15*(a^2 - 2*a*b + b^2)*(d*x + c) +
2*(23*b^2 + 15*(2*a*b + 3*b^2)*e^(8*d*x + 8*c) + 90*(a*b + b^2)*e^(6*d*x +
6*c) + 10*(9*a*b + 14*b^2)*e^(4*d*x + 4*c) + 10*(3*a*b + 7*b^2)*e^(2*d*x +
2*c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \, dx = x\left(a^2 + 2\,a\,b + b^2\right) - \frac{b^2\tanh(c + d\,x)}{d} - \frac{b^2\tanh(c + d\,x)^3}{3\,d} - \frac{b^2\tanh(c + d\,x)^5}{5\,d} - \frac{2\,a\,b\,\ln\left(\tanh(c + d\,x) + 1\right)}{d} - \frac{a\,b\tanh(c + d\,x)^2}{d}$$

```
[In] int((a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x))/d - (b^2*tanh(c + d*x)^3)/(3*d)
 - (b^2*tanh(c + d*x)^5)/(5*d) - (2*a*b*log(tanh(c + d*x) + 1))/d - (a*b*ta
nh(c + d*x)^2)/d
```

3.258 $\int \frac{1}{1+\tanh^3(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{x}{2} - \frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6(1+\tanh(x))}$$

```
[Out] 1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3742, 2099, 213, 632, 210}

$$\int \frac{1}{1+\tanh^3(x)}\,dx = -\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{x}{2} - \frac{1}{6(\tanh(x)+1)}$$

```
[In] Int[(1 + Tanh[x]^3)^(-1),x]
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)(1+x^3)}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)^2} - \frac{1}{2(-1+x^2)} + \frac{1}{3(1-x+x^2)}\right)dx, x, \tanh(x)\right)\\
&= -\frac{1}{6(1+\tanh(x))} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{x}{2} - \frac{1}{6(1+\tanh(x))} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2} - \frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{1+\tanh^3(x)}\,dx = -\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\operatorname{arctanh}(\tanh(x)) - \frac{1}{6(1+\tanh(x))}$$

```
[In] Integrate[(1 + Tanh[x]^3)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/(
6*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9} - \frac{\ln(\tanh(x)-1)}{4} - \frac{1}{6(1+\tanh(x))} + \frac{\ln(1+\tanh(x))}{4}$	41
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9} - \frac{\ln(\tanh(x)-1)}{4} - \frac{1}{6(1+\tanh(x))} + \frac{\ln(1+\tanh(x))}{4}$	41
risch	$\frac{x}{2} - \frac{e^{-2x}}{12} + \frac{i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\right)}{9} - \frac{i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\right)}{9}$	47

```
[In] int(1/(1+tanh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*3^(1/2)*arctan(1/3*(2*tanh(x)-1)*3^(1/2))-1/4*ln(tanh(x)-1)-1/6/(1+tanh
(x))+1/4*ln(1+tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{18\,x\cosh(x)^2 + 36\,x\cosh(x)\sinh(x) + 18\,x\sinh(x)^2 - 8\left(\sqrt{3}\cosh(x)^2 + 2\sqrt{3}\cosh(x)\sinh(x) + \sqrt{3}\sinh\right.}{36\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.68

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{9x\tanh(x)}{18\tanh(x)+18} + \frac{9x}{18\tanh(x)+18} + \frac{4\sqrt{3}\tanh(x)\operatorname{atan}\left(\frac{2\sqrt{3}\tanh(x)}{3}-\frac{\sqrt{3}}{3}\right)}{18\tanh(x)+18} + \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tanh(x)}{3}-\frac{\sqrt{3}}{3}\right)}{18\tanh(x)+18} - \frac{3}{18\tanh(x)+18}$$

```
[In] integrate(1/(1+tanh(x)**3),x)
```

```
[Out] 9*x*tanh(x)/(18*tanh(x) + 18) + 9*x/(18*tanh(x) + 18) + 4*sqrt(3)*tanh(x)*a
tan(2*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) + 4*sqrt(3)*atan(2*s
qrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) - 3/(18*tanh(x) + 18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{2}x - \frac{1}{12}e^{(-2x)}$$

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/2*x - 1/12*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}e^{(2\,x)}\right)+\frac{1}{2}\,x-\frac{1}{12}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 1/2*x - 1/12*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{\frac{x}{2}+\frac{\tanh(x)}{6}+\frac{x\tanh(x)}{2}}{\tanh\left(x\right)+1}+\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\tanh(x)-1\right)}{3}\right)}{9}$$

```
[In] int(1/(tanh(x)^3 + 1),x)
[Out] (x/2 + tanh(x)/6 + (x*tanh(x))/2)/(tanh(x) + 1) + (2*3^(1/2)*atan((3^(1/2)*
(2*tanh(x) - 1))/3))/9
```

3.259 $\int \tanh(x) \left(a+b\tanh^4(x)\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \tanh(x) \left(a+b\tanh^4(x)\right)^{3/2} dx = -\frac{1}{4}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}$$

```
[Out] 1/2*(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-
1/4*(3*a+2*b)*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))*b^(1/2)-1/4*
(a+b*tanh(x)^4)^(1/2)*(2*a+2*b+b*tanh(x)^2)-1/6*(a+b*tanh(x)^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3751, 1262, 749, 829, 858, 223, 212, 739}

$$\int \tanh(x) \left(a+b\tanh^4(x)\right)^{3/2} dx = \frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{4}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}$$

[In] Int[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]

[Out] -1/4*(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + ((a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/2 - ((2*(a + b) + b*Tanh[x]^2)*Sqrt[a + b*Tanh[x]^4])/4 - (a + b*Tanh[x]^4)^(3/2)/6

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x(a+bx^4)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \frac{(-a-bx)\sqrt{a+bx^2}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-ab(2a+b)-b^2(3a+2b)x}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{4b}\\
&= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}\\
&\quad + \frac{1}{2}(a+b)^2\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)\\
&\quad - \frac{1}{4}(b(3a+2b))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)
\end{aligned}
$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}-\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{1}{2}(a+b)^2\text{Subst}\left(\int\frac{1}{a+b-x^2}\,dx,x,\frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{4}(b(3a+2b))\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)$$

$$= -\frac{1}{4}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}-\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}$$

Mathematica [A] (verified)

Time = 4.55 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.34

$$\int\tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \frac{1}{12}\left(-6\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + 6(a+b)^{3/2}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b\tanh^4(x)}\left(8a+6b+3b\tanh^2(x)+2b\tanh^4(x)\right) - \frac{3\sqrt{a}}{}\right.$$

```
[In] Integrate[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]] + 6*
(a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])
] - Sqrt[a + b*Tanh[x]^4]*(8*a + 6*b + 3*b*Tanh[x]^2 + 2*b*Tanh[x]^4) - (3*
Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a]]*Sqrt[a + b*Tanh[x]^4])
/Sqrt[1 + (b*Tanh[x]^4)/a])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.00

method	result
derivativedivides	$-\frac{b\tanh(x)^4\sqrt{a+b\tanh(x)^4}}{6}-\frac{b\tanh(x)^2\sqrt{a+b\tanh(x)^4}}{4}-\frac{2\sqrt{a+b\tanh(x)^4}\,a}{3}-\frac{b\sqrt{a+b\tanh(x)^4}}{2}-\frac{\left(\frac{5}{3}ab+b^2\right.}{}$
default	$-\frac{b\tanh(x)^4\sqrt{a+b\tanh(x)^4}}{6}-\frac{b\tanh(x)^2\sqrt{a+b\tanh(x)^4}}{4}-\frac{2\sqrt{a+b\tanh(x)^4}\,a}{3}-\frac{b\sqrt{a+b\tanh(x)^4}}{2}-\frac{\left(\frac{5}{3}ab+b^2\right.}{}$

```
[In] int(tanh(x)*(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*tanh(x)^4*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/2
)-2/3*(a+b*tanh(x)^4)^(1/2)*a-1/2*b*(a+b*tanh(x)^4)^(1/2)-1/2*(5/3*a*b+b^2)
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/a^(1
/2)*b^(1/2))^(1/2),I)-3/4*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b
^(1/2)*a-1/2*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b^(3/2)-1/2*I*
(-7/5*a*b-b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(
x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)/b^(
1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(tanh(x)*(I/a
^(1/2)*b^(1/2))^(1/2),I))+1/2*a^2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*
a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+a*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh
(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/2*b^2/(a+b)^(1/2)*arctanh(1
/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/2*(-5/3*a*b-b^2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/a^(
1/2)*b^(1/2))^(1/2),I)-1/2*I*(7/5*a*b+b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/
2)/(a+b*tanh(x)^4)^(1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/
2),I)-EllipticE(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2646 vs. 2(101) = 202.

Time = 0.55 (sec) , antiderivative size = 11528, normalized size of antiderivative = 92.97

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

[Out] Too large to include

Sympy [F]

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx=\int \left(a+b\tanh^4{(x)}\right)^{\frac{3}{2}}\tanh{(x)}\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(3/2),x)
[Out] Integral((a + b*tanh(x)**4)**(3/2)*tanh(x), x)
```

Maxima [F]

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx=\int \left(b\tanh\left(x\right)^4+a\right)^{\frac{3}{2}}\tanh\left(x\right)\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

Giac [F]

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx=\int \left(b\tanh\left(x\right)^4+a\right)^{\frac{3}{2}}\tanh\left(x\right)\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx=\int \tanh(x)\,\left(b\tanh(x)^4+a\right)^{3/2}\,dx$$

```
[In] int(tanh(x)*(a + b*tanh(x)^4)^(3/2),x)
[Out] int(tanh(x)*(a + b*tanh(x)^4)^(3/2), x)
```

3.260 $\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = -\frac{1}{2}\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\,\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

```
[Out] -1/2*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((
a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))*(a+b)^(1/2)-1/2*(a+b*tanh
(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3751, 1262, 749, 858, 223, 212, 739}

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = -\frac{1}{2}\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\,\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] -1/2*(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + (Sqrt[a
 + b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/2 - S
qrt[a + b*Tanh[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a-bx}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)\\
&\quad - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\\
&\quad - \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\\
&= -\frac{1}{2}\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\\
&\quad + \frac{1}{2}\sqrt{a+b}\,\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^4(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \frac{1}{2}\left(-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\sqrt{a+b}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\right)$$

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] (-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + Sqrt[a + b
]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a +
 b*Tanh[x]^4])/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^4}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)}{2}+\frac{b\,\text{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}+\frac{a\,\text{arctanh}\left(\right.}{}$
default	$-\frac{\sqrt{a+b\tanh(x)^4}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)}{2}+\frac{b\,\text{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}+\frac{a\,\text{arctanh}\left(\right.}{}$

```
[In] int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*tanh(x)^4)^(1/2)-1/2*b^(1/2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x
)^4)^(1/2))+1/2*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(
a+b*tanh(x)^4)^(1/2))+1/2*a/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+
b)^(1/2)/(a+b*tanh(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(69) = 138$.

Time = 0.44 (sec) , antiderivative size = 5136, normalized size of antiderivative = 57.71

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{a+b\tanh^4{(x)}}\tanh{(x)}\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(x)**4)*tanh(x), x)
```

Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{b\tanh(x)^4+a}\tanh(x)\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Giac [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{b\tanh(x)^4+a}\tanh(x)\,dx$$

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx=\int \tanh(x)\ \sqrt{b\tanh{(x)}^4+a}\,dx$$

```
[In] int(tanh(x)*(a + b*tanh(x)^4)^(1/2),x)
[Out] int(tanh(x)*(a + b*tanh(x)^4)^(1/2), x)
```

3.261 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3751, 1262, 739, 212}

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x}{(1-x^2)\sqrt{a+bx^4}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a + 
b])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}$	37
default	$\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}$	37

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)
^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(32) = 64.

Time = 0.41 (sec) , antiderivative size = 1286, normalized size of antiderivative = 32.15

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*
```

```
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
+ 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
osh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqr
t(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*
a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(
x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sq
rt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*
cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*
cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh
(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(
a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*
b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b
 + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*co
sh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(
x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3
+ 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 -
b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)))/(a +
b)]
```

Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\tanh^4(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{a + b\tanh^4{(x)}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh\left(x\right)}{\sqrt{b\tanh\left(x\right)^4+a}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh\left(x\right)}{\sqrt{b\tanh\left(x\right)^4+a}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh\left(x\right)^4+a}}\,dx$$

```
[In] int(tanh(x)/(a + b*tanh(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a + b*tanh(x)^4)^(1/2), x)
```

3.262 $\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

```
[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(3/2)+
1/2*(-a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3751, 1262, 755, 12, 739, 212}

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
3/2)) - (a - b*Tanh[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tanh[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{(1-x^2)\,(a+bx^4)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\,(a+bx^2)^{3/2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}+\frac{\text{Subst}\left(\int \frac{a}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}
\end{aligned}$$

$$= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}$$

$$= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)}$$

$$= \frac{\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
[Out] (ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)^(3/
2) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.82

method	result
derivativedivides	$\frac{b\left(\frac{\tanh(x)^3}{4a(a+b)}+\frac{\tanh(x)^2}{4a(a+b)}+\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}} - \frac{-\frac{\text{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}} - \frac{\sqrt{1-\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\,\text{El}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a-}}}{2(a+b)}$
default	$\frac{b\left(\frac{\tanh(x)^3}{4a(a+b)}+\frac{\tanh(x)^2}{4a(a+b)}+\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}} - \frac{-\frac{\text{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}} - \frac{\sqrt{1-\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\,\text{El}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a-}}}{2(a+b)}$

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*(1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2+1/4/a/(a+b)*tanh(x)-1/4/(a+b
)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b
*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(
1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*
a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))+b*(-
1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2-1/4/a/(a+b)*tanh(x)-1/4/(a+b)/b
)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*ta
nh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2
)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(
1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(63) = 126$.

Time = 0.48 (sec) , antiderivative size = 3914, normalized size of antiderivative = 52.89

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)^7 + (a^2 + a*b
)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cosh(x)^2 + a^2 -
a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*cosh(x))*sinh(x
)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4 + 30*(a^2 - a*b
)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*cosh(x)^5 + 10*(a
^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*b)*cosh
(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^4 + 9*(a^2 + a*
b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 + a*b)*cosh(x)^7
+ 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*
sinh(x))*sqrt(a + b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b +
b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 -
 b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*
sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 +
15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)
*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh
```

```
(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x
)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a
^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cos
h(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4)) - 2*sqrt(2)*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*si
nh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a
^2 - b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 -
b^2)*cosh(x)^3 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x
)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*
a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^4 + 3*a^3*b + 3*a^2*
b^2 + a*b^3)*cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh
(x)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^8 + 4*(a^4 + a^3*b - a^
2*b^2 - a*b^3)*cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x
)^5 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 2*(35*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 +
30*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 + 10*
(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*cosh(x))*sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 4
*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b
^2 - a*b^3)*cosh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*cosh(x)^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^5 + 3*(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh
(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)
^7 + (a^2 + a*b)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cos
h(x)^2 + a^2 - a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*
cosh(x))*sinh(x)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4
+ 30*(a^2 - a*b)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*co
sh(x)^5 + 10*(a^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(
a^2 - a*b)*cosh(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^
4 + 9*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 +
a*b)*cosh(x)^7 + 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 -
 a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*
(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(
x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)
^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 +
3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cos
```

```
h(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*
cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(
7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2
 + 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 -
 b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*si
nh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15
*(a^2 - b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(
x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)
*cosh(x)^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)
)) + sqrt(2)*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^
2 - b^2)*sinh(x)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cos
h(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^
3 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^4 + (a + b)
*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh
(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^7 + (a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^8 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3
)*cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x)^5 + 6*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + 30*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 + 10*(a^4 + a^3*b
- a^2*b^2 - a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x
))*sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b^2 - a*b^3)*c
osh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)
^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^5 + 3*(a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x))
]
```

Sympy [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx=\int \frac{\tanh\left(x\right)}{\left(a+b\tanh^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(3/2),x)
[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\left(a + b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^4 + a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\left(a + b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^4 + a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\left(a + b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh\left(x\right)^4 + a\right)^{3/2}}\,dx$$

```
[In] int(tanh(x)/(a + b*tanh(x)^4)^(3/2),x)
[Out] int(tanh(x)/(a + b*tanh(x)^4)^(3/2), x)
```

3.263 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}$$

```
[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(5/2)+
1/6*(-3*a^2+b*(5*a+2*b)*tanh(x)^2)/a^2/(a+b)^2/(a+b*tanh(x)^4)^(1/2)+1/6*(-
a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3751, 1262, 755, 837, 12, 739, 212}

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx=-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}+\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}$$

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
5/2)) - (a - b*Tanh[x]^2)/(6*a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) - (3*a^2 -
b*(5*a + 2*b)*Tanh[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{(1-x^2)\,(a+bx^4)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\,(a+bx^2)^{5/2}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-2b+2bx}{(1-x)(a+bx^2)^{3/2}}\,dx, x, \tanh^2(x)\right)}{6a(a+b)}\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3a^2b}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{6a^2b(a+b)^2}\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)^2}\\
&= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^2}\\
&= \frac{\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \frac{1}{6}\left(\frac{3\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{5/2}} + \frac{-a^2(4a+b)+3ab(2a+b)\tanh^2(x)-3a^2b\tanh^4(x)+b^2(5a+2b)\tanh^6(x)}{a^2(a+b)^2\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]

[Out] ((3*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b)^(5/2) + (-(a^2*(4*a + b)) + 3*a*b*(2*a + b)*Tanh[x]^2 - 3*a^2*b*Tanh[x]^4 + b^2*(5*a + 2*b)*Tanh[x]^6)/(a^2*(a + b)^2*(a + b*Tanh[x]^4)^(3/2)))/6

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 637, normalized size of antiderivative = 5.40

method	result
derivativedivides	$-\frac{\left(-\frac{\tanh(x)^3}{6a(a+b)b}-\frac{\tanh(x)^2}{6a(a+b)b}-\frac{\tanh(x)}{6a(a+b)b}+\frac{1}{6(a+b)b^2}\right)\sqrt{a+b\tanh(x)^4}}{2\left(\tanh(x)^4+\frac{a}{b}\right)^2}+\frac{b\left(\frac{(3a+b)\tanh(x)^3}{8a^2(a+b)^2}+\frac{(5a+2b)\tanh(x)^2}{12a^2(a+b)^2}+\frac{(11a+5b)\tanh(}{24a^2(a+b)^2}\right.}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}$
default	$-\frac{\left(-\frac{\tanh(x)^3}{6a(a+b)b}-\frac{\tanh(x)^2}{6a(a+b)b}-\frac{\tanh(x)}{6a(a+b)b}+\frac{1}{6(a+b)b^2}\right)\sqrt{a+b\tanh(x)^4}}{2\left(\tanh(x)^4+\frac{a}{b}\right)^2}+\frac{b\left(\frac{(3a+b)\tanh(x)^3}{8a^2(a+b)^2}+\frac{(5a+2b)\tanh(x)^2}{12a^2(a+b)^2}+\frac{(11a+5b)\tanh(}{24a^2(a+b)^2}\right.}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}$

[In] int(tanh(x)/(a+b*tanh(x)^4)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(-1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2-1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/b)^2+b*(1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(x)^2+1/24/a^2*(11*a+5*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I

```
/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*
(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)))-1/2*(1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)
^2+1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/
b)^2+b*(-1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(
x)^2-1/24/a^2*(11*a+5*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)
^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^
(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(
1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^
(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8210 vs. $2(102) = 204$.

Time = 2.04 (sec) , antiderivative size = 16463, normalized size of antiderivative = 139.52

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh{\left(x \right)}}{\left(a+b\tanh^{4}{\left(x \right)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(5/2),x)
[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^{4}+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^4+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="giac")
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh\left(x\right)^4+a\right)^{5/2}}\,dx$$

```
[In] int(tanh(x)/(a + b*tanh(x)^4)^(5/2),x)
[Out] int(tanh(x)/(a + b*tanh(x)^4)^(5/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```